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Systematic Geometric Error Modeling for Workspace Volumetric Calibration of a 5-axis Turbine Blade Grinding Machine

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Abstract

A systematic geometric model has been presented for calibration of a newly designed 5-axis turbine blade grinding machine. This machine is designed to serve a specific purpose to attain high accuracy and high efficiency grinding of turbine blades by eliminating the hand grinding process. Although its topology is RPPPR (P: prismatic; R: rotary), its design is quite distinct from the competitive machine tools. As error quantification is the only way to investigate, maintain and improve its accuracy, calibration is recommended for its performance assessment and acceptance testing. Systematic geometric error modeling technique is implemented and 52 position dependent and position independent errors are identified while considering the machine as five rigid bodies by eliminating the set-up errors of workpiece and cutting tool. 39 of them are found to have influential errors and are accommodated for finding the resultant effect between the cutting tool and the workpiece in workspace volume. Rigid body kinematics techniques and homogenous transformation matrices are used for error synthesis.

Keywords: 5-axis machine tools; calibration; modeling; geometric errors; kinematics; homogenous transformation matrices

1. Introduction

Advent of 5-axis computer numerical control (CNC) machine tools made a great breakthrough in manufacturing industry due to its versatility and degree of freedom to machine complex parts with more ease. It reduces the processing time by up to 90% in comparison with conventional machine tools[1]. In addition, it requires less operator efforts and the use of CAD/CAM software improves its efficiency. Y. Takeuchi, et al.[2], H. Trankle[3], and E. E. Sprow[4] cast light on and explained some more advantages and features of 5-axis machine tools, which made it capable of providing a comprehensive solution for complex parts like turbine blade manufacturing and its meterage. 5-axis turbine blade grinding machine has the ability to machine and grind sculptured surfaces with high accuracy. It can produce the actual blade, with its entire complex freeform surfaces, filleting and rounding, the blade footing, and profile with high surface quality without waviness by eliminating the hand grinding process. To generate high accuracy profiles and surfaces completely depends on the accuracy of the machine tools, whereas research on CNC machine tools exhibits that accuracy deteriorates substantially even in an ideal environment[5].

A study by the Hewlett-Packard Company revealed that 88% of 57 purchased production machines were out of specifications upon installation, in which the foundation, mounting, alignment and temperature conditions on the shop floor are all critical to machine accuracy[6]. The correctness of a machine tool is one of the most important prerequisites for assurance of the product's quality whereas the machine tool accuracy must be consistent with the intended users or usage. Calibration is the only comprehensive indicator and is considered as one of the most important indices which depicts a detailed picture regarding accuracy of machine tools by assessing the quality and performance capability[7]. Furthermore it is inevitable tool for acceptance testing, periodic calibration, error characterization and its compensation[8-12].

In previous research some common calibration techniques for machine tools performance evaluation were reported[13] which can be categorized into three main methodologies. The first method is parametric method and quite popular for quantifying various errors terms independently and is mostly used for thoroughly checking the geometric errors of machine tool. The second method is volumetric calibration method and is able to quantify the error between actual and

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commanded motion at a specific desired point in workspace volume of a machine tool through kinematic reference standards[14-16] or artifacts[17-18]. As this method is apply to checking the combined effect of all errors, it is popular for acceptance testing. The third method is hybrid calibration method which bridges the gap between two methodologies. The volumetric workspace error at any particular point within the work space may then be calculated through populating parametric errors by modeling techniques[10-11,13,19-20] and is known as error synthesis calibration method whereas its inverse condition is identified as error separation method. The hybrid calibration method is the only well-known method which is reliable, authenticated and provides realistic information about elemental accuracy of machine tools and volumetric accuracy at a specific point in workspace volume as well. This method is the most popular and appreciated by the machine tool builders and users for error characterization and its compensation in 5-axis machine tools, although the determination of errors takes a longer time which is a drawback, the results are quite reliable. The geometric error identification and error modeling are dependent on a machine and vary from machine to machine or topology of machine. So this type of modeling is called a type dependent modeling of machine tools, which completely depends on the errors, machine type and machine topology. Error identification and their modeling techniques are more essential aspects of this calibration, which will be discussed in detail in this article.

The basic standard as a guideline for measuring the errors in machine tools are documented in International Organization of Standardization (ISO) 230 series[21-25] and American Society of Mechanical Engineers (ASME) B5.54. 1992[26] and some methodologies, techniques and information about instrumentation are already available and are in practice. These basic standards can be applied to any machine tool but there are some specific standards for the calibration of machines including the tolerance for deviations of errors from the measured values. ISO series 10791[27-32] and ISO 13041 are dedicated for geometric, interpolation and contouring performance tests of different types of machining centers. Unfortunately at present there is no specific standard, which covers the testing and calibration of 5-axis machine tools. On the other hand, no standard and specific documents are available as a guide line for modeling of 5-axis or multi-axis machine tools, so 5-axis machine tools still lack a validated and comprehensive documentation and methodology.

2. Literature Survey on Modeling Techniques

Substantial work has been done on modeling of 3-axis machine tools but only a little work is available on 5-axis machine tools. Investigators have addressed the machine tool error identification problem from different perspectives by using various modeling techniques. Although for a long time they have been trying to find out the resultant error of individual components in relation to tool workpiece point deviation, mostly two main techniques are well-known and widely used i.e. Denavit and Hartenberg (D-H) method and rigid body kinematics method with different perspectives. Error modeling technique provides a systematic and suitable way to establish the error model. The methods experienced by the researchers are error matrix method, second order method, neural network method, variational method, rigid body kinematics and D-H method. The development in the field of modeling is hereby presented through a timeline literature surveys with the methodologies used by the key researchers and gives an overview of how the research revolutionizes with passage of time.

2.1. Modeling techniques up to 1970s

The pioneer work was presented by J. Denavit, et al.[33] in form of ideal kinematic model based on homogenous transformation matrices (HTMs) later modified by P. P. Paul, considering a reference frame[34] and laid the analytical foundation of a generalized error model. In the 1960s, D. L. Leete[35] gave a new direction followed by D. French, et al.[36] by developing the trigonometric relationship for geometric errors' modeling. In 1973, W. J. Love, et al.[20] analyzed the volumetric errors by determining the combined effects through trigonometric technique. In 1977, R. Schultschik[19] introduced the close vector chain technique. In the same year, R. Hocken, et al.[11] developed a matrix translation method and presented a calibration technique. In 1979, R. Schultschik[37] further analyzed the machine tool error through volumetric error vector technique under load conditions.

2.2. Modeling techniques in 1980s

In 1981, the "error matrix" method was reported by P. Dufour, et al.[38] whereas in 1982 V. T. Portman[39] used rigid body kinematics for the geometric error of a mechanism. W. K. Veitschnegger, et al.[40] used kinematics for robotic manipulators by first and second order errors. The ideal and actual spatial relationship between the machine's components was described by homogeneous transform[33-34] whereas dominating error components on overall geometric error of the machine was outlined[41]. In 1985, A. Donmez[42] implemented the HTMs method to obtain the positioning errors and found the thermal loading errors in the same order of magnitude as the geometric errors. In 1986, M. A. Donmez, et al.[12,43] proposed a general methodology of error modeling and compensation. The method was implemented in several steps, and a kinematic error modeling method[12] for geometric and thermally induced errors was further proposed. In the same year, P. M. Ferreira, et al.[44-45] proposed an analytical quadratic model for the prediction of geometric errors. Unlike the previously proposed models, this method allowed for the variation of errors along the machine's joints, and the model is not true in most cases due to its limitations.

In 1986, Z. J. Han, et al.[46] modeled the positioning accuracy by using Fourier transforms. In 1987, T. Sata, et al.[47] assumed a quadratic relationship and built co-relational models from observations and used finite element model to predict structural changes with temperature. Later based on the Hocken method, T. Sata et al. presented a positioning error model by supposing a second-order relationship. In the same year, K. F. Eman, et al.[41] developed error model which accounted for errors due to inaccuracies in the geometry and mutual relationships and relative motion of the machine structural elements. In 1988, G. Zhang, et al.[13] proposed a displacement method to determine the machine's geometric errors. Based on the assumption of rigid bodies, M. Anjanappa, et al.[48] developed kinematic model to synthesize geometric errors. In 1990, A. K. Elshennaway, et al.[49] also used rigid body kinematics to develop a model for geometric positioning error. J. Jedrzejewski, et al.[50] used numerical methods to optimize the design of a machine tool based on finite element method (FEM). K. F. Ehmann[51] developed a kinematic modeling procedure for the volumetric errors. These modeling procedures, however, did not include errors associated with rotary joints and could not calculate orientation errors.

2.3. Modeling techniques in 1990s

In 1991, K. Kim et al. [52] extended R. Schultschik's work with assumption of small angle approximation. In 1992, J. A. Soons, et al.[53] presented a general error model based on tool and workpiece kinematic chain by direct kinematics and piecewise polynomials, however parameters and procedure of deriving error model were not well delineated. J. S. Chen, et al.[54] addressed the non-rigid body effects associated with the volumetric accuracy of a horizontal spindle machine tool. In 1993, J. Ni, et al.[55] formed a specific model for the hybrid on-off-line measurement system configurations for coordinate measuring machine (CMM). In 1993, P. D. Lin, et al.[56] proposed a generalized methodology for evaluating the position and orientation errors based on a kinematic modeling procedure by using the D-H conventions for robotics. Their work provided a basis for the automatic derivation of error synthesis models although the approach was very complicated and was quite difficult to decode the error model obtained by the direct analysis approach for physical meanings. In the same year, V. S. B. Kiridena, et al.[57] developed a kinematic model to compensate for both the position and orientation errors of a 5-axis machining center using the same convention. However, their model contained only five parametric errors (one positioning error for each axis).

In 1994, V. S. B. Kiridena, et al.[58] developed an *n*th order quasistatic error model which is a function of error components of each link. The main problem of their model was the estimation of the parameters. P. M. Ferreira, et al.[59] combined the parameters for volumetric errors by using rigid body kinematics. They expressed functions of 15 error measurements made at nine points on the edges of a cubical workspace. J. Mou[60] developed a model for error estimation based on well-known theories and assumed that the error terms for the link are position independent whereas the joint are position dependent. Applied his model to 4-axis machine and estimated the error coefficients through neural network, C. H. Lo, et al.[61] developed a method for kinematic model synthesis that allowed for measurement of the quasi-static errors of a machine tool at arbitrary locations in the work volume. S. Wang et al.[62] developed a compensation scheme model consisting of dimensional shape function to predict the error at tool tip, and implemented the model on 3-axis machine.

S. Wang, el al.[63] provided an automated tool for the evaluation of error propagation in machine tools considering the eleven families of error terms included in the model. The error model was used to predict the 3-D error distributions of a 3-axis machining center. V. S. B. Kiridena, et al.[64] predicted the quasi-static errors based on Taylor series' expansion for compensation. J. H. Cho, et al.[65] considered a closed loop volumetric accuracy by considering bi-cubic Bezier surface on a 3-axis machine. V. B. Kreng, et al.[66] developed a periodically updated error model to express the observed error in volumetric workspace by using a specially designed pallet. In 1995, A. Srivastava, et al.[67] used a method based on direct consideration of the shape and joint transformations by focusing on a specific machine type. S. H. Yang dedicated one chapter of his dissertation to the formulation of a generalized 5D error synthesis model[68] and considered only 27 geometric error components which were incomplete. Unlike the commonly acknowledged 21 parametric errors in 3-axis machines, more constant error components should be included in a 5-axis model, such as the squarness, parallelism and constant offset of a rotary axis, etc. Therefore, the main challenge in this work was the development of a robust 5-axis error model, which must be generic enough to handle most of the common 5-axis machine types.

In 1996, S. H. Yang, et al.[69] proposed a polynomial form of the volumetric error model to combine both the geometric and thermal errors. In 1997, R. M. D. Mahbubur, et al.[70] improved the positioning accuracy in 5-axis milling by identifying the angular errors through modeling. In the same year, X. B. Chen, et al.[71] developed a new error identification and accuracy improvement model by using a meshing concept to subdivide the workspace into smaller 3D elements. H. Tajbakhsh, et al.[72] extended the model of Kirienda and Ferreira by minimizing the L_∞ norm. This was obtained by using linear Chebyshev polynomials. A. J. Patel, et al.[73] presented an error model based on differentiation of the direct kinematic equations of a

Stewart platform and gave a sensitivity analysis which could be use for tolerance allocation during manufacturing. S. H. Suh, et al.[74] used the D-H representation in 1998 to develop a versatile path planning method by which 5-axis machining could be done. In 1999, K. G. Ahn, et al.[75] used a general rigid body kinematic model to find the total volumetric error. In 2000, A. C. Okafor, et al.[76] presented derivation of a general volumetric error model, which synthesized both geometric and thermal errors of a vertical milling machine by using HTMs and considering 21 geometric errors components.

2.4. Modeling techniques from 2001 to up-to-date

In 2001, G. H. J. Florussen, et al.[77] identified the geometric errors in 5-axis machining center based on double ball bar measurement. He measured the translational axis but ignored the rotational axis to simplify the model and finally 21 parameters were reduced to 12. In 2002, R. W. Bagshaw, et al.[78] mentioned that the total errors in the workpiece were due to not only the machine inaccuracy, but also the fixture and programming errors. He developed an expert system for diagnosis and removal of the errors. In the same year, Y. Abbaszadeh-Mir, et al.[79] categorized the rigid body geometric errors into position dependent and position independent groups, and identified 20 potential positions independent errors in a 5-axis machine tool in which only 8 were found linearly independent.

In 2003, Y. Lin, et al.[80] presented the matrix summation approach for modeling the geometric errors of 5-axis machine tools. This approach breaks down the kinematic equation into six components including the ideal tool tip position under nominal axis motions, and the contribution of the error motions of each axis. A generalized geometric error model associated with 5-axis machine was developed by B. K. Jha, et al.[81] with its experimental verification. They presented a scheme to compensate the geometric errors and analyzed the effect on the accuracy of a cam profile. M. Tsutsumi, et al.[82] identified the geometric errors in the two rotary axis of a 5-axis machine tool and considered the angular and positional deviations. Categorically eight deviations were identified including three angular errors, three positional errors of A-axis relative to the machine coordinate system (MCS). The rest of two errors were angular errors and distance errors between the two rotary axes. In the same year, Y. M. Cheng, et al.[83] studied contour errors of a complete CNC machine system which covered all groups of functions. J. W. Fan, et al.[84] proposed the kinematics of multi-body system (MBS) by adding movement error and positioning error items, and a universal way of how to make a kinematic model of numerical control (NC) machine tools was presented.

In 2002, E. L. J. Bohez[85] presented a valuable work on 5-axis machine tools, analyzed the systematic errors in the tool path generation of 5-axis machine tools and suggested that errors could be measured by direct method. In 2007, E. L. J. Bohez, et al.[86] researched on 5-axis milling machine and presented a new method to identify and compensate the systematic errors in a multi-axis machine tool. The mathematical model is based on a first order rigid body model of the machine tool. The total number of identified errors was reduced from 39 to 32.

The literature survey revealed that most researches are limited to 3-axis machine tools, especially on CMMs and very little work is available on 5-axis machine tools. Geometric error characterization and mapping is one of the most important problems to find a universal kinematic modeling. Most studies on 5-axis machines have commonly identified the need to develop a model to analyze the kinematic structure of the machine tool. Several approaches are proposed for this purpose and some of them came from robotics. However, most of the earlier approaches adopted analytic geometry[20] vector representation[11,19], error matrices[10, 87] and screw theory[34]. The most recent work is related to rigid body kinematics in connection with HTMs[45].

In geometric error modeling of 5-axis machine tools, most researchers did not consider all the error parameters due to lack of error identification, measuring instrument, meterage methods, or they tried to reduce the error parameters without any reasonable justification. Mostly, they reduced the parameters to solve them easily by avoiding numerical problems, and similarly they selected the degree and type of polynomial. Modeling and calibration were very much neglected in the past as most researches focused on the compensation. Due to lack of information about machine errors, most models were not formulated in a systematic way. So, the type-dependent model was formulated which is still in use and is only suitable for a specific machine or specific topology of machine. Error identification and assumption is based on the knowledge of the researcher, whereas it is a specialized field which needs dedicated work.

3. Systematic Geometric Errors and Their Sources in Machine Tools

Errors in machine tools can be classified into two main categories which are systematic errors and random errors. Systematic errors are consistent in nature and repetitive, recurring consistently every time the measurement is made, but varying very slowly with passage of time due to degradation of a machine system. A. H. Slocum defined these errors as repeatable and random errors[88] and explained that in repeatable errors numerical value and sign were constant for each manipulator configuration. Assembly error was the best example of this. In random errors numerical value or sign changed unpredictably and backlash was an example of random error. P. Anderson[89] explained and categorized the machine geometry, stiffness, load, and positioning, internal and external source of thermal effects, vibrations into systematic and random errors. An important source of legitimate systematic errors stems from the imprecise or improper calibration which depicts the accuracy of machine system and on the basis of that compensation is made for maintaining or improving the accuracy. Systematic errors are mainly due to geometrical irregularities in machine elements. However, a machine itself may introduce built-in errors resulting from incorrect design, fabrication or maintenance. Such errors may be caused by false element, like incorrect scale graduations, defective gearing and transmission system, linkage of wrong proportions. Certain electronic circuits and system faults also fall under the headings and can be corrected or identified through proper calibration. Machine calibration has become an important tool for assessing the machine accuracy and thus allowing the comparison of machine tool performance against the standard specifications. The calibration data can be used for error compensation purposes and also as a diagnostic aid for machine maintenance.

Systematic geometric errors are a combination of those errors which affect the geometry of the machine's components present in the machine's structural loop. American National Standards Institute (ANSI) and ASME standards define the structural loop as an assembly of mechanical components which maintain a relative position between specified objects. In 5-axis machine tools the structural loop consists of spindle, rotary table, shaft, bearing, housing, guideways, main body or base, drives, cutting tools and work holding fixtures. Change in geometry due to change in these structural elements generates errors in geometry and causes erroneous motion (kinematic error), which definitely affects the position and orientation of the end effector. The magnitude of these errors depends upon the sensitivity of the machine's structural loop on various error sources. Hence the error sources which affect the accuracy of the relative end effector's position and orientation[90-94] are kinematic errors, thermo-mechanical errors, loads, dynamic forces and motion control, and control software.

These error sources basically change dimension and geometry of the machine components and can therefore be considered as systematic geometric error components. 75% of initial errors of a new machine tool arise as a result of manufacture and assembly[95]. The systematic geometric error components of the machine tool directly affect the tool tip position because of dimensional and form errors of the kinematic linkage and linear/angular misalignment between them. These errors primarily comes from manufacturing or assembly defects, misalignment of the machine's axis, position and straightness error of the each axis, which are mainly because of guide ways misalignment and flatness errors, link length error, angular error including roll, pitch and yaw, straightness error, squarness error, parallelism error, perpendicularity error and zero position errors (offset error). These errors increase due to the gradual machine wear, static deflection of machine components, and deadweight of the moving machine slides, misalignment due to assembly, installation and soft machine foundation.

4. Topology of 5-axis Machine Tools and 5-axis Turbine Blade Grinding Machine

As per robotics theoretical concepts, 5-axis machine tools can be considered as robots having five degrees of freedom and composed of links and joints. The joints, mostly consist of prismatic (P) and rotary (R) or revolute joints, can be grouped into different arrangements as mentioned by E. L. J. Bohez[96], who explained that by permutations, theoretically 720 possible 5-axis machines can be made with one tool carrying axis. Practically, most 5-axis machine tools are composed of three prismatic and two rotary joints, which have the ability to simultaneously position and orientate the cutting tool in a coordinate system defined in workspace. Five degrees of freedom facilitates positioning the cutting tool exactly perpendicular to the workpiece in the workspace volume. Joint's well-known configurations are as follows.

(1) PPPRR: Three sequential prismatic joints and then two rotary joints configuration, in which cutting tool is supported by a double pan head, i.e. the tool has two rotational joints, one for rotation and the other for tilting. The feed motion may be located either at the tool or at the table.

(2) RPPPR: One rotary joint then three prismatic and subsequently one rotary joint in which the workpiece is supported by a rotary joint and the tool is supported by a single pan head i.e. the tool has one rotational degree of freedom. The other rotational joint is located at the rotational table.

(3) RRPPP: Two rotary joints and then three prismatic joints in which the workpiece is supported by double rotary joints constitute a turn table. As the work table has double rotary joints, their configuration can be formulated in two ways, firstly a rotational table on the tilting one and secondly a tilting table on the rotational one.

The 5-axis turbine blade grinding machine consists of three prismatic joints and two rotary joints with configuration of RPPPR. It has three prismatic joints, which represent the fixed Cartesian frame and their construction provide a fewer engineering problems by dictating/defining the machine structure in the similar frame and MCS. These three prismatic joints hence represent X, Y, Z axes and are called as X, Y, Z prismatic joints which generate liner movement in the similar axis direction. Two rotary joints are "A" and "B" rotary joints, which are designated as per EIA-267-B and ISO-841[97-98] and generate rotary motion around X axis and Y axis respectively. Machine structure of the turbine blade grinding machine is explained through Fig.1 whereas its kinematic chain diagram is elaborated in Fig.2.

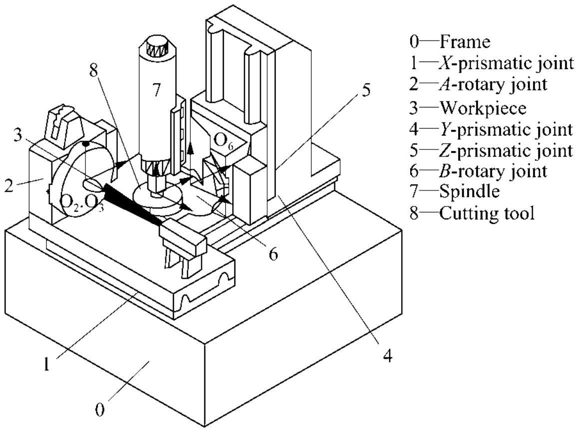

Fig.1 5-axis turbine blade grinding machine.

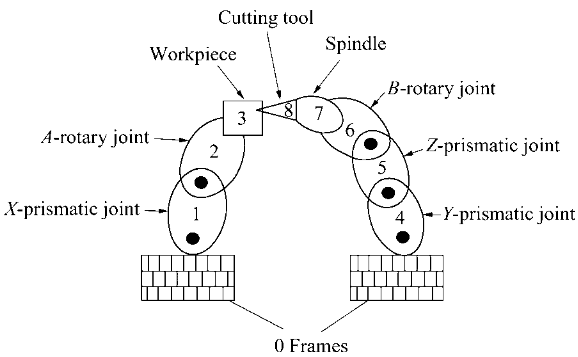

Fig.2 Kinematic chain diagram of 5-axis turbine blade grinding machine.

5. Geometric Error Identification in 5-axis Machine Tools and Turbine Blade Grinding Machine

The machine tools are composed of links and joints, which have unavoidable errors during their position and movements. These errors can be categorized into position dependent and position independent geometric errors[79]. Position independent errors are named as link errors such as joint misalignment, angular offsets and rotary axis separation errors, which describe the relative location of the machine's in successive rotary and prismatic joints. Position dependent errors are the six degrees of freedom errors of the joints during orientation and motions. In mechanics a rigid body is an idealization of a solid body having six degrees of freedom[99] while position or motion in three dimensional space has linear and angular error components. In a prismatic joint, the linear error components are three translations along the axis as one positioning error, and two straightness errors i.e. horizontal and vertical straightness errors, whereas angular error components are one roll error and two tilt errors called pitch and yaw as mentioned in Fig.3. In rotary joints similar six error components, two radial errors and one axial error motion are linear components, whereas angular components are one angular position error and two tilt error motion as shown in Fig.4 according to the assumption of rigid body behavior. These errors are function of the nominal movement only, and don't depend on the location of the other joints.

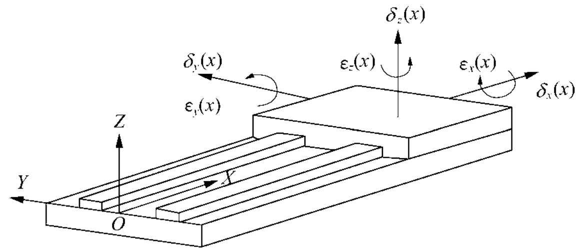

Fig.3 Six parametric errors of prismatic joint.

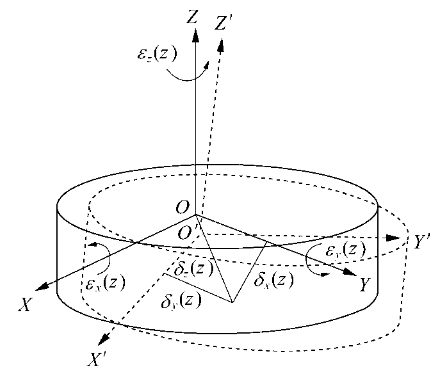

Fig.4 Six parametric errors in a rotary joint.

Position independent errors are also called location errors of a joint defined as an error from the nominal position and orientation of this joint in the machine coordinate system. A prismatic joint is defined by a vector with a zero position on the vector. Therefore, there are just three location errors for a linear movement, two orientation errors (*AOZ* and *BOZ*) and the zero position error (*ZOZ*) as explained through Fig.5. Similarly the location of a rotary joint with regard to the nominal position is expressed by five location errors which includes two position errors (*XOC* and *YOC*), two orientation errors (*AOC* and *BOC*), and in analogy to the zero position of linear axis—the zero angular position (*COC*) as explained through Fig.6. Three are location errors which define the machine coordinate system and include the machine axis, so in this way a 5-axis machine tool with two rotary and three prismatic joints has combined 52 potential geometric errors components while eliminating the workpiece and cutting tool setting errors.

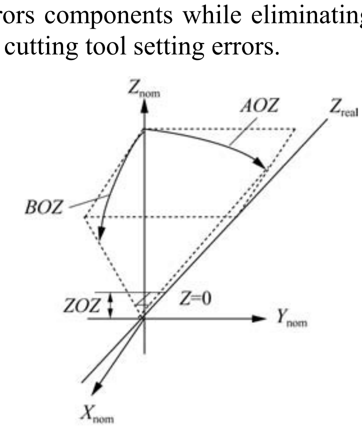

Fig.5 Location errors of prismatic joint.

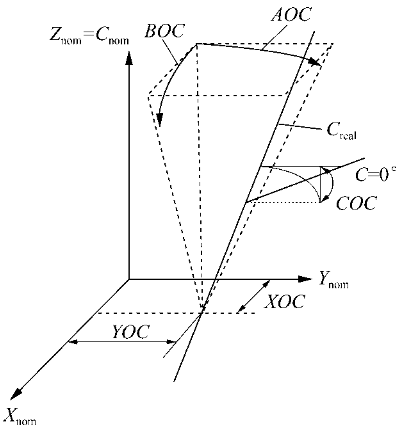

Fig.6 Location errors of rotary joint "C".

Error in the turbine blade grinding machine is identified as discussed under section of a geometric error identification system. However, location errors of the coordinate system are taken as negligible, as the machine has one position that is defined as the reference position, and one coordinate system is fixed to the machine frame and to each body in the kinematic chain. As the turbine blade grinding machine has five joints and each joint has six degrees of freedom errors, three linear (δ_x, δ_y and δ_z) and three angular (ε_x, ε_y and ε_z), the number of rigid body errors or position dependent errors of the joints is 6×5, i.e., 30. All joints assign a coordinate system whose X-axis aligns with the reference coordinate system so the ideal X-joint has no angular error or squarness error. The plane through real X-axis and real Y-axis is selected as the reference plane, so the real Y joint can have only one squarness error (ε_{xy}), whereas the real Z joint has two squarness errors i.e. ε_{zx} and ε_{yz}. A rotary joint must be parallel to the X-axis of X-joint so it might have two squarness or parallelism errors (ε_{xya}, ε_{xza}). Similarly, B rotary joint has two squarness or parallelism errors (ε_{xyb} and ε_{yzb}). Moreover, two location errors between the B-joint and A-joint which are offset errors, in case their axis does not coincide with the reference frame coordinates, and hence generate the location errors. So in this way the turbine blade grinding machine has 39 parametric geometric errors. Moreover during movement due to systematic geometric errors the machine generates small translation and rotation errors.

6. Workspace Volumetric Error Modeling Methodology

6.1. Workspace volumetric error concept

Before discussing the modeling methodology for 5-axis turbine blade grinding machine it is essential to clarify the volumetric workspace and its error concept. The volumetric workspace in 5-axis machine tool is the workspace in which the tool and workpiece reference point can meet, whereas the workspace volumetric error means the coincidence error between the tool and workpiece reference points or error of relative position between the tool and the workpiece which is affected by the systematic and random geometric error sources. It can be explained through Fig.7 in which OO' is expressed as the ideal position of the cutter contact point in relation to the workpiece reference point. Because of various error sources discussed already, the cutting tool structural loop (A', B' and C' structural element) of machine and workpiece structural loop (A, B and C structural element) of machine are distorted and their position is shifted as mentioned through D and D'. As a result the volumetric error is introduced in the work space volume that can be estimated by measuring DD'.

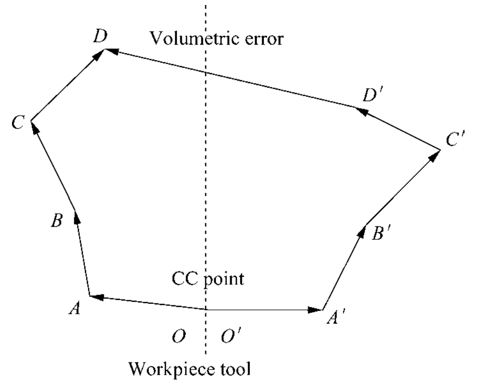

Fig.7 Volumetric error concept in 3D workspace.

6.2. Volumetric error modeling methodology

In methodology of volumetric error modeling, the error vector is formulated through the closed error vector chain in the machine tool structure which permits to calculate the errors at each and every desired point of the workspace volume on the basis of direct measurement of parametric or elemental errors. Link and joint errors can be represented through four vectors in structural loop for a joint in relation to its adjacent joint. Resultant vector can be taken from systematic arrangement and is represented in form of HTMs by plugging the errors. For the modeling arrangement, the structural loop of the machine is divided into two loops by considering the body of the machine as a reference. One structural loop is called "M" loop from the reference frame to the cutting tool, whereas the second one is named as "N" loop which is sequentially arranged from reference frame to the workpiece. Each joint of the body and main body of the machine is appointed a coordinate frame whose coordinate directions are as per the direction coordinate frame of the MCS which is further divided into four elemental frames called position, position error, motion and motion error coordinate frames. From these frames the error vectors i.e. position vector ${}_j^i\boldsymbol{T}_{\mathrm{p}}$, error position vector ${}_j^i\boldsymbol{T}_{\mathrm{pe}}$, motion vector ${}_j^i\boldsymbol{T}_{\mathrm{s}}$ and erroneous motion vector ${}_j^i\boldsymbol{T}_{\mathrm{se}}$ can be considered as mentioned in Fig.8 whereas the left side subscript and superscripts "j" and "i" denote the adjacent higher and lower joints respectively in the structural loop. "$\boldsymbol{T}$" describes the transformation in adjacent joints as explained in Fig.8 through joints "B_i" and "B_j". Due to link error and motion errors these coordinate frames have small translational and rotational errors during their position and translation, which can be represented by homogenous transformation matrices as stated. Homogenous transformation matrix rank is 4×4, representing the pose (position and orientation) of any one frame in which top left 3×3 of each matrix represents the directional cosines of the body, and top right 3×1 matrix represents the origin position of the body from the reference coordinate system, whereas left down 1×3 vector is called perspective vector and similarly at right down 1×1 row vector is placed as a scaling factor. As per rigid body concept, position and movement with their errors from one joint to its adjacent joint can be described through multiplication of the HTMs of relevant joints.

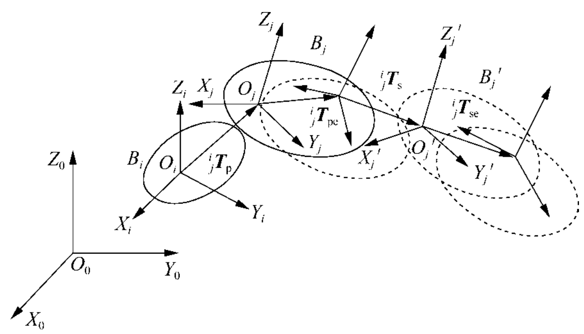

Fig.8 Error vector representation in adjacent joints B_i and B_j.

Position (p) and motion (s) characteristic matrix for prismatic and rotary joint can be selected from matrices 1-6 as per required motion or position suitability.

$$ {}^i_j\boldsymbol{T}\,(\delta_{xp}) = \begin{bmatrix} 1 & 0 & 0 & \delta_{xp} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

$$ {}^i_j\boldsymbol{T}(\delta_{yp}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \delta_{yp} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

$$ {}^i_j\boldsymbol{T}(\delta_{zp}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta_{zp} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

$$ {}^i_j\boldsymbol{T}\,(\varepsilon_{xp}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varepsilon_{xp} & -\sin\varepsilon_{xp} & 0 \\ 0 & \sin\varepsilon_{xp} & \cos\varepsilon_{xp} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

$$ {}^i_j\boldsymbol{T}\,(\varepsilon_{yp}) = \begin{bmatrix} \cos\varepsilon_{yp} & 0 & \sin\varepsilon_{yp} & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\varepsilon_{yp} & 0 & \cos\varepsilon_{yp} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

$$ {}^i_j\boldsymbol{T}(\varepsilon_{zp}) = \begin{bmatrix} \cos\varepsilon_{zp} & -\sin\varepsilon_{zp} & 0 & 0 \\ \sin\varepsilon_{zp} & \cos\varepsilon_{zp} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

Error matrix "$\boldsymbol{T}_e$" for position (pe) and motion (se) in prismatic and rotary joints can be resolved by multiplying the matrix as per Eqs.(1)-(6), and error matrix can be attained by taking small angle approximations shown in Eq.(7).

$$ {}^i_j\boldsymbol{T}_e = \begin{bmatrix} 1 & -\varepsilon_{zp} & \varepsilon_{yp} & \delta_{xp} \\ \varepsilon_{zp} & 1 & -\varepsilon_{xp} & \delta_{yp} \\ -\varepsilon_{yp} & \varepsilon_{xp} & 1 & \delta_{zp} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7} $$

Error vector between two adjacent joints can be calculated by the position, position error, motion and motion error matrices as mentioned in Eq.(8).

$$ {}^i_j\boldsymbol{T}_e = {}^i_j\boldsymbol{T}_p \cdot {}^i_j\boldsymbol{T}_{pe} \cdot {}^i_j\boldsymbol{T}_s \cdot {}^i_j\boldsymbol{T}_{se} \tag{8} $$

The turbine blade grinding machine has five degrees of freedom, and with RPPPR configuration each joint with respect to its adjacent lower joint was calculated as Eq.(8) and the structural loop division i.e. M, N. In "N" structural loop, reference point to the workpiece, the individual joint error vector transformation ${}^0_1\boldsymbol{T}$, ${}^1_2\boldsymbol{T}$ and ${}^2_3\boldsymbol{T}$ are for X-prismatic, A-rotary joints and for workpiece respectively. Whereas ${}^2_3\boldsymbol{T}$ is a workpiece coordinate with respect to A-axis and transformation of link and joint is mentioned in Eqs.(9)-(11).

$$ {}^0_1\boldsymbol{T} = {}^0_1\boldsymbol{T}_p \cdot {}^0_1\boldsymbol{T}_{pe} \cdot {}^0_1\boldsymbol{T}_s \cdot {}^0_1\boldsymbol{T}_{se} = \boldsymbol{I}_{4\times4} \cdot \boldsymbol{I}_{4\times4} \cdot $$

$$ \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\varepsilon_z(x) & \varepsilon_y(x) & \delta_x(x) \\ \varepsilon_z(x) & 1 & -\varepsilon_x(x) & \delta_y(x) \\ -\varepsilon_y(x) & \varepsilon_x(x) & 1 & \delta_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9} $$

$$ {}^1_2\boldsymbol{T} = {}^1_2\boldsymbol{T}_p \cdot {}^1_2\boldsymbol{T}_{pe} \cdot {}^1_2\boldsymbol{T}_s \cdot {}^1_2\boldsymbol{T}_{se} = \boldsymbol{I}_{4\times4} \cdot \begin{bmatrix} 1 & -\varepsilon_{xya} & \varepsilon_{xza} & 0 \\ \varepsilon_{xya} & 1 & 0 & 0 \\ -\varepsilon_{xza} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot $$

$$ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos A & -\sin A & 0 \\ 0 & \sin A & \cos A & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\varepsilon_z(a) & \varepsilon_y(a) & \delta_x(a) \\ \varepsilon_z(a) & 1 & -\varepsilon_x(a) & \delta_y(a) \\ -\varepsilon_y(a) & \varepsilon_x(a) & 1 & \delta_z(a) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10} $$

$$ {}_3^2\boldsymbol{T} = \begin{bmatrix} p_{wx} & p_{wy} & p_{wz} & 1 \end{bmatrix} \tag{11} $$

where p_{wx}, p_{wy} and p_{wz} are the workpiece point position coordinates in X, Y and Z directions respectively.

Similarly in "M" structural loop reference point to a cutting tool, the individual joint error vectors transformation ${}_4^0\boldsymbol{T}$, ${}_5^4\boldsymbol{T}$, ${}_6^5\boldsymbol{T}$, ${}_7^6\boldsymbol{T}$, ${}_8^7\boldsymbol{T}$ are for Y-prismatic joint, Z-prismatic joint, B-rotary joint, spindle and cutting tool respectively. The spindle is fixed with B-axis with no errors and the cutting tool has coordinate with reference to B-axis as mentioned in Eqs.(12)-(16).

$$ {}_4^0\boldsymbol{T} = {}_4^0\boldsymbol{T}_{\mathrm{p}} \cdot {}_4^0\boldsymbol{T}_{\mathrm{pe}} \cdot {}_4^0\boldsymbol{T}_{\mathrm{s}} \cdot {}_4^0\boldsymbol{T}_{\mathrm{se}} = \boldsymbol{I}_{4\times4} \cdot \begin{bmatrix} 1 & -\varepsilon_{xy} & 0 & 0 \\ \varepsilon_{xy} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot $$

$$ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\varepsilon_z(y) & \varepsilon_y(y) & \delta_x(y) \\ \varepsilon_z(y) & 1 & -\varepsilon_x(y) & \delta_y(y) \\ -\varepsilon_y(y) & \varepsilon_x(y) & 1 & \delta_z(y) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12} $$

$$ {}_5^4\boldsymbol{T} = {}_5^4\boldsymbol{T}_{\mathrm{p}} \cdot {}_5^4\boldsymbol{T}_{\mathrm{pe}} \cdot {}_5^4\boldsymbol{T}_{\mathrm{s}} \cdot {}_5^4\boldsymbol{T}_{\mathrm{se}} = \boldsymbol{I}_{4\times4} \cdot \begin{bmatrix} 1 & 0 & \varepsilon_{zx} & 0 \\ 0 & 1 & -\varepsilon_{yz} & 0 \\ -\varepsilon_{zx} & \varepsilon_{yz} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot $$

$$ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\varepsilon_z(z) & \varepsilon_y(z) & \delta_x(z) \\ \varepsilon_z(z) & 1 & -\varepsilon_x(z) & \delta_y(z) \\ -\varepsilon_y(z) & \varepsilon_x(z) & 1 & \delta_z(z) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13} $$

$$ {}_6^5\boldsymbol{T} = {}_6^5\boldsymbol{T}_{\mathrm{p}} \cdot {}_6^5\boldsymbol{T}_{\mathrm{pe}} \cdot {}_6^5\boldsymbol{T}_{\mathrm{s}} \cdot {}_6^5\boldsymbol{T}_{\mathrm{se}} = \boldsymbol{I}_{4\times4} \cdot \begin{bmatrix} 1 & -\varepsilon_{xyb} & 0 & 0 \\ \varepsilon_{xyb} & 1 & -\varepsilon_{yzb} & 0 \\ 0 & \varepsilon_{yzb} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot $$

$$ \begin{bmatrix} \cos B & 0 & \sin B & 0 \\ 0 & 1 & 0 & 0 \\ -\sin B & 0 & \cos B & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\varepsilon_z(b) & \varepsilon_y(b) & \delta_x(b) \\ \varepsilon_z(b) & 1 & -\varepsilon_x(b) & \delta_y(b) \\ -\varepsilon_y(b) & \varepsilon_x(b) & 1 & \delta_z(b) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14} $$

$$ {}_7^6\boldsymbol{T} = \boldsymbol{I}_{4\times4} \tag{15} $$

$$ {}_8^7\boldsymbol{T} = \begin{bmatrix} p_{tx} & p_{ty} & p_{tz} & 1 \end{bmatrix} \tag{16} $$

If the cutting tool position $\{p_{tx}, p_{ty}, p_{tz}\}$ is represented by a $\boldsymbol{t}$-vector $\boldsymbol{t}=[0 \quad 0 \quad l]^{\mathrm{T}}$ as the position of cutting tool is dependent on its length in Z-direction which can be taken as "l", and the workpiece is represented by a $\boldsymbol{w}$-vector $\boldsymbol{w}=[p_{wx} \quad p_{wy} \quad p_{wz}]^{\mathrm{T}}$ whereas the setting errors are neglected, then a formula can be derived for error (e_{tw}) calculation between a cutting tool and the workpiece due to link and joint errors and their effects on volumetric workspace as mentioned in Eq.(17).

$$ \begin{bmatrix} e_{tw} \\ 0 \end{bmatrix} = \begin{bmatrix} \prod_{M}^{0} {}_j^i\boldsymbol{T} \end{bmatrix} \begin{bmatrix} \boldsymbol{t} \\ 1 \end{bmatrix} - \begin{bmatrix} \prod_{N}^{0} {}_j^i\boldsymbol{T} \end{bmatrix} \begin{bmatrix} \boldsymbol{w} \\ 1 \end{bmatrix} \tag{17} $$

where $\prod_{M}^{0} {}_j^i\boldsymbol{T} = {}_4^0\boldsymbol{T} \cdot {}_5^4\boldsymbol{T} \cdot {}_6^5\boldsymbol{T} \cdot {}_7^6\boldsymbol{T} \cdot {}_8^7\boldsymbol{T}$ and $\prod_{N}^{0} {}_j^i\boldsymbol{T} = {}_1^0\boldsymbol{T} \cdot {}_2^1\boldsymbol{T} \cdot {}_3^2\boldsymbol{T}$.

Validation and authentication of the model are carried out through MATLAB software by testing the individual loops 0-M (0-8) and 0-N (0-3) and then further as a whole against the error determination through Eq.(17) in various combinations as a single and combined parameter checking method. When there is no error and no displacement is introduced, the individual loops exhibits an "$\boldsymbol{I}$" matrices and Eq.(17) shows zero error. If only displacement in the axes is introduced, the similar displacement relevant to the same joint direction is obtained, whereas by introducing the displacement and displacement errors in the elemental prismatic joint, a combined added effect of displacement and introduced errors are resulted. In case of pure rotation, similar magnitude of rotational effect is obtained, whereas by introducing an additional rotational error with a combination of rotation movement, the combined added effect of rotation and rotational error is resulted. Pair-wise movement and combinational movements are introduced along with some arbitrary errors and the parameters are also verified. Moreover the model is checked by analytical calculation in bits and as a whole, and no additional unforeseen error is observed. The testing methodology is quite simple and can be easily verified through any computational software like MATLAB which makes the systematic geometric error modeling technique more robust and error free. Testing the developed methodology proves the technique's verification and authentication for further utilization.

7. Conclusions

A systematic geometric volumetric error modeling technique has been proposed, and its implication is explained through a 5-axis turbine blade grinding machine. Research work on modeling of machine tools of various authors in chronological order is considered for better understanding and clarification of up-to-date discoveries in modeling techniques. Topologies of 5-axis machine tools are explained and error identification and systematic geometric error sources are discussed in detail. 39 position dependent and position independent systematic geometric errors out of 52 potential errors of 5-axis machine tools were identified and considered.

This modeling technique is quite simple, comprehensive, robust, and easy to calculate, analyze and synthesize the geometric errors of 5-axis machine tools for finding the volumetric workspace errors without unnecessary calculation and free from errors and mistakes. This proposed modeling methodology can provide valuable information for error avoidance due to contribution of each of the parametric errors to the workspace deviation, which gives a factual accuracy picture of a machine tool and increases the efficiency of modeling.

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