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A new method to predict fatigue crack growth rate of materials based on average cyclic plasticity strain damage accumulation

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Abstract By introducing a fatigue blunting factor, the cyclic elasto-plastic Hutchinson-Rice-Rosengren (HRR) field near the crack tip under the cyclic loading is modified. And, an average damage per loading-cycle in the cyclic plastic deformation region is defined due to Manson-Coffin law. Then, according to the linear damage accumulation theory—Miner law, a new model for predicting the fatigue crack growth (FCG) of the opening mode crack based on the low cycle fatigue (LCF) damage is set up. The step length of crack propagation is assumed to be the size of cyclic plastic zone. It is clear that every parameter of the new model has clearly physical meaning which does not need any human debugging. Based on the LCF test data, the FCG predictions given by the new model are consistent with the FCG test results of Cr2Ni2MoV and X12CrMoWVNbN 10-1-1. What's more, referring to the relative researches, the good predictability of the new model is also proved on six kinds of materials.

1. Introduction

Recently, with the rapid development of science and technology, damage tolerance analysis has become more and more important. The fatigue crack growth (FCG), as a key indicator of fatigue and fracture properties evaluation, plays an important role in structure life design, failure analysis and process optimization. At present, experiments and numerical simulations are the main methods to obtain the fatigue crack growth properties (FCGP) of materials. Many researchers have studied FCGP under different situations such as constant and variable load amplitude, different load ratios, different service environments, etc. As is known to all, FCG is a function of stress intensity factor, and the relationship can be divided into three stages in the log-log coordinates, including crack initiation stage, stable growth stage and unstable growth stage. Based on a number of test data, Paris and Erdogan supplied an empirical model to describe the relationship between the crack growth da/dN and the stress intensity factor amplitude ΔK near crack tip for the stable growth stage[1]:

$$da/dN = C \cdot \Delta K^m \tag{1}$$

where C and m are material constant parameters. Then, Forman[2] and Priddle[3] proposed empirical models to describe the whole stage respectively, where the effect of load ratio was under consideration.

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According to many researches, the initiation and growth of a crack result from the cyclic plastic deformation near the crack tip. Therefore, to some extent, the fatigue crack growth behavior can be converted from the low cyclic fatigue properties. Shen et al.[4] has verified it by studying the internal relationship between material's $s-N$, $\varepsilon-N$ and $\mathrm{d}a/\mathrm{d}N - \Delta K$ properties. Oh and Nam[5] considered that the failure of the crack tip happened in a fatigue process zone, in which the stress amplitude reached the fatigue strength, and every step of crack advancement was calculate as $\mathrm{d}a/\mathrm{d}N = \varepsilon_\mathrm{p} D_\mathrm{p}$. The parameter ε_p is the mean cyclic plastic strain, and D_p equals the fatigue process zone size. Therefore the relationship between the fatigue damage and the fracture process as well as the cyclic plastic strain near the crack tip was established. Castro[6] considered the material near the crack tip as a series of represent elements (like a micro low cyclic fatigue specimen), then a whole stage empirical predicting model was proposed based on low cyclic fatigue damage (LCFD) accumulation principle

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A[\Delta K - \Delta K_\mathrm{th}(R)]^m \left\{ \frac{K_\mathrm{c}}{K_\mathrm{c} - [\Delta K/(1-R)]} \right\} \tag{2}$$

where $m = 2$, A is a constant parameter related to the low cyclic fatigue properties, ΔK_th (R) the threshold value of stress intensity amplitude varying with loading ratio R, and K_c the fracture toughness of material. Liu and Iino[7] combined the Miner accumulative damage concept and the Manson-Coffin strain fatigue relationship, then, got a similar fatigue crack growth model with $m = 2$. However, lots of experimental results show that m is not constant and would vary with different materials. So, there are some limits to these fatigue crack growth models from Refs.[6,7]. Majumdar and Morrow[8] modified Liu's conclusion[7] by referring to a micro structure parameter ρ^* and a variable parameter m was predicted. But they ignored the damage proportion of these materials next to the crack tip, so the prediction was conservative. Meanwhile, Huang et al.[9] successfully predicted the fatigue crack growth behavior based on the low cyclic fatigue properties with finite element analysis (FEA). According to the above research backgrounds, this paper proposes a fatigue crack growth theoretical prediction model—FCG-LCFD model based on the low cyclic fatigue accumulative damage of all these materials along the crack growth direction in the cyclic plastic zone.

2. FCG-LCFD model for opening mode crack

2.1. Description of cyclic stress-strain amplitude near crack tip

According to the fine line in Fig. 1, the origin of coordinate was set at the crack tip without deformation. Based on the Hutchinson-Rice-Rosengren (HRR) field, the stress-strain field near the crack tip was modified by Schwalbe.[10]

$$\begin{cases} \Delta\varepsilon_\mathrm{p}(r) = \dfrac{2\sigma_\mathrm{yc}}{E}\left(\dfrac{\mathrm{PZ_c}}{r}\right)^{1/(1+n)} \\ \Delta\sigma(r) = 2k\left(\dfrac{\Delta\varepsilon_\mathrm{p}(r)}{2}\right)^n \end{cases} \tag{3}$$

where E is elastic modulus, σ_yc yield stress, k cyclic hardening coefficient, n cyclic hardening exponent, r the distance of a node from the crack tip, $\triangle\varepsilon_\mathrm{p}$ the plastic strain amplitude, $\triangle\sigma$ the stress amplitude and $\mathrm{PZ_c}$ the cyclic plastic zone size that can be calculated as follows.

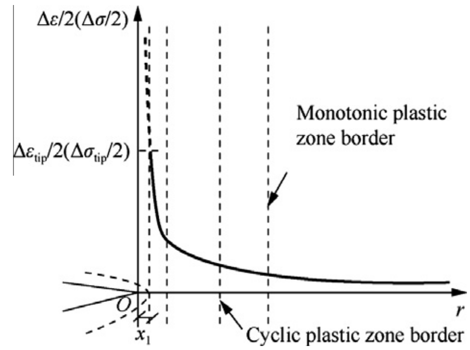

Fig. 1 Distribution of stress and strain amplitude along growth direction at fatigue crack tip.

$$\mathrm{PZ_c} = \frac{1}{4\pi\kappa^2(1+n)}\left(\frac{\Delta K}{\sigma_\mathrm{yc}}\right)^2 \tag{4}$$

where

$$\kappa = \begin{cases} 1 & \text{Plane stress} \\ \dfrac{1}{1-2v} & \text{Plane strain} \end{cases}$$

where v is the Poisson ratio.

In Eq. (3), an ideal sharp crack hypothesis is made, so the stress-strain field is mathematically singular. It means that the plastic strain amplitude will be infinite when r is close to zero. In fact, according to a lot of FEA, the curvature of the crack tip is non-zero, and the plastic strain of the crack tip is finite. In order to describe this physical phenomenon, a fatigue blunting factor x_1 is assumed in this paper. Then, by transferring the coordinate system with x_1 into the crack, the cyclic plastic strain amplitude can be described as

$$\Delta\varepsilon_\mathrm{p}(r + x_1) = \frac{2\sigma_\mathrm{yc}}{E}\left(\frac{\mathrm{PZ_c}}{r + x_1}\right)^{1/(1+n)} \tag{5}$$

So the final cyclic stress and strain fields near the crack tip of the opening crack model can be described as the heavy line in Fig. 1 (in which $\triangle\varepsilon_\mathrm{tip}$ and $\triangle\sigma_\mathrm{tip}$ are the strain and stress amplitudes on the crack tip), and the calculation of x_1 will be discussed in Section 2.2.

2.2. Definition of fatigue plastic strain damage

The existing researches show that the material near the crack tip can be considered as a series of fatigue elements which are under monotonic tensile cyclic loading.[10] The failure of these fatigue elements corresponds to the initiation and expansion of the fatigue crack. According to the fatigue theory, the relationship between fatigue life N_f and the cyclic plastic strain amplitude can be described as

$$\frac{\Delta\varepsilon_\mathrm{p}}{2} = \varepsilon_\mathrm{f}'(2N_\mathrm{f})^c \tag{6}$$

where ε_f' and c are plastic hardening coefficient and plastic hardening exponent, respectively.

Combining Eqs. (5) and (6) and defining a unit damage $D = 1/N_\mathrm{f}$, the distribution of the average damage of the nodes along the crack growth direction in the cyclic plastic zone can be described as

$$D(r+x_1) = 2\left(\frac{\sigma_{yc}}{E\varepsilon_f'}\right)^{-1/c}\left(\frac{PZ_c}{r+x_1}\right)^{-1/(1+n)c} \quad 0 < r \leqslant PZ_c - x_1 \tag{7}$$

According to the fatigue striations phenomenon of the fatigue fracture image, assume that each step of the crack advancement size equals to the cyclic plastic zone size $PZ_c - x_1$ along the growth direction. The plastic strain amplitude is much bigger than the elastic strain amplitude in the cyclic plastic zone, so the damage of elastic strain can be ignored. Then, the sum of the nodes' damages during one cycle is expressed as

$$\int_0^{PZ_c - x_1} D(r+x_1)\mathrm{d}r$$

And a unit average damage parameter is defined as

$$\overline{D} = \frac{\int_0^{PZ_c - x_1} D(r+x_1)\mathrm{d}r}{PZ_c - x_1}$$

According to the Miner accumulative damage theory, when $\overline{D} = 1$, the crack will grow one step. Therefore, the life and the rate of each step can be calculated as follows:

$$\begin{cases} N_{fi} = \dfrac{1}{\overline{D}_i} = \dfrac{PZ_{ci} - x_i}{\int_0^{PZ_{ci}-x_i} D_i(r+x_1)\mathrm{d}r} \\ \left(\dfrac{\mathrm{d}a}{\mathrm{d}N}\right)_i = \dfrac{PZ_{ci} - x_1}{N_{fi}} = \displaystyle\int_0^{PZ_{ci}-x_i} D_i(r+x_1)\mathrm{d}r \end{cases} \tag{8}$$

From Eq. (8), it is found that the fatigue crack growth equals the sum of the nodes' damages in the cyclic plastic zone. Combining Eqs. (7) and (8), a new FCG prediction model based on the mean plastic strain in the cyclic plastic zone can be given.

$$\left(\frac{\mathrm{d}a}{\mathrm{d}N}\right)_i = 2\left(\frac{E\varepsilon'}{\sigma_{yc}}\right)PZ_{ci}\cdot\frac{c+cn}{c+cn+1}\left[1-\left(\frac{x_1}{PZ_{ci}}\right)^{1+1/(c+cn)}\right] \tag{9}$$

As is known to all, at the initiation stage of the FCG curve where the stress intensity factor amplitude $\Delta K = \Delta K_{th}$, no crack growth occurs approximately. So, the fatigue blunting factor x_1 can be calculated by

$$x_1 = PZ_{cth} = \frac{1}{4\pi\kappa^2(1+n)}\left(\frac{\Delta K_{th}}{\sigma_{yc}}\right)^2 \tag{10}$$

From Eq. (10), the fatigue blunting factor equals the cyclic plastic zone size corresponding to ΔK_{th}. Then a zero range of cyclic plastic strain damage is obtained from Eqs. (10) and (7) nearby the crack initiation, which means there is no damage accumulated under this condition.

3. Predictability of FCG-LCFD model

3.1. LCF and FCG experiments

Two rotor materials Cr2Ni2MoV and X12CrMoWVNbN 10-1-1 are employed to the verification tests, where the basic mechanics properties of materials are showed in Table 1. According to the national test standard, the LCF specimen is of smooth straight shape, and the diameter is 8 mm. Compact tension (CT) specimens are used for the FCG test, and the shape of this specimen is shown in Fig. 2 (in which W is the width of the specimen, and B the thickness of the specimen). The experiment equipment contains MTS809 25KN, 632.61C strain extensometer and MTS632.02F-20. The scheme of FCG test is shown in Fig. 3.

The materials' LCF properties are also shown in Table 1. The load ratio of FCG test is 0.1, the test data are analyzed

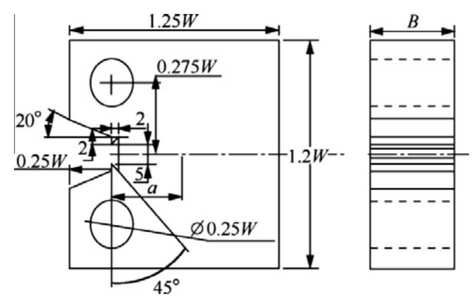

Fig. 2 Geometry of standard compact tension specimen.

Fig. 3 Scheme of fatigue crack growth test.

Table 1 Common mechanics properties and LCF properties of rotor materials.

Material	E (GPa)	σ_{yc} (MPa)	n	ε_f'	c	R	x_1 (μm)	ΔK_{th} (MPa·m)$^{1/2}$
Cr2Ni2MoV	214	853	0.0595	1.1005	−0.679	0.1	0.1681	4.2
X12CrMoWVNbN 10-1-1	226	765	0.0676	2.3989	−0.847	0.1	0.4739	4.5

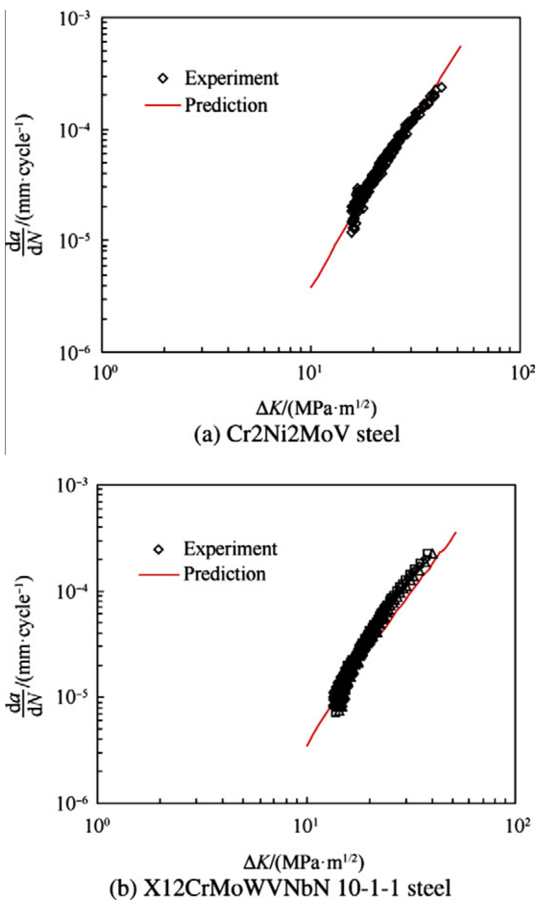

Fig. 4 Comparison between FCG results of theoretical model prediction and test for two rotor materials at room temperature.

according to test standard GBT6398-2000 and the FCG curves are shown in Fig. 4.

3.2. *Verification of the feasibility of new model*

Using the data shown in Table 1, the FCG behavior for the two rotor materials are successfully predicted by the new model, and the comparison between analytical results and test results is shown in Fig. 4.

According to Fig. 4, the test results are consistent for the same material, so the test data can reflect the physical nature of the material. Meanwhile, the predicted results from the new model are consistent with the experiment results. Therefore, the new model is practical and has high precision.

3.3. *Verification of the universality of new model*

Using the data from the Ref.[6] and Refs.[11–14] shown in Table 2, the FCG results of different materials are predicted by the new model as shown in Fig. 5. According to Fig. 5, the predicted result of each material's FCG behavior is consistent with the experimental result. So the new model proposed in this paper has a good universality.

4. Conclusions and outlook

(1) By introducing a fatigue blunting factor x_1 which equals the cyclic plastic zone size corresponding to ΔK_{th}, the cyclic HRR field is modified to describe the crack tip zone more properly. Then, combined with the Miner damage accumulative rule, a new FCG prediction model is obtained on the basis of the material plastic damage in the cyclic plastic zone along the growth direction (see Eq. (9)). All the parameters in the new model have definite meanings without any human debugging.

(2) By means of experiments, the LCF and FCG data of two rotor materials Cr2Ni2MoV and X12CrMoWVNbN 10-1-1 are obtained. And the predictions of the FCG behavior of the corresponding materials through the new model are consistent with the experimental results. Meanwhile, the correctness and applicability of the new model are verified with six kinds of different materials given by some references. Therefore, to some extent, the new model is suitable for the FCG prediction of the opening mode crack in engineering practice.

(3) However, the new model ignores the elastic damage proportion of the material. In order to obtain a more natural prediction model, the damage resulting from the stress should be introduced to increase the accuracy. Also, the new model ignores the distribution of the cyclic stress and strain amplitude along the thickness direction of the specimen which must be considered in the three-dimensional fracture problem. Therefore, when it is expanded to the three-dimensional situation, the equivalent fatigue element should be a volume.

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Table 2 Mechanics property parameters for FCG-LCFD model.

Material	E (GPa)	σ_{yc} (MPa)	n	ε_f'	c	R	x_1 (μm)	ΔK_{th} (MPa·m)$^{1/2}$
TA12[11]	113	903	0.1224	0.29	−0.662	0.1	1.122	8
1020[6]	205	270	0.2111	0.25	−0.54	0.1	27.920	11.6
Al 7075-T6[12]	71	469	0.0865	0.19	−0.52	0.5	0.1328	1.45
St-4340[12]	200	889	0.1407	0.64	−0.636	0.7	0.09356	2.26
A533-B1[13]	200	345	0.1635	0.32	−0.52	0.1	7.310	7.7
304[14]	197	285	0.4000	0.106	−0.373	0.1	3.1923	4

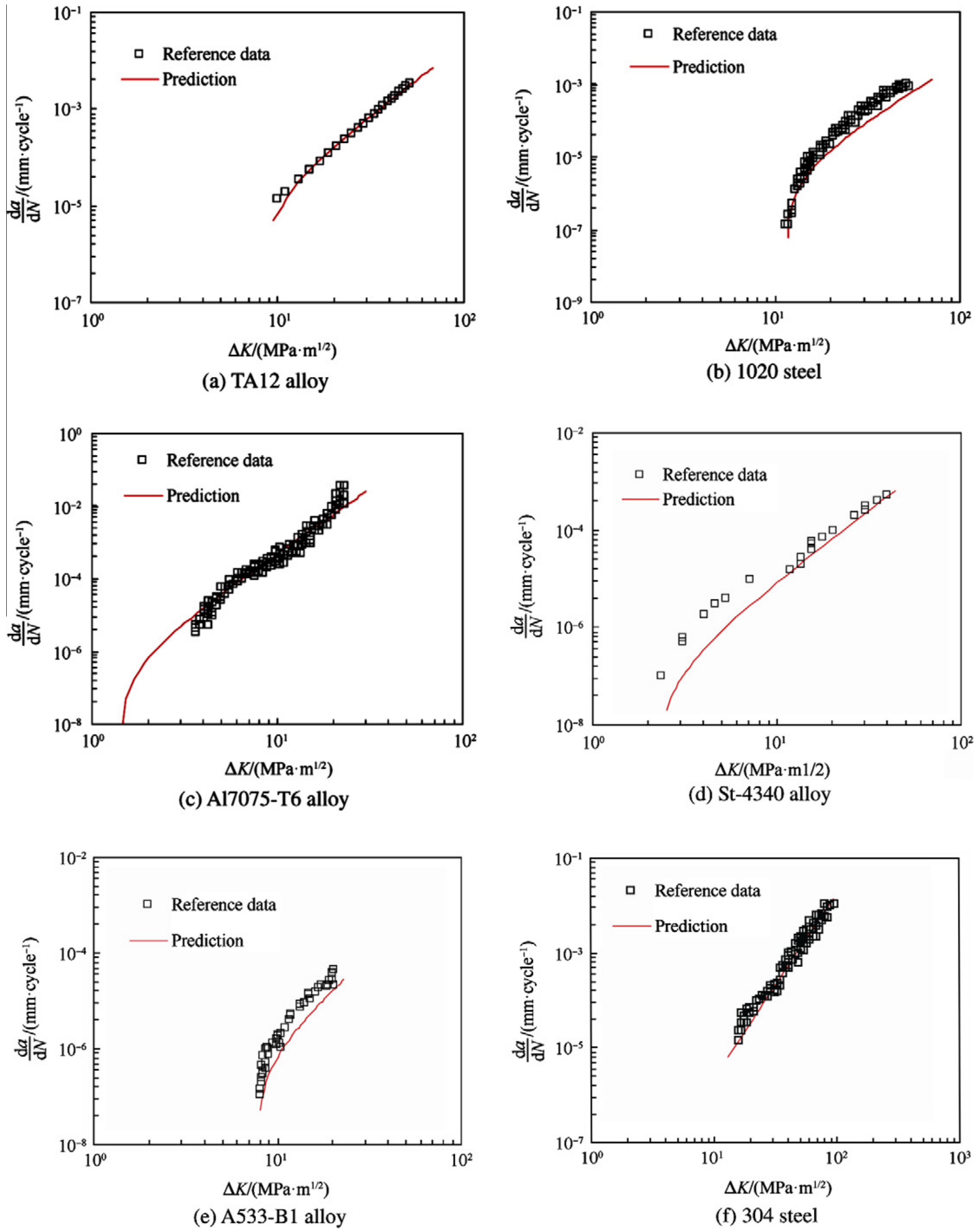

Fig. 5 Comparison between FCG results of theoretical model prediction and test for different materials at room temperature.

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