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Identification Method for RLG Random Errors Based on Allan Variance and Equivalent Theorem

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Abstract

An identification method using Allan variance and equivalent theorem is proposed to identify non-stationary sensor errors mixed out of different simple noises. This method firstly derives the discrete Allan variances of all component noises inherent in noise sources in terms of their different equations; then the variances are used to estimate the parameters of all component noise models; finally, the original errors are represented by the sum of the non-stationary component noise model and the equivalent model mixed out of the stationary and critically stationary component noises. Results of two examples for identification confirm the superiority of this approach regardless of the errors being stationary or not. The comparison of results of real ring laser gyro (RLG) errors processed by various methods shows that the proposed approach is more suited to depict the original noises than common ones.

Keywords: Allan variance; equivalent theorem; non-stationary; auto-regressive and moving average model; ring laser gyro

1. Introduction

The errors inherent in instruments have to be modeled and identified so that they can properly be compensated or filtered after their integration into a system[1-2]. Ring laser gyro (RLG) is an important instrument in inertial navigation system (INS) that has found broad applications in a variety of fields. Various methods to model RLG random errors have been studied until recently. In Ref.[3], RLG random errors were described by a non-stationary auto-regressive and moving average (ARMA) model with the parameters estimated through data differentials. In Ref.[4], C.N. Lawrenence, et al. applied the Allan variance method to model various RLG errors including deterministic and random noises. Ref.[5] proposed state space and Kalman filtering to identify RLG random errors.

Among the above-mentioned methods, models of Ref.[3] and Ref.[5], which fails to consider the inherent noise sources, could not properly describe the RLG errors while the model of Ref.[4], which is not estimated in time-domain, proves inconvenient for compensating online. To help overcome these shortcomings, this article proposes an effective approach to identify RLG errors, in which, after using Allan variance to estimate the parameters of the component noises contained in RLG errors, the equivalent theorem is applied to mix the stationary and critically stationary component noises.

For verification, the proposed method is applied to model a set of data collected from a real RLG and the data are compensated using extended Kalman filter (EKF) based on the model. By comparing the thus compensated results to those compensated with the method in Ref.[5], it is understood that the model established with the proposed method is capable of defining the original noises more precisely and effectively than that introduced by Ref.[5].

2. Equivalent ARMA Model Theorem

This section derives an equivalent ARMA model for a mixture of ARMA processes representing different forms of noises. The following lemma shows that a stationary process with finite non-zero autocorrelations can be simplified by a single moving average (MA) model.

2.1. Proof of lemma

Lemma Given a stationary process $\{X_t, -\infty < t < \infty\}$

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with zero-mean and autocorrelation function $r(\bullet)$ that meets

$$r(k)=0 \text{ when } |k|>p \quad \text{and} \quad r(p)\neq 0 \tag{1}$$

then $\{\boldsymbol{X}_t, -\infty<t<\infty\}$ satisfies

$$\boldsymbol{X}_t=\boldsymbol{W}_t+\sum_{i=1}^{p}\theta_i\boldsymbol{W}_{t-i}$$

where $\{\boldsymbol{W}_t,-\infty<t<\infty\}$ is a white noise process.

Proof First let $\boldsymbol{\Gamma}_t \triangleq \overline{\text{span}}\{\boldsymbol{X}_s,-\infty<s<t\}$, $\boldsymbol{\Gamma}_t^u \triangleq \overline{\text{span}}\{\boldsymbol{X}_s, t-u<s<t\}$, $t\in\mathbf{Z}$.

Let $\boldsymbol{W}_t=\boldsymbol{X}_t-P_{\boldsymbol{\Gamma}_t}\boldsymbol{X}_t$ and $\boldsymbol{W}_t^u=\boldsymbol{X}_t-P_{\boldsymbol{\Gamma}_t^u}\boldsymbol{X}_t$, where $P_{\boldsymbol{\Gamma}_t}\boldsymbol{X}_t$ or $P_{\boldsymbol{\Gamma}_t}\boldsymbol{X}_t$ is the projection of $\boldsymbol{X}_t$ in $\boldsymbol{\Gamma}_t$ or $\boldsymbol{\Gamma}_t^u$.

Then

$$\left.\begin{array}{l}\boldsymbol{W}_s\in\boldsymbol{\Gamma}_{s+1}\subset\boldsymbol{\Gamma}_t, E(\boldsymbol{W}_s\boldsymbol{W}_t)=0 \text{ when } s<t\\ \boldsymbol{W}_t^u\perp\boldsymbol{\Gamma}_t^u, \boldsymbol{W}_t^v\perp\boldsymbol{\Gamma}_t^v, \text{and } \boldsymbol{\Gamma}_t^v\subset\boldsymbol{\Gamma}_t^u \text{ when } u<v\end{array}\right\} \tag{2}$$

Therefore

$$\left\|P_{\boldsymbol{\Gamma}_t^u}\boldsymbol{X}_t-P_{\boldsymbol{\Gamma}_t^v}\boldsymbol{X}_t\right\|^2+\left\|P_{\boldsymbol{\Gamma}_t^v}\boldsymbol{X}_t\right\|^2=\left\|P_{\boldsymbol{\Gamma}_t^u}\boldsymbol{X}_t\right\|^2 \tag{3}$$

For $0<\left\|P_{\boldsymbol{\Gamma}_t^v}\boldsymbol{X}_t\right\|\le\left\|P_{\boldsymbol{\Gamma}_t^u}\boldsymbol{X}_t\right\|\le\left\|P_{\boldsymbol{\Gamma}_t}\boldsymbol{X}_t\right\|<\infty$,

$$\lim_{u\to\infty}\left\|P_{\boldsymbol{\Gamma}_t^u}\boldsymbol{X}_t\right\|=\left\|P_{\boldsymbol{\Gamma}_t}\boldsymbol{X}_t\right\| \tag{4}$$

It can be seen that $\{P_{\boldsymbol{\Gamma}_t^u}\boldsymbol{X}_t,-\infty<u<t,t\in\mathbf{Z}\}$ is a Cauchy sequence, so can be obtained

$$\lim_{u\to\infty}P_{\boldsymbol{\Gamma}_t^u}\boldsymbol{X}_t=P_{\boldsymbol{\Gamma}_t}\boldsymbol{X}_t \tag{5}$$

Since $\{\boldsymbol{X}_t,-\infty<t<\infty\}$ is stationary to Eq.(5),

$$\|\boldsymbol{W}_{t+1}\|=\left\|\boldsymbol{X}_{t+1}-P_{\boldsymbol{\Gamma}_{t+1}}\boldsymbol{X}_{t+1}\right\|=\lim_{u\to\infty}\left\|\boldsymbol{X}_{t+1}-P_{\boldsymbol{\Gamma}_{t+1}^u}\boldsymbol{X}_{t+1}\right\|=$$
$$\lim_{u\to\infty}\left\|\boldsymbol{X}_t-P_{\boldsymbol{\Gamma}_t^u}\boldsymbol{X}_t\right\|=\left\|\boldsymbol{X}_t-P_{\boldsymbol{\Gamma}_t}\boldsymbol{X}_t\right\|=\|\boldsymbol{W}_t\| \tag{6}$$

As a result, $\{\boldsymbol{W}_t,-\infty<t<\infty\}\sim N(0,\sigma^2)$, where $\sigma^2=\|\boldsymbol{W}_t\|^2$. Namely, $\{\boldsymbol{W}_t,-\infty<t<\infty\}$ is a white noise process with $E(\boldsymbol{W}_t)=0$ and $E(\boldsymbol{W}_t^2)=\sigma^2$.

$\boldsymbol{\Gamma}_t$ can be divided into two orthogonal subspaces $\boldsymbol{\Gamma}_{t-p}$ and $\overline{\text{span}}\{W_{t-p},\cdots,W_{t-1}\}$. From the supposition Eq. (1), can be obtained

$$\boldsymbol{X}_t\perp\boldsymbol{\Gamma}_{t-p} \tag{7}$$

Then

$$\boldsymbol{X}_t=\boldsymbol{W}_t+P_{\boldsymbol{\Gamma}_{t-p}}\boldsymbol{X}_t+P_{\overline{\text{span}}\{W_{t-p},\cdots,W_{t-1}\}}\boldsymbol{X}_t=$$
$$\mathbf{0}+\boldsymbol{W}_t+\sum_{i=1}^{p}\theta_i\boldsymbol{W}_{t-i} \tag{8}$$

where $\theta_i\triangleq\sigma^{-2}E(\boldsymbol{X}_t\boldsymbol{X}_{t-i})$, $i=1,2,\cdots,p$ and thus the proof is completed.

2.2. Equivalent theorem

Equivalent theorem Consider two ARMA processes, x and y, of which x with order (p_1, q_1) and y with order (p_2, q_2), which satisfy

$$x(n)=\frac{1+\sum_{i=1}^{q_1}b_iz^{-i}}{1+\sum_{i=1}^{p_1}a_iz^{-i}}\bullet u_1(n),\quad y(n)=\frac{1+\sum_{i=1}^{q_2}c_iz^{-i}}{1+\sum_{i=1}^{p_2}d_iz^{-i}}\bullet u_2(n)$$

where u_1 and u_2 are independent white processes, and z is a forward shift operator.

Denote $\varsigma(n)\triangleq x(n)+y(n)$, then

$$\left(\sum_{i=0}^{p_1}a_iz^{-i}\bullet\sum_{i=0}^{p_2}c_iz^{-i}\right)\varsigma(n)=$$
$$\sum_{i=0}^{q1}f_iz^{-i}\bullet u_1(n)+\sum_{i=0}^{q2}g_iz^{-i}\bullet u_2(n) \tag{9}$$

where $q1=p_2+q_1$, $q2=p_1+q_2$.

Obviously, the left part of Eq.(9) corresponds to the AR part of the equivalent ARMA model with an order of p_1+p_2. As for the right part of Eq.(9), define $s(n)=\sum_{i=0}^{q1}f_iz^{-i}\bullet u_1(n)+\sum_{i=0}^{q2}g_iz^{-i}\bullet u_2(n)$. According to the lemma, $s(n)$ can be represented by $s(n)=\sum_{i=0}^{q}\theta_iz^{-i}\bullet w(n)$, where $q=\max(q_1,q_2)$, and w is a white process.

It should be noted that the autocorrelations of $s(n)$ satisfy

$$\left.\begin{array}{l}r(0)\triangleq E(s^2(k))=\sigma^2\sum_{i=0}^{q}\theta_i^2\\ r(1)\triangleq E(s(k)s(k-1))=\sigma^2\sum_{i=0}^{q-1}\theta_i\theta_{i+1}\\ \vdots\\ r(q-1)\triangleq E(s(k)s(k-q+1))=\sigma^2(\theta_{q-1}\theta_0+\theta_q\theta_1)\\ r(q)\triangleq E(s(k)s(k-q))=\sigma^2\theta_q\theta_0\end{array}\right\} \tag{10}$$

Taking generality into consideration, let $\sigma=1$. Define

$$\left.\begin{array}{l}f_1\triangleq\sum_{i=0}^{q}\theta_i^2-r(0)\\ f_2\triangleq\sum_{i=0}^{q-1}\theta_i\theta_{i+1}-r(1)\\ \vdots\\ f_q\triangleq\theta_{q-1}\theta_0+\theta_q\theta_1-r(q-1)\\ f_{q+1}\triangleq\theta_q\theta_0-r(q)\\ \boldsymbol{F}(\boldsymbol{\theta})\triangleq[f_1\quad f_2\quad\cdots\quad f_q\quad f_{q+1}]^{\mathrm{T}}\\ \boldsymbol{\theta}\triangleq[\theta_0\quad\theta_1\quad\cdots\quad\theta_{q-1}\quad\theta_q]^{\mathrm{T}}\end{array}\right\} \tag{11}$$

To solve Eq.(10), Newton algorithm is used as follows:

① Let $\boldsymbol{\theta}^{(0)}=\left[\sqrt{r(0)}\quad 0\quad\cdots\quad 0\quad 0\right]^{\mathrm{T}}$.

② Calculate $f_k^{(i)}$, k=1,2,⋯,q, q+1 according to Eq. (11), which yields

$$\boldsymbol{A}_i \triangleq \frac{\partial \boldsymbol{F}(\boldsymbol{\theta}^{(i)})}{\partial \boldsymbol{\theta}^{(i)\prime}} = \begin{bmatrix} \frac{\partial f_1^{(i)}}{\partial \theta_0^{(i)}} & \frac{\partial f_1^{(i)}}{\partial \theta_1^{(i)}} & \cdots & \frac{\partial f_1^{(i)}}{\partial \theta_q^{(i)}} \\ \frac{\partial f_2^{(i)}}{\partial \theta_0^{(i)}} & \frac{\partial f_2^{(i)}}{\partial \theta_1^{(i)}} & \cdots & \frac{\partial f_2^{(i)}}{\partial \theta_q^{(i)}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_{q+1}^{(i)}}{\partial \theta_0^{(i)}} & \frac{\partial f_{q+1}^{(i)}}{\partial \theta_1^{(i)}} & \cdots & \frac{\partial f_{q+1}^{(i)}}{\partial \theta_q^{(i)}} \end{bmatrix}$$

③ Update $\boldsymbol{\theta}^{(i)}$ according to the equation: $\boldsymbol{\theta}^{(i+1)} = \boldsymbol{\theta}^{(i)} - \boldsymbol{A}_i^{-1}\boldsymbol{F}(\boldsymbol{\theta}^{(i)})$.

④ Set threshold value to be ε and criterion function $\left|\frac{\theta_k^{(i+1)} - \theta_k^{(i)}}{\theta_k^{(i+1)}}\right| \le \varepsilon$. Once $\boldsymbol{\theta}^{(i)}$ satisfies the criterion function, stop calculating and obtain $\boldsymbol{\theta}=\boldsymbol{\theta}^{(i)}$; otherwise, go to ② and continue.

Up to now, it is shown that the mixture of any two ARMA processes can be represented by a single equivalent ARMA model, and the parameters of the equivalent model can be obtained through Newton algorithm. This result can be easily extended to the mixture of more than two ARMA processes.

3. Identification of RLG Random Errors

In order to illustrate the application of the results obtained in Section 2, this section presents a procedure for identifying RLG errors.

3.1. Four main noises of RLG errors

As mentioned in Ref.[6], the four important forms of discrete RLG random errors include white noise, quantization noise, random walks and first-order Markov processes, which will be introduced in some detail as follows:

(1) White noise, also known as angle random walk, results from integrating a wideband rate power spectral density (PSD) noise. It is the major source of errors of RLGs that employ randomized dither as an anti-lock approach. This form of noise is mainly caused by randomized dither or spontaneous emission of photons.

(2) Quantization noise is strictly blamed for the digital nature of RLG outputs since readouts of an electronic device are all in the digital form. The quantization noise represents the minimum resolution level of a sensor.

(3) Rate random walk results from integrating wideband acceleration PSD. Without a clear reason, this may stem from a random process of uncertain origin, possibly a limiting case of an exponentially correlated noise with a very long correlation time.

(4) As a common form of noise found in RLG, exponentially correlated first-order Markov process is described by an exponentially decaying function with a limited correlation time. The potential source of this noise component is randomized mechanical dither stemming from the resonant nature of the dither mechanism that does not allow all frequencies with an equal amplitude being transferred to the gyro body.

Let y_1, y_2, y_3 and y_4 represent the above four forms of noises, which satisfy[6]

$$\left.\begin{aligned} y_1(n) &= w_1(n) \\ y_2(n) &= w_2(n) - w_2(n-1) \\ y_3(n) &= y_3(n-1) + w_3(n) \\ y_4(n) &= a(n)y_4(n-1) + w_4(n) \end{aligned}\right\} \tag{12}$$

where w_1, w_2, w_3 and w_4 are independent white noises with variances $\sigma_{w_1}^2$, $\sigma_{w_2}^2$, $\sigma_{w_3}^2$ and $\sigma_{w_4}^2$, respectively. $a(n)$ is the time-varying coefficient of the Markov process. Considering that randomized mechanical dither may be a potential source of Markov process, the coefficient $a(n)$ is generally not a constant, but a series satisfying[6]

$$a(n) = a + e(n)$$

where e is a white noise with small variance σ_e^2.

RLG errors are non-stationary, which could be imputed to, on one hand, the critically stationary random walk y_3 and, on the other, the non-stationary first-order Markov process when $\sigma_e^2 \ne 0$. Consequently, RLG errors are better depicted by auto-regressive integrated moving average (ARIMA) model or time-varying ARMA (TARMA) model rather than ARMA. Table 1 summarizes equivalent ARMA models mixed out of four possible forms of RLG noises.

Table 1 Equivalent ARMA models

Noise	The model's order	Noise	The model's order
y_1+y_2	ARMA(0,1)	$y_1+y_2+y_3$	ARIMA(0,2,1)
y_1+y_3	ARIMA(0,1,1)	$y_1+y_2+y_4$	ARMA(1,2) TARMA (1,2)^2
y_1+y_4	ARMA(1,1) TARMA(1,1)^2	$y_1+y_3+y_4$	ARIMA(1,2,1) TARMA (2,2)^2
y_2+y_3	ARIMA(0,2,1)	$y_2+y_3+y_4$	ARIMA(1,3,1) TARMA (2,3)^2
y_2+y_4	ARMA(1,2) TARMA(1,2)^2	$y_1+y_2+y_3+y_4$	ARIMA(1,3,1) TARMA (2,3)^2
y_3+y_4	ARIMA(1,1,1) TARMA (2,1)^2		

2 Orders of the models are decided by the condition of $\sigma_e^2 \ne 0$.

3.2. Discrete Allan variances of the four main noise forms

The Allan variance is defined as[7]

$$A\sigma^2(\tau) = \frac{1}{2\tau^2}\left\langle \{x(t) - 2x(t+\tau) + x(t+2\tau)\}^2 \right\rangle$$

where $\langle\ \rangle$ denotes an ensemble average or a sample average, $x(t)$ a time series of time differences between two clocks spaces τ seconds apart.

Suppose $x(t)$ is sampled as $\{x(k\tau_0),\ k=1,\ 2,\ \cdots,\ N\}$, with the sample time being τ_0 and the notation simplified by writing $x(k\tau_0)=x_k$. Then the Allan variance becomes

$$A\sigma^2(\tau)=\frac{1}{2\tau^2(N-2m)}\sum_{k=1}^{N-2m}(x_{k+2m}-2x_{k+m}+x_k)^2$$

where $\tau=m\tau_0$.

Commonly, the main forms of RLG noise are all represented with PSD when Allan variances are applied to identify them. Since they are now separately described by differential equations, the identification algorithm must be rededuced as follows.

For y_1, the Allan variance is

$$A\sigma_{y_1}^2(n)=\frac{1}{2}\left\langle\left[\frac{1}{n}\left(\sum_{i=1}^{n}(y_1(i)-y_1(n+i))\right)\right]^2\right\rangle=$$
$$\frac{1}{2n^2}E\left(\sum_{i=1}^{n}(w_1(i)-w_1(n+i))\right)^2=\frac{1}{n}\sigma_{w_1}^2 \quad (13)$$

For y_2, the Allan variance is

$$A\sigma_{y_2}^2(n)=\frac{1}{2n^2}E[2w_2(n)-w_2(0)-w_2(2n)]^2=\frac{3}{n^2}\sigma_{w_2}^2 \quad (14)$$

For y_3, the Allan variance is

$$A\sigma_{y_3}^2(n)=\frac{1}{2n^2}\left\langle\left[\left(\sum_{i=1}^{n}(y_3(i)-y_3(n+i))\right)\right]^2\right\rangle=$$
$$\frac{1}{2n^2}E\left(\sum_{i=2}^{n}(i-1)(w_3(i)+w_3(n+i))+nw_3(n+1)\right)^2=$$
$$\frac{2n^2+1}{6n}\sigma_{w_3}^2 \quad (15)$$

And for y_4, the autocorrelations satisfy

$$R(n+1,n+1)=(a^2+\sigma_e^2)R(n,n)+\sigma_{w_1}^2$$
$$R(n,i)=R(i,n)=a^{n-i}R(i,i) \quad \text{for } i\le n$$

then

$$A\sigma_{y_4}^2=\frac{1}{2n^2}\left\langle\left[\sum_{i=1}^{n}(y_4(i)-y_4(n+i))\right]^2\right\rangle=$$
$$\frac{1}{2n^2}\left\langle 2\left[\left(\sum_{i=1}^{n}y_4(i)\right)^2+\left(\sum_{i=n+1}^{2n}y_4(i)\right)^2\right]-\left(\sum_{i=1}^{2n}y_4(i)\right)^2\right\rangle=$$
$$\frac{-a^{2n+1}+4a^{n+1}+(1-a^2)n-3a}{n^2(1-a)^2(1-a^2-\sigma_e^2)}\sigma_{w_4}^2 \quad (16)$$

Taking into account the fact that values of $\sigma_{w_4}^2$ and σ_e^2 are not independent, it is necessary to calculate Allan variances of both RLG errors $y(n)$ and $y(n)\ (1-z^{-1})$ in order to determine the values of $\sigma_{w_4}^2$ and σ_e^2. At the same time, in the errors $y(n)(1-z^{-1})$, the four main forms will become

$$y_1'(n)\triangleq y_1(n)(1-z^{-1})=w_1(n)-w_1(n-1)$$
$$y_2'(n)\triangleq y_2(n)(1-z^{-1})=w_2(n)-2w_2(n-1)+w_2(n-2)$$
$$y_3'(n)\triangleq y_3(n)(1-z^{-1})=w_3(n)$$
$$y_4'(n)\triangleq y_4(n)(1-z^{-1})=a(n)y_4'(n-1)+w_4(n)-w_4(n-1)$$

Then their Allan variances will satisfy

$$A\sigma_{y_1'}^2(n)=\frac{3}{n^2}\sigma_{w_1}^2 \quad (17)$$
$$A\sigma_{y_2'}^2(n)=\frac{6}{n^2}\sigma_{w_2}^2 \quad (18)$$
$$A\sigma_{y_3'}^2(n)=\frac{1}{n}\sigma_{w_3}^2 \quad (19)$$
$$A\sigma_{y_4'}^2=(3-a^2-\sigma_e^2)A\sigma_{y_4}^2+\frac{\sigma_{w_4}^2}{n} \quad (20)$$

From Eqs.(13)-(20), it is seen that the Allan variances based on differential equations are not continuous, rather, they are discrete according to the sampling frequency that has been normalized with identification algorithm.

3.3. Determination of parameters of four noise forms

According to Eqs.(13)-(20), the parameters of the four main noise forms can be obtained by using least square method or Newton method to fit the Allan variances $A\sigma_y^2(n)$ and $A\sigma_{y'}^2(n)$. The fitting equations should satisfy

$$A\sigma_y^2=A\sigma_{y_1}^2+A\sigma_{y_2}^2+A\sigma_{y_3}^2+A\sigma_{y_4}^2 \quad (21)$$
$$A\sigma_{y'}^2=A\sigma_{y_1'}^2+A\sigma_{y_2'}^2+A\sigma_{y_3'}^2+A\sigma_{y_4'}^2 \quad (22)$$

Eq.(16) and Eq.(20) are too complicated to calculate in curve-fitting process. Because $|a|<1$, could be found

$$A\sigma_{y_4}^2=\begin{cases}\dfrac{-a^{2n+1}+4a^{n+1}+(1-a^2)n-3a}{n^2(1-a)^2(1-a^2-\sigma_e^2)}\sigma_{w_4}^2 & (n<n_0)\\[2ex] \dfrac{(1-a^2)n-3a}{n^2(1-a)^2(1-a^2-\sigma_e^2)}\sigma_{w_4}^2 & (n\ge n_0)\end{cases} \quad (23)$$

$$A\sigma_{y_4'}^2=(3-a^2-\sigma_e^2)A\sigma_{y_4}^2+\frac{\sigma_{w_4}^2}{n} \quad (24)$$

where n_0 generally equals 3 or 4.

3.4. Determination of MA part of equivalent model of stationary and critically stationary components

It has been shown that the necessary condition of the equivalent theorem is that all noises should be either stationary or critically stationary. However, since y_4 is non-stationary as mentioned in Section 3.1, it is necessary to divide the RLG noises into two parts

$$y(n)=y^{(1)}(n)+y^{(2)}(n) \tag{25}$$

where $y^{(1)}(n)=y_1(n)+y_2(n)+y_3(n)$, $y^{(2)}(n)=y_4(n)$.

Then the TARMA model shown in Table 1 should be replaced by the sum of a stationary (when $\sigma^2_{w_3}=0$) or a critically stationary (when $\sigma^2_{w_3}\neq 0$) ARMA model $y^1(n)$ and a non-stationary one $y^2(n)$. As mentioned in Section 2, the AR part of the equivalent model is very easy to acquire while the MA part should be resolved through Newton algorithm. For the RLG errors, the MA part's order of $y^1(n)$ does not exceed 2. The precise equivalent models of MA(1) and MA(2) can be calculated as follows.

For MA(1) model, assuming that the autocorrelations are $r(0)$ and $r(1)$, the parameters of the equivalent model are

$$\left.\begin{aligned}\theta_0&=\frac{\sqrt{r(0)+2r(1)}+\sqrt{r(0)-2r(1)}}{2}\\ \theta_1&=\frac{\sqrt{r(0)+2r(1)}-\sqrt{r(0)-2r(1)}}{2}\end{aligned}\right\} \tag{26}$$

For MA(2) model, assuming that the autocorrelations are $r(0)$, $r(1)$ and $r(2)$, then

$$\left.\begin{aligned}\theta_0&=\frac{1}{2}\left[\frac{r(1)}{\theta_1}+\sqrt{\left(\frac{r(1)}{\theta_1}\right)^2-4r(2)}\right]\\ \theta_1&=\frac{1}{2}\left(\sqrt{r(0)+2r(2)+2r(1)}-\sqrt{r(0)+2r(2)-2r(1)}\right)\\ \theta_2&=\frac{1}{2}\left[\frac{r(1)}{\theta_1}-\sqrt{\left(\frac{r(1)}{\theta_1}\right)^2-4r(2)}\right]\end{aligned}\right\} \tag{27}$$

4. Examples and Verification of the Equivalent ARMA Model

Example 1 To verify the effectiveness of the proposed method, was simulated a stationary ARIMA model $y(n)$ that satisfies $\sigma^2_{w_1}=1$, $\sigma^2_{w_2}=1$, $\sigma^2_{w_3}=1$, $\sigma^2_{w_4}=0$.

According to Eq.(18), $y(n)=y(n-1)+2.369\,2w(n)-1.791\,3w(n-1)+0.422\,1w(n-2)$.

Two different methods were applied to estimate the parameters of ARIMA(0, 2, 1) model $y(n)$. From the estimation results shown in Table 2, it is confirmed that both models produced by the methods accord with simulated noise quite well. Common method in Table 2 estimates parameters by solving Yule-Walker equations and the orders of the model are decided by Akaike information criterion (AIC).

Table 2 Comparison of estimation results by using different methods

Parameter	Precise value	Proposed method (Value/Error)	Normal method (Value/Error)
$\sigma^2_{w_1}$	1	1.026 9/ 2.69%	
$\sigma^2_{w_2}$	1	1.022 0/ 2.20%	
$\sigma^2_{w_3}$	1	0.971 1/ 2.89%	
$\sigma^2_{w_4}$	0	0/0%	
θ_0	2.369 2	2.377 6/ 0.35%	2.397 1/ 1.18%
θ_1	−1.791 3	−1.821 9/ 1.71%	−1.715 2/ 4.25%
θ_2	0.422 1	0.429 9/ 1.85%	0.421 0/ 0.26%

Example 2 The common methods, such as Durbin-Levinson or double Levinson algorithm, are not effective to estimate non-stationary noises, but the proposed one is. To verify this, from HG4195 was recorded an RLG noise, of which the input and output data have been collected from a static rate table test. No rate input was applied to the gyro. Output data were compensated by the earth rate input and a constant bias.

The ensuing parameters of all component models are estimated according to Allan variances with the unit of the original RLG noise being (°)/h.

$$\sigma^2_{w_1}=1.541\times10^{-3},\ \sigma^2_{w_2}=3.475\times10^{-6}$$
$$\sigma^2_{w_3}=5.796\times10^{-5},\ \sigma^2_{w_4}=8.552\times10^{-6}$$
$$\sigma^2_{e}=9.476\times10^{-8},\ a=0.901$$

Then the RLG noise can be written in the following forms according to Section 3.4

$$y(n)=y^{(1)}(n)+y^{(2)}(n) \tag{28}$$
$$y^{(1)}(n)=y^{(1)}(n-1)+w^{(1)}(n)-0.826\,3w^{(1)}(n-1)+0.001\,850w^{(1)}(n-2)$$
$$y^{(2)}(n)=(0.901+e(n))y^{(2)}(n-1)+w_4(n)$$

According to the equivalent theorem and Eq.(27), $\sigma^2_{w^{(1)}}$ =0.001 878 can be obtained.

The above model can be expressed as

$$\begin{bmatrix}\boldsymbol{X}_1(n)\\ \boldsymbol{X}_2(n)\end{bmatrix}=\begin{bmatrix}\boldsymbol{\Phi}_1(n) & 0\\ 0 & \boldsymbol{\Phi}_2(n)\end{bmatrix}\begin{bmatrix}\boldsymbol{X}_2(n-1)\\ \boldsymbol{X}_2(n-1)\end{bmatrix}+\begin{bmatrix}\boldsymbol{W}_1(n)\\ \boldsymbol{W}_2(n)\end{bmatrix}$$

$$\boldsymbol{Y}(n)=\begin{bmatrix}1 & 0 & 0 & 1 & 0\end{bmatrix}\boldsymbol{X}(n)$$

$$\boldsymbol{X}_1(n)=\begin{bmatrix}y^{(1)}(n) & w^{(1)}(n) & w^{(1)}(n-1)\end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{\Phi}_1(n)=\begin{bmatrix}1 & -0.826\,3 & 0.001\,85\\ 0 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

$$\boldsymbol{W}_1(n)=\left[w^{(1)}(n)\quad w^{(1)}(n)\quad 0\right]^{\mathrm{T}}$$

$$\boldsymbol{\Phi}_2(n)\boldsymbol{X}_2(n-1)=\begin{bmatrix}(0.901+e(n))y^{(2)}(n-1)\\ 0\end{bmatrix}$$

$$\boldsymbol{W}_2(n)=\left[w_4(n)\quad e(n)\right]^{\mathrm{T}}$$

EKF is applied to filter the RLG noise based on the above non-linear state equation[8]. Fig.1 shows the result curve as a continuous line. Another result curve drawn with Kalman filter (KF) method based on the ARIMA(0, 2, 2) model appears to be a broken line[5]. The variances of the two filtered noises are 0.000 142 1 and 0.000 255 6, respectively. The ARIMA (0, 2, 2) can be expressed as

$$y(n)=2y(n-1)-y(n-2)+w(n)-0.913\,1w(n-1)+0.011\,32w(n-2) \tag{29}$$

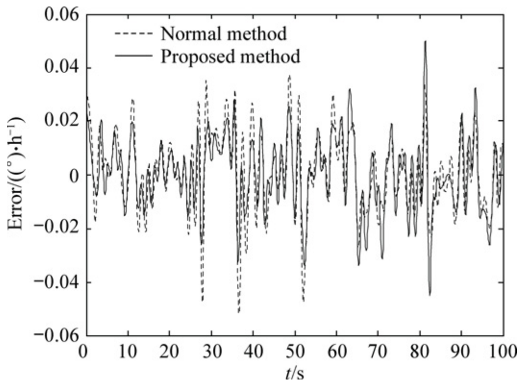

Fig.1 Comparison of result curves filtered with the two methods.

From Fig.1 and the variances, it can be confirmed that the first filtered result is much better than the second, which evidences the model Eq.(28) is more suited to depict original RLG noise than model Eq.(29). One reason may be that the proposed model Eq.(25) is based on the physical characteristics of RLG, and the other may be that the model Eq.(25) includes the model ARIMA(0, 2, 2) (when a = 1 and $\sigma_w^2=0$) which is the base of the common method.

5. Conclusions

An effective approach is proposed for modeling random noises inherent in inertial sensors. First, the discrete Allan variances of the four component ARMA models contained in original RLG random errors are deduced. Then the four components are identified with the discrete Allan variances. Finally, the stationary and critically stationary ARMA noises are mixed with an equivalent ARIMA model. It is shown that RLG random errors could be represented by the sum of an equivalent ARIMA model and a non-stationary one by using Allan variance method and equivalent theorem.

Applying the proposed method to an ARIMA model bears witness to that it is as effective as the common method when the random noise is critical stationary. When applying the proposed method to model the real test data of RLG, the result shows that the equivalent model generated by the proposed method is more suited to depict the non-stationary noise than the one by common method. The superiority of the proposed method is reflected not only in smaller filtered variances acquired with the model established with the proposed method than those with the common method, but also in its discovery of the RLG noise possessive of non-stationary characteristic the common method fails to display. This method can also be used to model other sensors' noises mixed out of random ones.

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