Form-finding methods for deployable mesh reflector antennas

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Abstract Deployable high-frequency mesh reflector antennas for future communications and observations are required to obtain high gain and high directivity. In order to support these new missions, reflectors with high surface accuracy are widely required. The form-finding analysis of deployable mesh reflector antennas becomes more vital which aims to determine the initial surface profile formed by the equilibrium prestress distribution in cables to satisfy the surface accuracy requirement. In this paper, two form-finding methods for mesh reflector antennas, both of which include two steps, are proposed. The first step is to investigate the prestress design only for the cable net structure as the circumferential nodes connected to the supporting truss are assumed fixed. The second step is to optimize the prestress distribution of the boundary cables connected directly to the supporting truss considering the elastic deformation of the antenna structure. Some numerical examples are carried out and the simulation results demonstrate the proposed form-finding methods can warrant the deformed antenna reflector surface matches the one by design and the cable tension forces fall in a specified range.

1. Introduction

Satellite systems have become more sophisticated over the past two decades and evolved new applications requiring much higher flux densities. These new requirements to provide high data rate service to very small user terminals have led to the need for large aperture reflectors with higher gain in turn.

Conventional metal parabolic reflectors have become too massive to support these new missions in a cost-effective manner and have also posed the problem of fitting within the constrained volumes of launch vehicles. Some new design options for reflector antennas have thus been widely explored including inflatable antennas using polyimide materials,1 antenna systems constructed of piezo-electric materials,2 phased array and deployable antenna systems constructed of wire mesh and cable net structures.3,4 Deployable mesh reflector antennas have the advantages of high packaging efficiency, low mass, thermal stability, and large scale, and recently gain wide acceptance in the fields of space missions.3,6

Cable net structures belong to the family of flexible tension structures characterized by strong geometric nonlinearities. The initial stiffness and shape of cable net structures can be achieved by prestressing cables. The form-finding analysis is to find prestressable configurations, which is an extremely vital
and challenging topic for surface accuracy of mesh reflectors. It aims to obtain the equilibrium pretension forces in cables to satisfy the required reflecting surface accuracy, that is, the reflecting surface of a deployed antenna under tension loads should be sufficiently close to a desired profile. The prestress distribution in cables plays vital effects on structural rigidity, fundamental frequency, and surface accuracy of mesh reflector antennas.8,9

The form-finding analysis of mesh reflector antennas encounters many difficulties resulting from a composite structure of the supporting truss and the flexible cable net structure. For example, (1) before the form-finding analysis, we only know the designed nodal locations of a mesh reflector in a state of equilibrium. The equilibrium configuration of the deployed antenna structure and tension forces in cables are both under investigation owing to their deformations; (2) the deformation between the supporting truss and the cable net structure are coupled. Therefore, the effects of the coupled deformation on surface accuracy must be considered in the form-finding analysis of mesh reflectors. However, the existing form-finding methods10,11 are only used for cable net structures and do not take the coupled deformation into account. Some studies on form-finding analysis of cable net structures have recently been conducted. Shape control concepts for mesh reflectors have been explored by two ways based on the force density method12–15, boundary cables of front cable net and tension tie cables. A contoured beam mesh reflector was researched in Ref. 16, in which, the reflector surface was designed using the plane wave synthesis and the force density method. However, these existing form-finding methods based on the force density method are only suitable for cable net and tensegrity structures including tension cables and/or compression elements, but not for mesh reflector antenna structures including tension cables and bending beam elements. The main purpose of this paper is to propose two new form-finding methods for determining the initial surface profile of mesh reflector antennas which are complex combination structures with supporting trusses and flexible cable nets.

2. Form-finding methods for mesh reflector antennas

2.1. Composition of a mesh reflector antenna

A deployed mesh reflector antenna is conceived with the concept of a tension truss, which is a light and inherently stiff structure that can be precisely and repeatedly deployed regardless of environment. As illustrated in Fig. 1, it is divided into three parts: a supporting truss, boundary cables, and a cable net reflector including surface cables and tension tie cables. The cable net reflector and the boundary cables together are named as the deployable cable net structure. From the geometric viewpoint, a cable net structure can also be divided into: a front cable net, a back cable net, and tension tie cables, as shown in Fig. 2. Front and back cable nets are both doubly curved geodesic nets that are placed back-to-back. Tension tie cables evenly apply approximately normal forces between front and back cable nets to permanently preload them in tension to maintain the surface profile of the reflector.

2.2. Form-finding method I

As a whole, the coupled deformation between the cable net structure and the supporting truss has a great influence on the prestress distribution of the cable net structure and the surface accuracy of the reflector. Two form-finding methods considering the coupled deformation are proposed by altering prestress distribution of boundary cables. Form-finding method I is based on two steps shown in Fig. 3. In the first step, the supporting truss is assumed rigidized and provides fixed boundary conditions for the cable net structure, as shown in Fig. 3(a). In this case, the prestress design method of the cable net structure will be described in Section 2.4, which is derived from nodal force equilibrium equations by the minimum norm method solving linear equations, and can obtain a set of prestresses of cables satisfying the required surface accuracy of the reflector. In fact, the supporting truss will deform under the force of boundary cables, which in turn will change the prestress distribution of cable nets and cause a worse surface accuracy of the reflector. Thus, considering the supporting truss’s deformation, the second step aims to keep the nodes of surface cables located in the equilibrium positions designed in the first step, which can be achieved by optimally adjusting prestress distribution of boundary cables to make the inner nodes of boundary cables (connected to the cable net reflector) fixed. That is, the optimized object is shown in Fig. 3(b), and the optimized model can be expressed as follows:

\[
\begin{aligned}
& \text{Min} \quad \Delta \mathbf{F} = (\|\Delta \mathbf{F}_1\|, \|\Delta \mathbf{F}_2\|, \ldots, \|\Delta \mathbf{F}_i\|, \ldots, \|\Delta \mathbf{F}_n\|) \\
& \text{s.t.} \quad F_{bi} > 0 \quad 1 \leq i \leq n
\end{aligned}
\]
where $F_{bi}$ is the optimal tension force vector of the $i$th boundary cable matching the deformed supporting truss; $n$ is the number of boundary cables; $\|\Delta F_{j}\|$ is the unbalanced force applied to the $j$th inner node of boundary cables; $d$ is the number of the inner nodes of boundary cables; $F_{bi} > 0$ denotes the cable only can bear tension.

Eq. (1) is a multi-objective optimization problem, which can be solved by the quadratic programming or min–max optimization scheme.\textsuperscript{17}

2.3. Form-finding method II

Form-finding method II is also based on two steps shown in Fig. 4. The first step is the same as that of method I. Considering the supporting truss’s deformation, the second step aims to make the reflector surface error minimized by optimally adjusting prestress distribution of boundary cables. That is, the optimized object is shown in Fig. 4(b), and the optimized model can be expressed as follows:

\[
\text{Find } F_b = (F_{b1}, F_{b2}, \cdots, F_{bn})
\]

\[
\text{Min } \text{RMS} = \sqrt{\frac{\sum_{j=1}^{m} \Delta S_{j}^2}{m}}
\]

\[
\text{s.t. } F_{bi} > 0 \quad 1 \leq i \leq n
\]

where RMS is the root mean square, $\Delta S_{j}$ is the displacement of the $j$th free node in the cable net reflector and $m$ is the total number of free nodes in the cable net reflector.

Form-finding method II is a single-objective optimization problem, which can be solved by the quadratic programming.

2.4. Prestress design of a cable net structure

The prestress design of a cable net structure aims to determine cable tension distribution to obtain the required reflector surface accuracy.\textsuperscript{7} A unit of a cable net structure is shown in Fig. 5, where node $i$ is connected to node $j$ by a cable. The equilibrium equation of node $i$ can be derived as follows:

\[
\sum_{j} F_{ij} \frac{x_{i} - x_{j}}{l_{ij}} = 0
\]

\[
\sum_{j} F_{ij} \frac{y_{i} - y_{j}}{l_{ij}} = 0
\]

\[
\sum_{j} F_{ij} \frac{z_{i} - z_{j}}{l_{ij}} = 0
\]

\[
\text{(3)}
\]
where $F_i$ and $l_i$ are the tension force and the length of cable $ij$, respectively.

Because of the assumption of nodes located in the required parabolic surface, the nodal locations and topology of the cable net structure can be determined. Thus, Eq. (3) can be rewritten in the form of matrix as follows:

$$M_{nk}xF = 0$$  

(4)

where $M$ is a coefficient matrix; $F = [F_1, F_2, \cdots, F_k]^T$ is the tension force vector of cables; $k$ is the total number of free nodes, and $r$ the total number of cables in the cable net structure.

As $3k < r$ for a cable net structure, Eq. (4) is statically indeterminate and the solution is not unique. Some restrictions need to be added, and the optimal solution can be obtained by an optimization method.

In the process of prestress design, the sum of squared deviations of cable tensions is chosen as an evaluation index of prestress distribution in a cable net structure. The mean value of $\frac{1}{k}t_i$ can be specified by designers to restrict the magnitude of tension forces in cables. The solution method of Eq. (12) is discussed in the following.

$$\begin{align*}
Q\Delta F &= Q(F - F^0) = b - QF^0 \\
\min \quad u &= \|F - F^0\|_2^2 \\
\text{s.t.} \quad F_i &> 0, \quad i = 1, 2, \cdots, r
\end{align*}$$  

(13)

The minimum norm solution of Eq. (13) is

$$\begin{align*}
\Delta F &= Q^*(b - QF^0) \\
F &= F^0 + Q^*(b - QF^0)
\end{align*}$$  

(14)

where $Q^*$ is the Moore–Penrose pseudo-inverse of $Q$.

Substituting Eq. (14) into Eq. (7), the optimal solution of tension forces in cables is

$$F = F^0 + Q^*(b - QF^0)$$  

(15)

It can be noted that $\hat{F}$ in Eq. (15) includes unknown $F^0_i$ and $F^0_s$. Thus, the optimization model of Eq. (11) must be further modified using design variables $F^0_f$ and $F^0_b$ as follows:

$$\begin{align*}
\text{Find} \quad F^0_f, F^0_b \\
\min \quad u &= \|F - F^0\|_2^2 \\
\text{s.t.} \quad F = F^0 + Q^*(b - QF^0) \\
F_i &> 0, \quad i = 1, 2, \cdots, r
\end{align*}$$  

(16)

The optimization model is solved by the pattern search algorithm. Then the solution of Eq. (16) is substituted into Eq. (15) to obtain the optimal solution of tension forces, $\hat{F}$, in cables.

3. Numerical simulations

In order to illustrate the proposed form-finding methods, two numerical examples are discussed here. One is a plane cable-beam structure, and the other is a spatial mesh reflector antenna.

3.1. Form-finding of a plane cable-beam structure

A plane cable-beam structure shown in Fig. 6 consists of 4 beams and 20 cables with 4 boundary cables and 16 surface cables. The node 10 is fixed and the others are free. The elastic modulus of each cable is 20 GPa and the cross sectional area is 3.14 mm². The beams are aluminum alloy tubes which form a square and the parameters are listed in Table 1. The diagonal cables of the square are divided into equal segments by nodes 1–9. The form-finding analysis of this structure is to determine the equilibrium configuration to satisfy these nodes remaining in the designed positions. The RMS error for displacements of nodes 1–9 is used to evaluate the form-finding accuracy.
The equilibrium tension forces of the plane cable net structure are obtained by the first step of the form-finding methods, and the RMS is $8.2 \times 10^{-5}$ mm. After the plane cable net structure is attached to the beam structure, the RMS changes to 3.6 mm.

The second step is conducted by adjusting tension forces of boundary cables from 1 to 4. The results of form-finding method I are shown in Figs. 7 and 8. The results of form-finding method II are shown in Figs. 9 and 10. It can be observed that the surface accuracy is warranted by the proposed form-finding methods. The acquired RMS error by form-finding method II is reduced sharply from 3.6 mm to 0.14 mm, which is lower than 0.33 mm RMS obtained by method I. This is because from the viewpoints of physics, the optimization problem of Eq. (1) in form-finding method I is harder than that of Eq. (2) in form-finding method II which is an efficient trade-off scheme. Furthermore, from the viewpoints of mathematics, form-finding method I is a multi-objective optimization problem, and form-finding method II is a single-objective optimization problem.

<table>
<thead>
<tr>
<th>Table 1 Material and geometrical parameters of beams.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Length (mm)</td>
</tr>
<tr>
<td>Inner diameter (mm)</td>
</tr>
<tr>
<td>Outer diameter (mm)</td>
</tr>
<tr>
<td>Elastic (GPa)</td>
</tr>
</tbody>
</table>

Fig. 6 A plane cable-beam structure and its composition.

Fig. 7 Iteration process of tension forces of boundary cables using form-finding method I.

Fig. 8 Iteration process of RMS using form-finding method I.

Fig. 9 Iteration process of tension forces of boundary cables using form-finding method II.

Fig. 10 Iteration process of RMS using form-finding method II.
3.2. Form-finding of a spatial mesh reflector antenna

An offset parabolic antenna with a spatial mesh reflector is shown in Fig. 1. The antenna specifications are as follows:

- Diameter of aperture: 10000 mm
- Focal length of front cable net: 5000 mm
- Focal length of back cable net: 30000 mm
- Offset distance: 6000 mm
- Number of surface cables $2r_s$: 1200 ($=600 \times 2$)
- Number of boundary cables $n$: 120 ($=60 \times 2$)
- Number of tension tie cables $r_v$: 211
- Number of free nodes $k$: 434 ($=217 \times 2$)
- Height: 2000 mm
- Type of facets: triangular
- Elastic modulus of cables: 20 GPa
- Cross sectional area of cables: 3.14 mm$^2$
- Material of truss member: CFRP
- Inner diameter of truss member: 24 mm
- Outer diameter of truss member: 25 mm
- Elastic modulus of truss member: 150 GPa

The mean value of tension forces in tension tie cables is specified as 5 N. The prestress distribution of the cable net structure found in the first step of the form-finding methods is shown in Table 2. The RMS error of the front cable net is 0.23 mm and its surface error is shown in Fig. 11.

After the spatial cable net structure is attached to the supporting truss, the surface error of the front cable net changes as shown in Fig. 12 with 9.15 mm RMS. The maximum nodal displacement is 16.12 mm. It can be observed that the RMS error becomes extremely poor due to the coupled deformation. Thus, the second step of the form-finding analysis must be carried out. Form-finding method II is adopted because of its effectiveness verified in the form-finding analysis of the plane cable-beam structure.

Fig. 13 shows the changes of the RMS error of the front cable net in the iteration process. It can be seen that the RMS error is lowered from 9.15 mm to 2.05 mm, as shown in Fig. 14. The statistics of the prestress distribution in cables are summarized in Table 3. It can be noted from Tables 2 and 3 that the prestress distribution of the front cable net before the second step is worse than that in the first step, while it is ameliorated after the second step.

<table>
<thead>
<tr>
<th>Item</th>
<th>Prestress values in cables (N)</th>
<th>Maximum/minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Minimum</td>
</tr>
<tr>
<td>Front cable net</td>
<td>71.00</td>
<td>18.32</td>
</tr>
<tr>
<td>Back cable net</td>
<td>315.12</td>
<td>99.41</td>
</tr>
<tr>
<td>Tension tie cables</td>
<td>6.20</td>
<td>4.23</td>
</tr>
</tbody>
</table>

Fig. 11 Surface error of the front cable net by the first step of form-finding analysis.

Fig. 12 Surface error of the front cable net considering the coupled deformation.

Fig. 13 Iteration process of RMS error of the front cable net using form-finding method II.

Fig. 14 RMS error of the front cable net after the second step of form-finding analysis.


4. Conclusions

(1) Two form-finding methods are proposed to address the cable prestress design problem of mesh reflector antennas considering the coupled deformation between the cable net structure and the supporting truss.

(2) Both form-finding methods can systematically determine the cable prestress distribution and an initial profile of the reflector, and render a desired deformed surface with a minimum surface error and warrant the cable tension distribution falling in a specified range.

(3) Some numerical examples are carried out and the results demonstrate that the proposed form-finding methods are effective not only for mesh reflector antennas but also for common cable-beam structures.

Acknowledgment

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References

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Table 3 Results of the prestress design.

<table>
<thead>
<tr>
<th>Item</th>
<th>Before the second step</th>
<th>After the second step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum (N)</td>
<td>Minimum (N)</td>
</tr>
<tr>
<td>Front cable net</td>
<td>66.85</td>
<td>4.23</td>
</tr>
<tr>
<td>Back cable net</td>
<td>271.30</td>
<td>75.86</td>
</tr>
<tr>
<td>Tension tie cables</td>
<td>4.78</td>
<td>2.18</td>
</tr>
<tr>
<td>Front cable net</td>
<td>80.34</td>
<td>9.20</td>
</tr>
<tr>
<td>Back cable net</td>
<td>275.33</td>
<td>84.68</td>
</tr>
<tr>
<td>Tension tie cables</td>
<td>6.78</td>
<td>3.26</td>
</tr>
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