# Application of fuzzy soft set in decision making problems based on grey theory 

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#### Abstract

There are many uncertain problems in practical production and life which need decisions made with soft sets and fuzzy soft sets. However, the basis of evaluation of the decision method is single and simple, the same decision problem can obtain different results from using a different evaluation basis. In this paper, in order to obtain the right result, we discuss fuzzy soft set decision problems. A new algorithm based on grey relational analysis is presented. The evaluation bases of the new algorithm are multiple. There is more information in a decision result based on multiple evaluation bases, which is more easily accepted and logical to one's thinking. For the two cases examined, the results show that the new algorithm is efficient for solving decision problems.


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## 1. Introduction

Molodtsov [1] proposed a completely new approach for modeling vagueness and uncertainty soft set theory. Most of its applications have already been demonstrated in [2]. Fuzzy soft set theory has been proposed [3] and has potential applications.

In recent years, soft set and fuzzy soft set theories have been proved to be useful in many different fields, such as decision making [4-14], data analysis [15], forecasting [16], simulation [17], evaluation of sound quality [18] and rule mining [19]. The study of hybrid models combining soft sets or fuzzy soft sets with other mathematical structures and new operations are emerging as an active research topic of soft set theory [20-39]. Maji et al. considered the reduct soft set with the help of rough set approach [4] and discussed soft set theory [20]. Roy et al. discussed score value as the evaluation basis to make decisions in fuzzy soft sets [5]. Zhi Kong et al. analyzed two decision evaluation bases, choice value and score value, and used a counter example to discuss the two methods [8], Naim Çaǧman et al. presented soft matrix theory and uni-int decision making [6,7]. Yuncheng Jiang et al. introduced two methods, semantic decision making using ontology and intuitionistic fuzzy soft sets decision making $[9,10]$, and extended soft sets with description logics, discussing interval-valued intuitionistic fuzzy soft set properties [27,28]. Feng Feng et al. presented an adjustable approach by means of level soft sets and interval-value fuzzy soft sets [11,12], and soft semirings and soft rough sets [22-24]. Xibei Yan et al. introduced the concept of interval-valued fuzzy soft sets and discussed its operations [13]. Ke Gong et al. discussed the bijective soft set and its operations [14]. Hacı Aktaş et al. discussed soft sets and soft groups [21]. Hailong Yang presented kernels and closures of soft set relations and soft set relation mappings [25]. Pinaki Majumdar et al. introduced generalized fuzzy soft sets [26]. Young Bae Jun et al. and Jianming Zhan et al. discussed algebras soft sets [29-33]. Wei Xu et al. presented vague soft sets and their properties [34]. Muhammad Irfan Ali et al. discussed some new operations in soft set theory and approximation space associated with each parameter in a soft set [35,36]. Babitha et al. presented soft set relations and functions [37]. Ummahan Acar et al. presented soft sets and soft rings [38]. Zhi Xiao et al. introduced exclusive soft sets [39]. Chen et al. [40] presented a definition of parameterization

[^0]reduction in soft set theory, and compared this definition to the related concept of attributes reduction in rough set theory. Kong et al. [41] introduced the definition of normal parameter reduction to overcome adding parameters and suboptimal choice problem and an algorithm has been given.

According to the previous results for decision making in fuzzy soft sets, researchers focus on a direction as the evaluation basis, such as choice value (the sum of all membership grades for one object), score value (the number of parameters of relatively larger membership value of an object), or other evaluation bases. The evaluation methods are completely different. For the same problem, the results may be inconsistent. Decision making problems may use different factors, so it is not easy to judge which result is right. Furthermore, we don't know which method should be chosen for making decisions. Two examples (a) and (b) are listed to make decisions depending on two different evaluation bases, choice values and score values. There are 6 objects $\left\{o_{1}, \ldots, o_{6}\right\}$ and we compute the choice values and score values of them, respectively. (a): choice value sequence $c=\{1.0,1.0,1.1,1.2,2.1,2.2\}$ and score value sequence $s=\{-13,-5,-6,2,16,6\}$. For problem (a), the result is $o_{6}$ with the choice value method, while the result is $o_{5}$ with the score value method. We don't know which answer is right, because the two answers are analyzed from two different factors. For objects $0_{5}$ and $o_{6}$, we know $c_{5}=2.1$, $c_{6}=2.2, s_{5}=16, s_{6}=6$. So the values $c_{5}=2.1$ and $c_{6}=2.2$ are similar, while the values $s_{5}=16$ and $s_{6}=6$ are very different. Therefore, synthesizing two group values of $0_{5}$ and $o_{6}$, we generally choose $o_{5}$ as the answer because of the suboptimal value $c_{5}=2.1$ and optimal value $s_{6}=6$.(b): choice value sequence $c=\{1.4,1.3,1.3,1.8,1.9,2.9\}$ and score value sequence $s=\{-1,-7,-4,-5,8,7\}$. For problem (b), the result is $o_{6}$ with the choice value method, while the result is $0_{5}$ with the score value method. Because of the different evaluation bases, the results are inconsistent. Considering the choice value and score value together, for problem (b), $0_{6}$ is better.

For the above analysis, different results are obtained from using a different single evaluation basis. However, the single evaluation basis is simple, one-sided and has a shortage of information. Only one evaluation basis is considered and an extreme decision result may be obtained. To improve the decision quality many evaluation bases are comprehensively analyzed. There is more information in the decision result of multiple evaluation bases, which is more general. So we present a new algorithm to solve this problem based on grey theory. Many evaluation bases are considered in the new algorithm and a comprehensive decision result is obtained.

The key to this problem is how to operate two or more different methods together. Grey theory can help us to solve this problem. The grey relational analysis method is an important method in grey system theory, which was initiated by Deng in 1982 [42]. It is a quantitative analysis to explore the similarity and dissimilarity among factors in developing dynamic processes. The theory proposes a dependence to measure the correlation degree of factors; the more similarities develop, the more factors correlate [43]. Grey relational analysis of grey theory is a well known approach that is utilized for generalizing estimates under small samples and uncertain conditions. We apply grey relational analysis of grey theory to combine multiple evaluation methods into a single evaluation value. Finally, we use the evaluation value to make the decision.

In this paper, we take choice value evaluation and score value evaluation as an example. Here we combine the two evaluation methods to make decisions in a fuzzy soft set based on grey relational analysis and compute the correlation degree for every object. Finally, we use the relational grade as the evaluation basis. In this paper, a new algorithm is presented based on evaluation bases. Applying the new algorithm, the above two problems can be solved well.

The new algorithm in this paper is suitable for soft set and fuzzy soft set decision making problems and unconstrained for evaluation methods. Whatever kind of evaluation methods we choose, as long as every evaluation sequence is given, such as choice value sequence, score value sequence, or other sequences, then we can make the decision using this new algorithm. The evaluation sequence associates to a kind of evaluation method, so this new algorithm is a hybrid evaluation method for comprehensive evaluation. Also, this new algorithm is objective, numerical and accurate, and we can compare the value to obtain the optimal choice, suboptimal choice, and so on.

The paper is organized as follows. In Section 2, we provide some basic definitions in soft sets and fuzzy soft sets. In Section 3, we discuss the present two methods in decision making problems in fuzzy soft sets, and two examples are listed to illustrate the two problems. In Section 4, a new algorithm is presented to solve the two problems. In Section 5, an example is analyzed using the uni-int decision making method and the new algorithm. In Section 6 , we end this paper with some conclusions.

## 2. Definitions and notation

In this section, we review useful notions of soft sets and fuzzy soft sets. Let $U$ be an initial universe set and let $E$ be a set of parameters.

Definition 2.1 (See [2]). A pair ( $F, E$ ) is called a soft set (over $U$ ) if and only if $F$ is a mapping of $E$ into the set of all subsets of the set $U$, i.e., $F: E \rightarrow P(U)$, where $P(U)$ is the power set of $U$, and $E$ is a set of parameters.

The soft set is a parameterized family of subsets of the set $U$. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of $\varepsilon$-elements of the soft set $(F, E)$, or as the $\varepsilon$-approximate elements of the soft set. As an illustration, some examples such as fuzzy sets and topological spaces were listed in [2]. The way of setting (or describing) any object in soft set theory differs in principle from the way it is used in classical mathematics. In classical mathematics, a mathematical

Table 1
Soft set table.

| $U$ | $e_{1}$ | $e_{2}$ | $e_{3}$ | $e_{4}$ | Choice value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $h_{1}$ | 1 | 1 | 1 | 0 | 3 |
| $h_{2}$ | 0 | 1 | 0 | 1 | 2 |
| $h_{3}$ | 1 | 0 | 1 | 1 | 3 |
| $h_{4}$ | 1 | 0 | 0 | 1 | 2 |

Table 2
Fuzzy soft set table.

| $U$ | $e_{1}$ | $\ldots$ | $e_{m}$ |
| :--- | :--- | :--- | :--- |
| $0_{1}$ | $0_{11}$ | $\ldots$ | $o_{1 m}$ |
| $\ldots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $o_{n}$ | $o_{n 1}$ | $\cdots$ | $o_{n m}$ |

model of an object is usually constructed for which it is too complicated to find the exact solution. Therefore the notion of an approximate solution has been introduced in soft set theory, whose approach is the opposite of classical mathematics.

The absence of any restrictions on the approximate description in soft set theory makes it very convenient and easy to apply in practice. We can use any parameterization with the help of words and sentences, real numbers, functions, mappings, and so on.

To illustrate this idea, let us consider the following example.
Example 2.1. Let universe $U=\left\{h_{1}, h_{2}, h_{3}, h_{4}\right\}$ be a set of houses, $E=\left\{e_{1}, e_{2}, e_{3}, e_{4}\right\}$ be a set of statuses of houses, which stand for the parameters "beautiful", "cheap", "in green surroundings", and "in good location" respectively. Consider the mapping $F$ to be a mapping of $E$ into the set of all subsets of the set $U$. Now consider a soft set $(F, E)$ that describes the "attractiveness of houses for purchase". According to the data collected, the soft set $(F, E)$ is given by

$$
\{F, E\}=\left\{\left(e_{1},\left\{h_{1}, h_{3}, h_{4}\right\}\right),\left(e_{2},\left\{h_{1}, h_{2}\right\}\right),\left(e_{3},\left\{h_{1}, h_{3}\right\}\right),\left(e_{4},\left\{h_{2}, h_{3}, h_{4}\right\}\right)\right\}
$$

where $F\left(e_{1}\right)=\left\{h_{1}, h_{3}, h_{4}\right\}, F\left(e_{2}\right)=\left\{h_{1}, h_{2}\right\}, F\left(e_{3}\right)=\left\{h_{1}, h_{3}\right\}$, and $F\left(e_{4}\right)=\left\{h_{2}, h_{3}, h_{4}\right\}$. In order to store a soft set in computer, a two-dimensional table is used to represent the soft set $(F, E)$. Table 1 is the table form of the soft set $(F, E)$. If $h_{i} \in F\left(e_{j}\right)$, then $h_{i j}=1$, otherwise $h_{i j}=0$, where $h_{i j}$ are the entries.

Suppose that Mr. X is interested in buying a house on the basis of his choice parameters "beautiful", "cheap", "in green surroundings", etc. According to the choice value, Mr. X can choose $h_{1}$ or $h_{3}$.

Definition 2.2 (See [5]). Let $\Psi(U)$ denote the set of all fuzzy sets of $U, E$ be a set of parameters. Let $A_{i} \subset E$. A pair $\left(F_{i}, A_{i}\right)$ is called a fuzzy soft set over $U$, where $F_{i}$ is the mapping given by $F_{i}: A_{i} \rightarrow \Psi(U)$.

Definition 2.3 (See [5]). For two fuzzy soft sets $(F, A)$ and $(G, B)$ over a common universe $U,(F, A)$ is a fuzzy soft subset of $(G, B)$ if (i) $A \subset B$, and (ii) $\forall \varepsilon \in A, F(\varepsilon)$ is a fuzzy subset of $G(\varepsilon)$. We write $(F, A) \widetilde{C}(G, B)$. $(F, A)$ is said to be a fuzzy soft super set of $(G, B)$, if $(G, B)$ is a fuzzy soft subset of $(F, A)$. We denote it by $(F, A) \widetilde{\partial}(G, B)$.

The tabular representation of a fuzzy soft set is given in Table 2. $o_{i j}$ is the membership degree of $o_{i}$ in a parameter set $\left\{e_{j}\right\}, o_{i j} \in[0,1]$. At present there are many methods to make decisions by fuzzy soft set theory. In this paper, two main methods are introduced, one uses choice value as the evaluation basis, the other uses score value as the evaluation basis. In Section 3, the two methods are described in detail.

Definition 2.4 (See [2]). Assume that we have a binary operation, denoted by $*$, for subsets of the set $U$. Let $(F, A)$ and ( $G, B$ ) be soft sets over $U$. Then, the operation $*$ for soft sets is defined in the following way: $(F, A) *(G, B)=(H, A \times B)$, where $H(\alpha, \beta)=F(\alpha) * G(\beta), \alpha \in A, \beta \in B$, and $A \times B$ is the Cartesian product of the sets $A$ and $B$.

## 3. Fuzzy soft set in decision making problems

At present there are two methods to handling decision making problems in fuzzy soft sets. In [5] the decision depends on the score $s_{i}$, where $s_{i}$ signifies the number of parameters of relatively larger membership value of object $o_{i}$. In [8] the decision depends on choice value $c_{i}$, where $c_{i}$ signifies the sum of the membership values of all parameters of object $o_{i}$. The decision that results is not always the same according to the two methods, the score value and choice value. However, sometimes the same decision may be obtained. The reason is that, for an object, possibly the more the number of parameters of the relatively larger membership values is, the greater the probability of the membership sum values of all parameters is. The algorithm for decision making problems in fuzzy soft sets [5] is as follows.

Algorithm [5].

1. Input the fuzzy-soft-set $(F, A),(G, B)$ and $(H, C)$.
2. Input the parameter set $P$ as observed by the observer.

Table 3
Fuzzy soft set table of case 1.

| $U$ | $e_{1}$ | $e_{2}$ | $e_{3}$ | $e_{4}$ | $e_{5}$ | Choice value |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| $o_{1}$ | 0.1 | 0.1 | 0.1 | 0.1 | 0.6 | $c_{1}=1.0$ |
| $o_{2}$ | 0.3 | 0.2 | 0.1 | 0.2 | 0.2 | $c_{2}=1.0$ |
| $o_{3}$ | 0.1 | 0.3 | 0.1 | 0.1 | 0.5 | $c_{3}=1.1$ |
| $o_{4}$ | 0.3 | 0.2 | 0.2 | 0.2 | 0.3 | $c_{4}=1.2$ |
| $o_{5}$ | 0.1 | 0.3 | 0.4 | 0.4 | 0.9 | $c_{5}=2.1$ |
| $o_{6}$ | 0.9 | 0.2 | 0.1 | 0.2 | 0.8 | $c_{6}=2.2$ |

Table 4
Comparison table of case 1.

|  | $o_{1}$ | $o_{2}$ | $o_{3}$ | $o_{4}$ | $0_{5}$ | $o_{6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $o_{1}$ | 5 | 2 | 4 | 1 | 1 | 1 |
| $o_{2}$ | 4 | 5 | 3 | 3 | 1 | 3 |
| $o_{3}$ | 4 | 3 | 5 | 2 | 2 | 2 |
| $o_{4}$ | 4 | 5 | 3 | 5 | 1 | 3 |
| $0_{5}$ | 5 | 4 | 5 | 4 | 5 | 4 |
| $o_{6}$ | 5 | 5 | 4 | 4 | 1 | 5 |

Table 5
Score table of case 1.

|  | Row sum | Column sum | Score value |
| :--- | :--- | :--- | :---: |
| $o_{1}$ | 14 | 27 | -13 |
| $o_{2}$ | 19 | 24 | -5 |
| $o_{3}$ | 18 | 24 | -6 |
| $o_{4}$ | 21 | 19 | 2 |
| $0_{5}$ | 27 | 11 | 16 |
| $o_{6}$ | 24 | 18 | 6 |

3. Compute the corresponding resultant-fuzzy-soft-set $(S, P)$ from the fuzzy soft sets $(F, A),(G, B)$ and $(H, C)$ and place it in tabular form.
4. Construct the Comparison table of the fuzzy-soft-set $(S, P)$ and compute $r_{i}$ and $t_{i}$ for $o_{i}$, $\forall i$.
5. Compute the score of $o_{i}, \forall i$.
6. The decision is $S_{k}$ if $S_{k}=\max _{i} S_{i}$.
7. If $k$ has more than one value then any one of $o_{k}$ may be chosen.

Algorithm [8].
From Step 4 the algorithm is revised as below: $c_{i j}$ and $r_{i}$ should be redesigned as

$$
\begin{aligned}
c_{i j} & =\sum_{k=1}^{m}\left(f_{i k}-f_{j k}\right) \\
r_{i} & =\sum_{j=1}^{m} c_{i j}
\end{aligned}
$$

where $f_{i k}$ is the membership value of object $o_{i}$ for the $k^{t h}$ parameter, $m$ is the number of parameters. Step 5: the decision is $k$ if $r_{k}=\max _{i} r_{i}$.

The above two methods in decision making problems are considered from different aspects. Consider the following two cases. Let $U=\left\{o_{1}, o_{2}, \ldots, o_{n}\right\}$ be the object set, $C=\left\{c_{1}, c_{2}, \ldots, c_{n}\right\}$ be the choice value sequence, $S=\left\{s_{1}, s_{2}, \ldots, s_{n}\right\}$ be the score sequence. Suppose $c_{i}$ is the optimal value, $c_{j}$ is the suboptimal value in the choice value sequence; $s_{j}$ is the optimal value, $s_{i}$ is the suboptimal value in the score sequence. Case 1 : if the choice values of two objects are almost equal $\left(c_{i}>c_{j}\right)$, but the scores differ widely $\left(s_{j}>s_{i}\right)$, in general, one usually chooses $o_{j}$ as the optimal object, not $o_{i}$. Case 2 : if the scores of two objects are almost equal, but the choice values differ widely, in general, one usually chooses $o_{i}$ as the optimal object, not $o_{j}$.

However, when we want to make a decision based on choice value and score, which one is the optimal object? In this section, the problem is discussed. There are two examples in the following.

Let $U=\left\{o_{1}, o_{2}, o_{3}, o_{4}, o_{5}, o_{6}\right\}$ be the set of objects. The parameter set $E=\left\{e_{1}, e_{2}, e_{3}, e_{4}, e_{5}\right\}$. The tabular representation of the fuzzy soft set and corresponding choice values of objects are as follows.

Example 3.1. See Table 3.
The choice value is $c_{i}=\sum_{j=1}^{5} o_{i j}$. The Comparison table of the above fuzzy-soft-set is given in Table 4.

Table 6
Fuzzy soft set table of case 2.

| $U$ | $e_{1}$ | $e_{2}$ | $e_{3}$ | $e_{4}$ | $e_{5}$ | Choice value |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| $o_{1}$ | 0.2 | 0.3 | 0.2 | 0.3 | 0.4 | $c_{1}=1.4$ |
| $o_{2}$ | 0.6 | 0.2 | 0.3 | 0.1 | 0.1 | $c_{2}=1.3$ |
| $o_{3}$ | 0.1 | 0.2 | 0.4 | 0.2 | 0.4 | $c_{3}=1.3$ |
| $o_{4}$ | 0.6 | 0.1 | 0.1 | 0.9 | 0.1 | $c_{4}=1.8$ |
| $o_{5}$ | 0.5 | 0.3 | 0.4 | 0.3 | 0.4 | $c_{5}=1.9$ |
| $o_{6}$ | 0.1 | 0.9 | 0.9 | 0.9 | 0.1 | $c_{6}=2.9$ |

Table 7
Comparison table of case 2.

|  | $o_{1}$ | $o_{2}$ | $o_{3}$ | $o_{4}$ | $o_{5}$ | $o_{6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $o_{1}$ | 5 | 3 | 4 | 3 | 3 | 2 |
| $o_{2}$ | 2 | 5 | 2 | 4 | 1 | 2 |
| $o_{3}$ | 2 | 4 | 5 | 3 | 2 | 2 |
| $o_{4}$ | 2 | 3 | 2 | 5 | 2 | 3 |
| $0_{5}$ | 5 | 4 | 5 | 3 | 5 | 2 |
| $o_{6}$ | 3 | 4 | 4 | 4 | 3 | 5 |

Table 8
Score table of case 2.

|  | Row sum | Column sum | Score value |
| :--- | :--- | :--- | :---: |
| $o_{1}$ | 20 | 19 | 1 |
| $o_{2}$ | 16 | 23 | -7 |
| $o_{3}$ | 18 | 22 | -4 |
| $o_{4}$ | 17 | 22 | -5 |
| $0_{5}$ | 24 | 16 | 8 |
| $o_{6}$ | 23 | 16 | 7 |

The Comparison table is a square table in which the number of rows and number of columns are equal, the rows and columns both are labelled by the object names $o_{1}, o_{2}, \ldots, o_{6}$ of the universe, and the entries are $c_{i j} i, j=1,2, \ldots, 6$. $c_{i j}$ indicates a numerical measure, which is an integer and $o_{i}$ dominates $o_{j}$ in $c_{i j}$ number of parameters out of $k$ parameter [5]. Next we compute the row sum, column sum, and the score for each $o_{i}$ as shown in Table 5.

The row sum of an object $o_{i}$ is denoted by $r_{i}$ and is calculated by using the formula $r_{i}=\sum_{j=1}^{6} c_{i j}$. $r_{i}$ indicates the total number of parameters in which $o_{i}$ dominates all the numbers of $U$. The column sum of an object $o_{j}$ is denoted by $t_{i}$ and may be computed as $t_{i}=\sum_{i=1}^{6} c_{i j}$. The score value $s_{i}=r_{i}-t_{i}[5]$.

Example 3.2. See Table 6.
The Comparison table of the above fuzzy-soft-set is given in Table 7.
Next we compute the row sum, column sum, and the score for each $o_{i}$ as shown in Table 8.

## 4. Algorithm

From Tables $3-5$, we see the choice value sequence is $\left\{c_{1}, c_{2}, \ldots, c_{6}\right\}=\{1.0,1.0,1.1,1.2,2.1,2.2\}$ and the score sequence is $\left\{s_{1}, s_{2}, \ldots, s_{6}\right\}=\{-13,-5,-6,2,16,6\}$, where $c_{5}=2.1, c_{6}=2.2, s_{5}=16, s_{6}=6$. In general, $o_{5}$ is the optimal choice when considering the choice value and score value together. From Tables 6-8, we see the choice value sequence is $\left\{c_{1}, c_{2}, \ldots, c_{6}\right\}=\{1.4,1.3,1.3,1.8,1.9,2.9\}$ and the score sequence is $\left\{s_{1}, s_{2}, \ldots, s_{6}\right\}=$ $\{1,-7,-4,-5,8,7\}$, where $c_{5}=1.9, c_{6}=2.9, s_{5}=8, s_{6}=7$. In general, $o_{6}$ is the optimal choice. We now present an algorithm to solve the above problems.

Algorithm.
Step 1.
Input some kind of evaluation requirements, for example, the choice value sequence $\left\{c_{1}, c_{2}, \ldots, c_{n}\right\}$ and the score sequence $\left\{s_{1}, s_{2}, \ldots, s_{n}\right\}$, where $c_{i}$ and $s_{i}$ are associate with object $o_{i}$.
Step 2.
Grey relational generating

$$
\begin{aligned}
c_{i}^{\prime} & =\frac{c_{i}-\operatorname{Min}\left\{c_{i}, i=1, \ldots, n\right\}}{\operatorname{Max}\left\{c_{i}, i=1, \ldots, n\right\}-\operatorname{Min}\left\{c_{i}, i=1, \ldots, n\right\}}, \\
s_{i}^{\prime} & =\frac{s_{i}-\operatorname{Min}\left\{s_{i}, i=1, \ldots, n\right\}}{\operatorname{Max}\left\{s_{i}, i=1, \ldots, n\right\}-\operatorname{Min}\left\{s_{i}, i=1, \ldots, n\right\}} .
\end{aligned}
$$

## Step 3.

Reorder sequence. $\left\{c_{1}^{\prime}, s_{1}^{\prime}\right\}, \ldots,\left\{c_{n}^{\prime}, s_{n}^{\prime}\right\}$, where $\left\{c_{i}^{\prime}, s_{i}^{\prime}\right\}$ is associated with object $o_{i}$.

## Step 4.

Difference Information.

$$
\begin{aligned}
& c_{\max }=\operatorname{Max}\left\{c_{i}^{\prime}, i=1, \ldots, n\right\}, \quad s_{\max }=\operatorname{Max}\left\{s_{i}^{\prime}, i=1, \ldots, n\right\}, \quad \Delta c_{i}^{\prime}=\left|c_{\max }-c_{i}^{\prime}\right|, \quad \Delta s_{i}^{\prime}=\left|s_{\max }-s_{i}^{\prime}\right| . \\
& \Delta_{\max }=\operatorname{Max}\left\{\Delta c_{i}^{\prime}, \Delta s_{i}^{\prime}, i=1, \ldots, n\right\}, \quad \Delta_{\min }=\operatorname{Min}\left\{\Delta c_{i}^{\prime}, \Delta s_{i}^{\prime}, i=1, \ldots, n\right\} .
\end{aligned}
$$

Step 5.
Grey relative coefficient.

$$
\begin{aligned}
& \gamma\left(c, c_{i}\right)=\frac{\Delta_{\min }+\xi * \Delta_{\max }}{\Delta c_{i}^{\prime}+\xi * \Delta_{\max }} \\
& \gamma\left(s, s_{i}\right)=\frac{\Delta_{\min }+\xi * \Delta_{\max }}{\Delta s_{i}^{\prime}+\xi * \Delta_{\max }}
\end{aligned}
$$

where $\xi$ is the distinguishing coefficient, $\xi \in[0,1]$. The purpose of the distinguishing coefficient is to expand or compress the range of the grey relational coefficient. In this paper, $\xi=0.5$.
Step 6.
Grey relational grade. $\gamma\left(o_{i}\right)=\left(\omega_{1} * \gamma\left(c, c_{i}\right)+\omega_{2} * \gamma\left(s, s_{i}\right)\right)$, where $\omega_{i} i=1,2$ is the weight of evaluation factor, $\omega_{1}+\omega_{2}=1$. In this paper, $\omega_{1}=\omega_{2}=0.5$.
Step 7.
Decision making. The decision is $o_{k}$ if $o_{k}=\max \gamma\left(o_{k}\right)$. Optimal choices have more than one object if there are more objects corresponding to the maximum.

Using the above algorithm, we discuss Examples 3.1 and 3.2 in the following.
Decision for Example 3.1.
Step 1.
The choice value sequence $\left\{c_{1}, c_{2}, \ldots, c_{6}\right\}=\{1.0,1.0,1.1,1.2,2.1,2.2\}$ and the score sequence is $\left\{s_{1}, s_{2}, \ldots, s_{6}\right\}=$ $\{-13,-5,-6,2,16,6\}$.
Step 2.

$$
\begin{aligned}
& \operatorname{Min}\left\{c_{i}, i=1, \ldots, 6\right\}=1.0, \quad \operatorname{Max}\left\{c_{i}, i=1, \ldots, 6\right\}=2.2, \quad c_{i}^{\prime}=\left(c_{i}-1.0\right) / 1.2 \\
& \left\{c_{1}^{\prime}, c_{2}^{\prime}, \ldots, c_{6}^{\prime}\right\}=\{0,0,0.083,0.167,0.917,1\} . \quad\left\{s_{1}^{\prime}, s_{2}^{\prime}, \ldots, s_{6}^{\prime}\right\}=\{0,0.276,0.241,0.517,1,0.655\}
\end{aligned}
$$

Step 3.

$$
\begin{aligned}
& \left\{c_{1}^{\prime}, s_{1}^{\prime}\right\}=\{0,0\}, \quad\left\{c_{2}^{\prime}, s_{2}^{\prime}\right\}=\{0,0.276\}, \quad\left\{c_{3}^{\prime}, s_{3}^{\prime}\right\}=\{0.083,0.241\}, \quad\left\{c_{4}^{\prime}, s_{4}^{\prime}\right\}=\{0.167,0.517\}, \\
& \left\{c_{5}^{\prime}, s_{5}^{\prime}\right\}=\{0.917,1\}, \quad\left\{c_{6}^{\prime}, s_{6}^{\prime}\right\}=\{1,0.655\} .
\end{aligned}
$$

Step 4.

$$
\begin{array}{lll}
c_{\max }=1, & s_{\max }=1, & \Delta c_{1}^{\prime}=1, \quad \Delta c_{2}^{\prime}=1, \quad \Delta c_{3}^{\prime}=0.917, \quad \Delta c_{4}^{\prime}=0.833, \quad \Delta c_{5}^{\prime}=0.083 \\
\Delta c_{6}^{\prime}=0 ; & \Delta s_{1}^{\prime}=1, & \Delta s_{2}^{\prime}=0.724, \quad \Delta s_{3}^{\prime}=0.759, \quad \Delta s_{4}^{\prime}=0.483, \quad \Delta s_{5}^{\prime}=0, \quad \Delta s_{6}^{\prime}=0.345 \\
\Delta_{\max }=1, & \Delta_{\min }=0
\end{array}
$$

Step 5.

$$
\begin{aligned}
& \gamma\left(c, c_{1}\right)=0.333, \quad \gamma\left(c, c_{2}\right)=0.333, \quad \gamma\left(c, c_{3}\right)=0.353, \quad \gamma\left(c, c_{4}\right)=0.375, \quad \gamma\left(c, c_{5}\right)=0.858, \\
& \gamma\left(c, c_{6}\right)=1 ; \quad \gamma\left(s, s_{1}\right)=0.333, \quad \gamma\left(s, s_{2}\right)=0.408, \quad \gamma\left(s, s_{3}\right)=0.397, \quad \gamma\left(s, s_{4}\right)=0.509, \\
& \gamma\left(s, s_{5}\right)=1, \quad \gamma\left(s, s_{6}\right)=0.529 .
\end{aligned}
$$

Step 6.

$$
\begin{aligned}
& \gamma\left(o_{1}\right)=0.333, \quad \gamma\left(o_{2}\right)=0.371, \quad \gamma\left(o_{3}\right)=0.375, \quad \gamma\left(o_{4}\right)=0.442, \quad \gamma\left(o_{5}\right)=0.929, \\
& \gamma\left(o_{6}\right)=0.796 .
\end{aligned}
$$

Step 7. The decision is $o_{5}$.
For Example 3.2, with the same steps we can obtain.
Step 1. The choice value sequence $\left\{c_{1}, c_{2}, \ldots, c_{6}\right\}=\{1.4,1.3,1.3,1.8,1.9,2.9\}$ and the score sequence is $\left\{s_{1}, s_{2}, \ldots, s_{6}\right\}=$ $\{1,-7,-4,-5,8,7\}$.

Step 2.

$$
\begin{aligned}
& \operatorname{Min}\left\{c_{i}, i=1, \ldots, 6\right\}=1.3, \quad \operatorname{Max}\left\{c_{i}, i=1, \ldots, 6\right\}=2.9 \\
& c_{i}^{\prime}=\left(c_{i}-1.3\right) / 1.6 . \quad\left\{c_{1}^{\prime}, c_{2}^{\prime}, \ldots, c_{6}^{\prime}\right\}=\{0.063,0,0,0.313,0.375,1\} \\
& \left\{s_{1}^{\prime}, s_{2}^{\prime}, \ldots, s_{6}^{\prime}\right\}=\{0.533,0,0.2,0.133,1,0.933\}
\end{aligned}
$$

Step 3.

$$
\begin{aligned}
& \left\{c_{1}^{\prime}, s_{1}^{\prime}\right\}=\{0.063,0.533\}, \quad\left\{c_{2}^{\prime}, s_{2}^{\prime}\right\}=\{0,0\}, \quad\left\{c_{3}^{\prime}, s_{3}^{\prime}\right\}=\{0,0.2\}, \quad\left\{c_{4}^{\prime}, s_{4}^{\prime}\right\}=\{0.313,0.133\}, \\
& \left\{c_{5}^{\prime}, s_{5}^{\prime}\right\}=\{0.375,1\}, \quad\left\{c_{6}^{\prime}, s_{6}^{\prime}\right\}=\{1,0.933\}
\end{aligned}
$$

Step 4.

$$
\begin{array}{lllll}
c_{\max }=1, & s_{\max }=1, \quad \Delta c_{1}^{\prime}=0.937, & \Delta c_{2}^{\prime}=1, & \Delta c_{3}^{\prime}=1, \quad \Delta c_{4}^{\prime}=0.687, \quad \Delta c_{5}^{\prime}=0.625 \\
\Delta c_{6}^{\prime}=0 ; & \Delta s_{1}^{\prime}=0.467, \quad \Delta s_{2}^{\prime}=1, & \Delta s_{3}^{\prime}=0.8, & \Delta s_{4}^{\prime}=0.867, \quad \Delta s_{5}^{\prime}=0, & \Delta s_{6}^{\prime}=0.067 \\
\Delta_{\max }=1, & \Delta_{\min }=0 .
\end{array}
$$

Step 5.

$$
\begin{aligned}
& \gamma\left(c, c_{1}\right)=0.348, \quad \gamma\left(c, c_{2}\right)=0.333, \quad \gamma\left(c, c_{3}\right)=0.333, \quad \gamma\left(c, c_{4}\right)=0.421, \quad \gamma\left(c, c_{5}\right)=0.444, \\
& \gamma\left(c, c_{6}\right)=1 ; \quad \gamma\left(s, s_{1}\right)=0.517, \quad \gamma\left(s, s_{2}\right)=0.333, \quad \gamma\left(s, s_{3}\right)=0.385, \quad \gamma\left(s, s_{4}\right)=0.366, \\
& \gamma\left(s, s_{5}\right)=1, \quad \gamma\left(s, s_{6}\right)=0.882 .
\end{aligned}
$$

Step 6.

$$
\begin{aligned}
& \gamma\left(o_{1}\right)=0.433, \quad \gamma\left(o_{2}\right)=0.333, \quad \gamma\left(o_{3}\right)=0.359, \quad \gamma\left(o_{4}\right)=0.393, \quad \gamma\left(o_{5}\right)=0.722, \\
& \gamma\left(o_{6}\right)=0.941 .
\end{aligned}
$$

Step 7. The decision is $0_{6}$.

## 5. Example

The primary motivation for designing the algorithm is to solve decision making problems by using grey theory. To illustrate the basic idea of the algorithm, we apply it to soft set based decision making problems. Let us consider Example 4 in [7].

Example 4 [7]. Assume that a company wants to fill a position. There are 48 candidates who fill in a form in order to apply formally for the position. There are two decision makers; one of them is from the department of human resources and the other one is from the board of directors. They want to interview the candidates, but it is very difficult to do this for all of them. Therefore, by using the uni-int decision making method, the number of candidates are reduced to a suitable one.

Assume that the set of candidates is $U=\left\{u_{1}, u_{2}, \ldots, u_{48}\right\}$, which may be characterized by a set of parameters $E=\left\{x_{1}, x_{2}, \ldots, x_{7}\right\}$. For $i=1,2, \ldots, 7$, the parameters $x_{i}$ stand for "experience", "computer knowledge", "training", "young age", "higher education", "marriage status" and "good health", respectively. See Table 9.

Now, apply the uni-int- $\wedge$ as follows:
Step 1: The decision makers consider the sets of parameters, $A=\left\{x_{1}, x_{2}, x_{4}, x 7\right\}$ and $B=\left\{x_{1}, x_{2}, x_{5}\right\}$, respectively, to evaluate the candidates.
Step 2: The decision makers seriously investigate the CV of the candidates. After a serious discussion each candidate is evaluated from the point of view of the goals and the constraint according to a chosen subset $A, B \subseteq E$. Then the decision makers construct the following two soft sets over $U$ according to their parameters, respectively.

$$
\begin{aligned}
F_{A}= & \left\{\left(x_{1},\left\{u_{4}, u_{7}, u_{13}, u_{21}, u_{28}, u_{31}, u_{32}, u_{36}, u_{39}, u_{41}, u_{43}, u_{44}, u_{48}\right\}\right),\right. \\
& \left(x_{2},\left\{u_{1}, u_{3}, u_{13}, u_{18}, u_{19}, u_{21}, u_{22}, u_{24}, u_{28}, u_{32}, u_{36}, u_{42}, u_{44}, u_{46}\right\}\right), \\
& \left(x_{4},\left\{u_{2}, u_{3}, u_{13}, u_{15}, u_{18}, u_{23}, u_{25}, u_{28}, u_{30}, u_{33}, u_{36}, u_{38}, u_{42}, u_{43}\right\}\right), \\
& \left.\left(x_{7},\left\{u_{1}, u_{5}, u_{12}, u_{13}, u_{17}, u_{20}, u_{24}, u_{28}, u_{29}, u_{34}, u_{36}, u_{41}, u_{45}, u_{47}\right\}\right)\right\} \\
F_{B}= & \left\{\left(x_{1},\left\{u_{3}, u_{4}, u_{5}, u_{8}, u_{14}, u_{21}, u_{22}, u_{26}, u_{27}, u_{34}, u_{35}, u_{37}, u_{40}, u_{42}, u_{46}\right\}\right),\right. \\
& \left(x_{2},\left\{u_{1}, u_{4}, u_{7}, u_{10}, u_{11}, u_{13}, u_{15}, u_{21}, u_{29}, u_{30}, u_{32}, u_{36}, u_{42}, u_{43}, u_{45}\right\}\right), \\
& \left.\left(x_{5},\left\{u_{2}, u_{4}, u_{8}, u_{9}, u_{12}, u_{13}, u_{14}, u_{16}, u_{17}, u_{21}, u_{23}, u_{28}, u_{36}, u_{42}, u_{44}\right\}\right)\right\} .
\end{aligned}
$$

Step 3: Now, we can find the $\wedge$-production $F_{A} \wedge F_{B}$ of the soft sets $F_{A}$ and $F_{B}$ as follows:

$$
\begin{aligned}
& \left\{\left(\left(x_{1}, x_{1}\right),\left\{u_{4}, u_{21}\right\}\right),\left(\left(x_{1}, x_{2}\right),\left\{u_{4}, u_{7}, u_{13}, u_{21}, u_{32}, u_{36}, u_{43}\right\}\right)\right. \\
& \left.\quad\left(\left(x_{1}, x_{5}\right),\left\{u_{4}, u_{13}, u_{21}, u_{28}, u_{36}, u_{44}\right\}\right),\left(\left(x_{2}, x_{1}\right),\left\{u_{3}, u_{21}, u_{22}, u_{42}, u_{46}\right\}\right)\right\} \\
& \left\{\left(\left(x_{2}, x_{2}\right),\left\{u_{1}, u_{13}, u_{21}, u_{32}, u_{36}, u_{42}\right\}\right),\left(\left(x_{2}, x_{5}\right),\left\{u_{13}, u_{21}, u_{28}, u_{36}, u_{42}, u_{44}\right\}\right)\right\} \\
& \left\{\left(\left(x_{4}, x_{1}\right),\left\{u_{3}, u_{42}\right\}\right),\left(\left(x_{4}, x_{2}\right),\left\{u_{13}, u_{15}, u_{30}, u_{36}, u_{42}, u_{43}\right\}\right)\right\} \\
& \left\{\left(\left(x_{4}, x_{5}\right),\left\{u_{2}, u_{13}, u_{23}, u_{28}, u_{36}, u_{42}\right\}\right),\left(\left(x_{7}, x_{1}\right),\left\{u_{5}, u_{34}\right\}\right)\right\} \\
& \left\{\left(\left(x_{7}, x_{2}\right),\left\{u_{1}, u_{13}, u_{29}, u_{36}, u_{45}\right\}\right),\left(\left(x_{7}, x_{5}\right),\left\{u_{12}, u_{13}, u_{17}, u_{28}, u_{36}\right\}\right)\right\} .
\end{aligned}
$$

Table 9
Score table A and B.

|  | $u_{1}$ | $u_{2}$ | $u_{3}$ | $u_{4}$ | $u_{5}$ | $u_{6}$ | $u_{7}$ | $u_{8}$ | $u_{9}$ | $u_{10}$ | $u_{11}$ | $u_{12}$ | $u_{13}$ | $u_{14}$ | $u_{15}$ | $u_{16}$ | $u_{17}$ | $u_{18}$ | $u_{19}$ | $u_{20}$ | $u_{21}$ | $u_{22}$ | $u_{23}$ | $u_{24}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $c_{\mathrm{Ai}}$ | 2 | 1 | 2 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 4 | 0 | 1 | 0 | 1 | 2 | 1 | 1 | 2 | 1 | 1 | 2 |
| $c_{\mathrm{Bi}}$ | 1 | 1 | 1 | 3 | 1 | 0 | 1 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 3 | 1 | 1 | 0 |
|  | $u_{25}$ | $u_{26}$ | $u_{27}$ | $u_{28}$ | $u_{29}$ | $u_{30}$ | $u_{31}$ | $u_{32}$ | $u_{33}$ | $u_{34}$ | $u_{35}$ | $u_{36}$ | $u_{37}$ | $u_{38}$ | $u_{39}$ | $u_{40}$ | $u_{41}$ | $u_{42}$ | $u_{43}$ | $u_{44}$ | $u_{45}$ | $u_{46}$ | $u_{47}$ | $u_{48}$ |
| $c_{\mathrm{Ai}}$ | 1 | 0 | 0 | 4 | 1 | 1 | 1 | 2 | 1 | 1 | 0 | 4 | 0 | 1 | 1 | 0 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| $c_{\mathrm{Bi}}$ | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 2 | 1 | 0 | 0 | 1 | 0 | 3 | 1 | 1 | 1 | 1 | 0 | 0 |

Table 10
Grey relational generation table $A$ and $B$.

|  | $u_{1}$ | $u_{2}$ | $u_{3}$ | $u_{4}$ | $u_{5}$ | $u_{6}$ | $u_{7}$ | $u_{8}$ | $u_{9}$ | $u_{10}$ | $u_{11}$ | $u_{12}$ | $u_{13}$ | $u_{14}$ | $u_{15}$ | $u_{16}$ | $u_{17}$ | $u_{18}$ | $u_{19}$ | $u_{20}$ | $u_{21}$ | $u_{22}$ | $u_{23}$ | $u_{24}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $c_{\mathrm{A}}^{\prime}$ | 0.5 | 0.25 | 0.5 | 0.25 | 0.25 | 0 | 0.25 | 0 | 0 | 0 | 0 | 0.25 | 1 | 0 | 0.25 | 0 | 0.25 | 0.5 | 0.25 | 0.25 | 0.5 | 0.25 | 0.25 | 0.5 |
| $c_{\mathrm{Bi}}^{\prime}$ | 0.33 | 0.33 | 0.33 | 1 | 0.33 | 0 | 0.33 | 0.67 | 0.33 | 0.33 | 0.33 | 0.33 | 0.67 | 0.67 | 0.33 | 0.33 | 0.33 | 0 | 0 | 0 | 1 | 0.33 | 0.33 | 0 |
|  | $u_{25}$ | $u_{26}$ | $u_{27}$ | $u_{28}$ | $u_{29}$ | $u_{30}$ | $u_{31}$ | $u_{32}$ | $u_{33}$ | $u_{34}$ | $u_{35}$ | $u_{36}$ | $u_{37}$ | $u_{38}$ | $u_{39}$ | $u_{40}$ | $u_{41}$ | $u_{42}$ | $u_{43}$ | $u_{44}$ | $u_{45}$ | $u_{46}$ | $u_{47}$ | $u_{48}$ |
| $c_{\mathrm{A}}^{\prime}$ | 0.25 | 0 | 0 | 1 | 0.25 | 0.25 | 0.25 | 0.5 | 0.25 | 0.25 | 0 | 1 | 0 | 0.25 | 0.25 | 0 | 0.5 | 0.5 | 0.5 | 0.5 | 0.25 | 0.25 | 0.25 | 0.25 |
| $c_{\mathrm{Bi}}^{\prime}$ | 0 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0 | 0.33 | 0 | 0.33 | 0.33 | 0.67 | 0.33 | 0 | 0 | 0.33 | 0 | 1 | 0.33 | 0.33 | 0.33 | 0.33 | 0 | 0 |

Step 4: Finally, we can find a decision set uni-int $\left(F_{A} \wedge F_{B}\right)$ as follows:

$$
\left.\begin{array}{rl}
\operatorname{uni}_{x} \operatorname{int}_{y}\left(F_{A} \wedge F_{B}\right)= & \cup_{x \in A}\left(\cap \cap_{y \in B}\left(f_{A \wedge B}(x, y)\right)\right) \\
= & \left\{\begin{array}{l}
\cap\left\{u_{4}, u_{21}\right\},\left\{u_{4}, u_{7}, u_{13}, u_{21}, u_{32}, u_{36}, u_{43}\right\}, \\
\left.\left\{u_{4}, u_{13}, u_{21}, u_{28}, u_{36}, u_{44}\right\}\right\} \\
\cap\left\{u_{3}, u_{21}, u_{22}, u_{42}, u_{46}\right\},\left\{u_{1}, u_{13}, u_{21}, u_{32}, u_{36}, u_{42}\right\} \\
\left.\left\{u_{13}, u_{21}, u_{28}, u_{36}, u_{42}, u_{44}\right\}\right\} \\
\cap\left\{\left\{u_{3}, u_{42}\right\},\left\{u_{13}, u_{15}, u_{30}, u_{36}\right.\right. \\
\left.\left\{u_{2}, u_{13}, u_{23}, u_{28}, u_{36}, u_{43}\right\}\right\} \\
\cap\left\{u_{5}, u_{34}\right\},\left\{u_{1}, u_{13}, u_{29}, u_{36}, u_{45}\right\}, \\
\left.\left\{u_{12}, u_{13}, u_{17}, u_{28}, u_{36}\right\}\right\}
\end{array}\right\}
\end{array}\right\},
$$

and

$$
\begin{aligned}
\operatorname{uni}_{y} \operatorname{int}_{x}\left(F_{A} \wedge F_{B}\right)= & \cup_{y \in B}\left(\cap \cap_{x \in A}\left(f_{A \wedge B}(x, y)\right)\right) \\
= & \left\{\begin{array}{l}
\cap\left\{\left\{u_{4}, u_{21}\right\},\left\{u_{3}, u_{21}, u_{22}, u_{42}, u_{46}\right\},\left\{u_{3}, u_{42}\right\},\left\{u_{5}, u_{34}\right\}\right\}, \\
\cap\left\{u_{4}, u_{7}, u_{13}, u_{21}, u_{32}, u_{36}, u_{43}\right\},\left\{u_{1}, u_{13}, u_{21}, u_{32}, u_{36}, u_{42}\right\}, \\
\left.\left\{u_{13}, u_{15}, u_{30}, u_{36}, u_{42}, u_{43}\right\},\left\{u_{1}, u_{13}, u_{29}, u_{36}, u_{45}\right\}\right\}, \\
\cap\left\{\left\{u_{4}, u_{13}, u_{21}, u_{28}, u_{36}, u_{44}\right\},\left\{u_{13}, u_{21}, u_{28}, u_{36}, u_{42}, u_{44}\right\},\right. \\
\left.\left\{u_{2}, u_{13}, u_{23}, u_{28}, u_{36}, u_{42}\right\},\left\{u_{12}, u_{13}, u_{17}, u_{28}, u_{36}\right\}\right\}
\end{array}\right\} \\
= & \cup\left\{\emptyset,\left\{u_{13}, u_{36}\right\},\left\{u_{13}, u_{28}, u_{36}\right\}\right\}=\left\{u_{13}, u_{28}, u_{36}\right\} .
\end{aligned}
$$

Hence, the decision makers to make interviewing invite the candidates which are the elements of the following uni-int decision set.

$$
\begin{aligned}
\operatorname{uni}-\operatorname{int}\left(F_{A} \wedge F_{B}\right) & =\operatorname{uni}_{x} \operatorname{int}_{y}\left(F_{A} \wedge F_{B}\right) \cup \text { uni }_{y} \operatorname{int}_{x}\left(F_{A} \wedge F_{B}\right)=\left\{\left\{u_{4}, u_{21}, u_{42}\right\} \cup \underline{\left\{u_{13}, u_{28}, u_{36}\right\}}\right\} \\
& =\underline{\left\{u_{4}, u_{13}, u_{21}, u_{28}, u_{36}, u_{42}\right\}} .
\end{aligned}
$$

Note: Underline marked sets in the above algorithm process are revised because of clerical error in Example 4 of paper [7]. Therefore, one of $u_{4}, u_{13}, u_{21}, u_{28}, u_{36}, u_{42}$ is the optimal choice.

Now, suppose the decision makers evaluate the parameter sets $A=\left\{x_{1}, x_{2}, x_{4}, x_{7}\right\}$ and $B=\left\{x_{1}, x_{2}, x_{5}\right\}$ using choice values and apply soft set decision making based on grey theory as follows:
Step 1. Input two kinds of evaluation criteria.
Step 2. Grey relational generating (see Table 10).
Step 3. Reorder sequence. $\left\{c_{A 1}^{\prime}, c_{B 1}^{\prime}\right\}, \ldots,\left\{c_{A 48}^{\prime}, c_{B 48}^{\prime}\right\}$.
Step 4. Difference information (see Table 11).
Step 5. Grey relative coefficient (see Table 12).
Step 6. Grey relational grade. $\omega_{1}=\omega_{2}=0.5$ (see Table 13).
Step 7. Decision making. From the grey relational grade, we can see $u_{13}$ and $u_{36}$ are the optimal choices.

Table 11
Difference information table $A$ and $B$.

|  | $u_{1}$ | $u_{2}$ | $u_{3}$ | $u_{4}$ | $u_{5}$ | $u_{6}$ | $u_{7}$ | $u_{8}$ | $u_{9}$ | $u_{10}$ | $u_{11}$ | $u_{12}$ | $u_{13}$ | $u_{14}$ | $u_{15}$ | $u_{16}$ | $u_{17}$ | $u_{18}$ | $u_{19}$ | $u_{20}$ | $u_{21}$ | $u_{22}$ | $u_{23}$ | $u_{24}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\Delta c_{\mathrm{Ai}}^{\prime}$ | 0.5 | 0.75 | 0.5 | 0.75 | 0.75 | 1 | 0.75 | 1 | 1 | 1 | 1 | 0.75 | 0 | 1 | 0.75 | 1 | 0.75 | 0.5 | 0.75 | 0.75 | 0.5 | 0.75 | 0.75 | 0.5 |  |
| $\Delta c_{\mathrm{Bi}}^{\prime}$ | 0.67 | 0.67 | 0.67 | 0 | 0.67 | 1 | 0.67 | 0.33 | 0.67 | 0.67 | 0.67 | 0.67 | 0.33 | 0.33 | 0.67 | 0.67 | 0.67 | 1 | 1 | 1 | 0 | 0.67 | 0.67 | 1 |  |
|  | $u_{25}$ | $u_{26}$ | $u_{27}$ | $u_{28}$ | $u_{29}$ | $u_{30}$ | $u_{31}$ | $u_{32}$ | $u_{33}$ | $u_{34}$ | $u_{35}$ | $u_{36}$ | $u_{37}$ | $u_{38}$ | $u_{39}$ | $u_{40}$ | $u_{41}$ | $u_{42}$ | $u_{43}$ | $u_{44}$ | $u_{45}$ | $u_{46}$ | $u_{47}$ | $u_{48}$ |  |
| $\Delta c_{\mathrm{Ai}}^{\prime}$ | 0.75 | 1 | 1 | 0 | 0.75 | 0.75 | 0.75 | 0.5 | 0.75 | 0.75 | 1 | 0 | 1 | 0.75 | 0.75 | 1 | 0.5 | 0.5 | 0.5 | 0.5 | 0.75 | 0.75 | 0.75 | 0.75 |  |
| $\Delta c_{\mathrm{Bi}}^{\prime}$ | 1 | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 1 | 0.67 | 1 | 0.67 | 0.67 | 0.33 | 0.67 | 1 | 1 | 0.67 | 1 | 0 | 0.67 | 0.67 | 0.67 | 0.67 | 1 | 1 |  |

Table 12
Grey relative coefficient table $A$ and $B$.

| $u_{1}$ | $u_{2}$ | $u_{3}$ | $u_{4}$ | $u_{5}$ | $u_{6}$ | $u_{7}$ | $u_{8}$ | $u_{9}$ | $u_{10}$ | $u_{11}$ | $u_{12}$ | $u_{13}$ | $u_{14}$ | $u_{15}$ | $u_{16}$ | $u_{17}$ | $u_{18}$ | $u_{19}$ | $u_{20}$ | $u_{21}$ | $u_{22}$ | $u_{23}$ | $u_{24}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\gamma\left(c, c_{\mathrm{Ai}}\right)$ | 0.5 | 0.4 | 0.5 | 0.4 | 0.4 | 0.33 | 0.4 | 0.33 | 0.33 | 0.33 | 0.33 | 0.4 | 1 | 0.33 | 0.4 | 0.33 | 0.4 | 0.5 | 0.4 | 0.4 | 0.5 | 0.4 | 0.4 |
| $\gamma\left(c, c_{\mathrm{Bi}}\right)$ | 0.43 | 0.43 | 0.43 | 1 | 0.43 | 0.33 | 0.43 | 0.6 | 0.43 | 0.43 | 0.43 | 0.43 | 0.6 | 0.6 | 0.43 | 0.43 | 0.43 | 0.33 | 0.33 | 0.33 | 1 | 0.43 | 0.43 |
|  | $u_{25}$ | $u_{26}$ | $u_{27}$ | $u_{28}$ | $u_{29}$ | $u_{30}$ | $u_{31}$ | $u_{32}$ | $u_{33}$ | $u_{34}$ | $u_{35}$ | $u_{36}$ | $u_{37}$ | $u_{38}$ | $u_{39}$ | $u_{40}$ | $u_{41}$ | $u_{42}$ | $u_{43}$ | $u_{44}$ | $u_{45}$ | $u_{46}$ | $u_{47}$ |
| $u_{48}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\gamma\left(c, c_{\mathrm{Ai}}\right)$ | 0.4 | 0.33 | 0.33 | 1 | 0.4 | 0.4 | 0.4 | 0.5 | 0.4 | 0.4 | 0.33 | 1 | 0.33 | 0.4 | 0.4 | 0.33 | 0.5 | 0.5 | 0.5 | 0.5 | 0.4 | 0.4 | 0.4 |
| $\gamma\left(c, c_{\mathrm{Bi}}\right)$ | 0.33 | 0.43 | 0.43 | 0.43 | 0.43 | 0.43 | 0.33 | 0.43 | 0.33 | 0.43 | 0.43 | 0.6 | 0.43 | 0.33 | 0.33 | 0.43 | 0.33 | 1 | 0.43 | 0.43 | 0.43 | 0.43 | 0.33 |

Table 13
Grey relation grade table $A$ and $B$.

| $u_{1}$ | $u_{2}$ | $u_{3}$ | $u_{4}$ | $u_{5}$ | $u_{6}$ | $u_{7}$ | $u_{8}$ | $u_{9}$ | $u_{10}$ | $u_{11}$ | $u_{12}$ | $u_{13}$ | $u_{14}$ | $u_{15}$ | $u_{16}$ | $u_{17}$ | $u_{18}$ | $u_{19}$ | $u_{20}$ | $u_{21}$ | $u_{22}$ | $u_{23}$ | $u_{24}$ |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\gamma\left(u_{i}\right)$ | 0.46 | 0.41 | 0.46 | 0.7 | 0.41 | 0.33 | 0.41 | 0.47 | 0.38 | 0.38 | 0.38 | 0.41 | 0.8 | 0.47 | 0.41 | 0.38 | 0.41 | 0.42 | 0.37 | 0.37 | 0.75 | 0.41 | 0.41 | 0.42 |
| $u_{25}$ | $u_{26}$ | $u_{27}$ | $u_{28}$ | $u_{29}$ | $u_{30}$ | $u_{31}$ | $u_{32}$ | $u_{33}$ | $u_{34}$ | $u_{35}$ | $u_{36}$ | $u_{37}$ | $u_{38}$ | $u_{39}$ | $u_{40}$ | $u_{41}$ | $u_{42}$ | $u_{43}$ | $u_{44}$ | $u_{45}$ | $u_{46}$ | $u_{47}$ | $u_{48}$ |  |
| $\gamma\left(u_{i}\right)$ | 0.37 | 0.38 | 0.38 | 0.72 | 0.41 | 0.41 | 0.37 | 0.46 | 0.37 | 0.41 | 0.38 | 0.8 | 0.38 | 0.37 | 0.37 | 0.38 | 0.42 | 0.75 | 0.46 | 0.46 | 0.41 | 0.41 | 0.41 | 0.37 |

Remark. If we select $\gamma\left(u_{i}\right) \geq 0.5$, the objects $u_{4}=0.7, u_{13}=0.8, u_{21}=0.75, u_{28}=0.72, u_{36}=0.8$ and $u_{42}=0.75$ meet the requirements, which is consistent with the result of the uni-int decision making method. But the uni-int decision making method is a preprocessing method and cannot give the final decision. Furthermore, the uni-int decision making method can only deal with soft set decision problems. For fuzzy soft set decision problems, it does not work well. The algorithm in this paper is suitable for soft set and fuzzy soft set decision making problems and unconstrained for evaluation methods. Whatever kind of evaluation methods we choose, as long as every evaluation sequence is given, such as choice value sequence, score value sequence, or other sequences, then we can make the decision using this new algorithm. The evaluation sequence associates to a kind of evaluation method, so this new algorithm is a hybrid evaluation method for comprehensive evaluation. Also, this new algorithm is objective, numerical and accurate, and we can compare the value to obtain the optimal choice, suboptimal choice, and so on.

## 6. Conclusions

In this paper, we use multiple evaluation bases to make decisions. A new algorithm for multiple evaluation bases for the fuzzy soft set decision making problem is presented based on grey theory. Using this new algorithm the decision result is general and efficient.

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