Procedia Computer Science
Volume 51, 2015, Pages 875–884
ICCS 2015 International Conference On Computational Science

Variations of Ant Colony Optimization for the solution of the structural damage identification problem

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Abstract

In this work the inverse problem of identification of structural stiffness coefficients of a damped spring-mass system is tackled. The problem is solved by using different versions of Ant Colony Optimization (ACO) metaheuristic solely or coupled with the Hooke-Jeeves (HJ) local search algorithm. The evaluated versions of ACO are based on a discretization procedure to deal with the continuous domain design variables together with different pheromone evaporation and deposit strategies and also on the frequency of calling the local search algorithm. The damage estimation is evaluated using noiseless and noisy synthetic experimental data assuming a damage configuration throughout the structure. The reported results show the hybrid method as the best choice when both rank-based pheromone deposit and a new heuristic information based on the search history are used.

Keywords: Ant Colony Optimization, Damage Identification, Inverse Problem, Metaheuristic, Hybrid Method

1 Introduction

In recent decades, a large research effort has been dedicated to the system identification subject for different reasons. One of the most important applications of system identification techniques is the monitoring of physical integrity of engineering structures by identifying damage. It is well known that the existence of damage in a given structure changes the dynamic response of it. At same time, identification of changes from measurements in the vibration behavior can be associated with changes in the mechanical properties of the structure [5].

The damage identification problem is classified as an inverse problem in vibration, since the evaluation of the damage can be obtained by determining variations in the stiffness coefficients, or by determining themselves [25]. Due to the characteristics of this class of problems, it is observed that small perturbations in the input data, such as random errors inherent to experimental data used in the inverse analysis, can cause large variations in the final solution.

Selection and peer-review under responsibility of the Scientific Programme Committee of ICCS 2015

doi:10.1016/j.procs.2015.05.218

Such behavior characterizes an ill-posed problem [11]. In general, the ill-posed inverse problem can be solved by using a well-posed functional form whose solution is obtained through the use of optimization methods.

Based on these considerations, different experimental, numerical and analytical techniques have been proposed to solve the problem of structural damage identification [10]. The different methods can be classified into different categories, such as local and global strategies, linear and non-linear schemes [6], time and frequency domains methods or deterministic and stochastic approaches [4].

Since the localization and quantification of structural damage, placed as an optimization problem, is a highly complex problem subject to a large number of local optima, becomes mandatory the use of non-conventional optimization techniques such as those based on stochastic search [19, 26].Within the set of stochastic approaches used for the solution of inverse vibration problems, heuristic methods have been used successfully. Thus, both single-solution and population-based strategies have been proposed to localize and quantify structural damages, such as Simulated Annealing [16], Tabu Search [1], Genetic Algorithm [2], Particle Swarm Optimization [21], Ant Colony Optimization [19, 26] but also hybrid approaches which combine optimization techniques to achieve better results [15, 12].

Regarding the stochastic approach inspired by the behavior of ant colonies, the Ant System paradigm was originally proposed to solve discrete optimization problem [7]. An incremental constructive mechanism is used to build up a complete path which represents a solution in the context of the problem [9]. If the solution for the optimization problem can be defined by a set of discrete points, inherently observed in the context of combinatorial optimization problems [8], the solution of this kind of problem is naturally related to the path constructed by the ants. However, when continuous optimization problems are faced, the natural relation to the floating values of design variables is no longer direct.

Proposed adaptations found in literature [17, 24] are usually related to the use of a probability density function (PDF instead of a discrete probability distribution for choosing the next component in the solution construction process. Some works have already used successfully the Ant Colony Optimization (ACO) algorithm to solve the inverse problem of identification of structural stiffness coefficients of different structures [26, 19, 18]. Nevertheless, the original idea of Ant Colony algorithms to deal with discrete values is still a possible choice to be used for identification of structural damage.

With this in mind, one of the contributions of this work is the use of a discretization process of the design variables domain which allows the solution of damage identification problems by the native Ant Colony Optimization algorithm rather than an adapted version of it. Furthermore, different from the ACO approaches found in literature [26, 19, 18], the present ACO approach uses all the components present in the original version of ACO proposed by Dorigo et al. [8]. Among these resources, the heuristic information stands out. Thereby, an Ant System with heuristic information is proposed, as well as other variations of ACO, such as elitist AS and rank-based AS. Besides, a hybrid approach characterized by the use of ACO method combined with the Hooke-Jeeves (HJ) local search method [13] is also proposed.

Simulations were performed assuming damaged elements throughout the structure, which is generally not reported in the literature, assuming different levels of damage and considering both noiseless and noisy synthetic experimental displacement data. Among the evaluated versions of the ACO, the hybrid approach with the heuristic information provided the best results for the solution of the damage identification problem.

2 The Damage Identification Problem

The determination of system vibratory response, either in time or frequency domain, can be obtained based on the structural parameters (mass, stiffness and damping), initial and boundary conditions and external forces applied to the system. This problem is identified as a Direct Problem in vibration. On the other hand, when one or more structural parameters are to be determined from measurements of displacements and/or natural frequencies and vibration modes, it is identified as an Inverse Problem in vibration.

In this work, the inverse problem of structural stiffness identification of a damped mass-spring system will be addressed. Therefore, it will be assumed known the other structural parameters (mass and damping rates) of a N degrees-of-freedom (DOF) system, external forces and also the time history of structural displacement.

2.1 Direct Problem

From the analysis and application of Newton's second law, the dynamic equation of motion for a vibratory system is described by the following ordinary differential equation [20]:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(t)\,, \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are matrices denoting structural parameters of mass, stiffness and damping, and $\mathbf{x}(t)$ is the displacement vector and $\mathbf{f}(t)$ is the vector of the external forces applied to the system. In this work, the solution for the direct problem was computed numerically by employing the Runge-Kutta 4th order method [14].

2.2 Inverse Problem

As mentioned earlier, for the solution of damage identification problems where the damage localization and quantification are to be determined, data corresponding to the vibrational response of the structure under study are needed. In this work it will be assumed available experimental data and information related to the time history of structural displacements.

Since inverse problems are classified as ill-posed problems, its solution was obtained through the minimization of a well-posed functional form. For a generic N-DOF system, the following objective function is defined:

$$OF = \sum_{j=1}^{N} \sum_{i=1}^{N_t} \left[\mathbf{x}_j^{Mod}(\mathbf{K}, t_i) - \mathbf{x}_j^{Exp}(\mathbf{K}, t_i) \right]^2 \,, \tag{2}$$

where estimated values for the stiffness coefficients are obtained by the minimization of the squared difference between the displacement computed by the mathematical model ($\mathbf{x}^{Mod}$) and the experimental displacement ($\mathbf{x}^{Exp}$). The parameter N is the number of degrees of freedom of the system and N_t is the number of time steps used in the integration process of the numerical method.

Despite the functional form be defined as a difference of displacements, it must be emphasized that there is an implicit constraint between displacements and the damaged stiffness. This constraint is imposed by the relationship defined in equation 1.

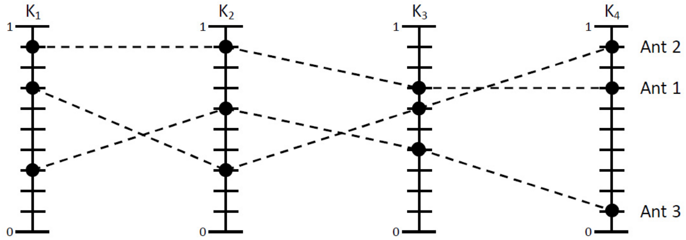

Figure 1: Forager ants perform random search among the integrity values of structural springs.

3 Optimization Methodology

The Ant System algorithm is a population-based metaheuristic inspired by the foraging behavior of ants. In nature, many species of ants are almost completely blind and the communication among them is performed through a chemical substance called pheromone. When walking, the pheromone is deposited by the ants on the performed paths, forming trails of pheromone. The following ants by perceiving the pheromone presence, choose the trail with higher concentration of it. These trails help population members to find food sources, as well as the way back. This makes the ants able to find the shortest path between their nest and the food source.

In the damage identification problem, the ants forage randomly within the search space and are evaluated according to their Objective Function (OF). The adopted modeling assumes each spring K_s ($s = 1, \ldots, n_s = N$) with integrity value from the interval $[0, 1]$ which is split in n_p discrete points, where the value 1 means no damage and value 0 means completely damaged spring (Figure 1). These discrete points represent the nodes of a hypothetical graph on which the ants will generate their paths.

3.1 Ant Colony algorithm and variations

In order to study the effectiveness of the ACO method in solving the damage identification problem, some variations of the original procedure of two phases are proposed. In the original procedure assumed in this work, the forward phase is characterized by ants choosing probabilistically the next values of stiffness integrity to construct a complete solution. The probability of a specific ant $k = 1, \ldots, n_a$ of choosing the position j, from position i, is computed according to pheromone (ϕ_{ij}) and heuristic information (η_{ij}), given by equation (3). This equation represents the ratio between the weight of using arc (i, j) and the sum of all weights of using all the possibilities in the vicinity $\mathcal{N}_i^k$:

$$p_{ij}^k = \frac{(\phi_{ij})^\alpha (\eta_{ij})^\beta}{\sum_{l \in \mathcal{N}_i^k} (\phi_{il})^\alpha (\eta_{il})^\beta}\,, \quad \text{for} \quad i = 1, \ldots, n_p\,, \quad k = 1, \ldots, n_a\,, \quad j \in \mathcal{N}_i^k\,. \tag{3}$$

The parameters α and β determine the proportion between the amount of pheromone associated with each arc and the quantity related to the heuristic information provided by the problem. Following, the backward phase is performed characterized by the pheromone updating, which is carried out in two steps: Evaporation and Deposit.

The pheromone updating is defined by the following expression:

$$\phi_{ij} = (1 - \rho)\phi_{ij} + \Delta\phi_{ij}^k\,, \quad \text{for} \quad k = 1, \ldots, n_a\,, \quad \forall (i, j) \in \mathcal{A}, \tag{4}$$

where ρ is the evaporation rate and $\Delta\phi_{ij}^k$ is a constant amount of pheromone that every ant k

in the current iteration deposits on the visited arcs (i, j) from the set $\mathcal{A}$ of all arcs. Algorithm 1 presents the pseudocode of the standard ACO method used in this work.

Algorithm 1: Template of the ACO.

```
Input: Initial values of pheromone trails.
Output: Best solution found or a set of solutions.
repeat
    for each ant do
        Solution construction using the pheromone trail;
        Update the pheromone trails:
            Evaporation;
            Reinforcement;
    end
until Stopping criteria;
```

All the variations presented in this work are based on the same operating principle, differing only with respect to the way the probability of choosing a particular arc is computed and how the pheromone information associated to each one of the arcs is updated.

3.1.1 Ant System (AS)

The Ant System is known as the first ACO algorithm. In this work, the AS method assumes the probability of choosing j as next position computed by equation (3), with $\alpha = 1$ and $\beta = 0$ as assumed in [19], since the shape of the heuristic information is not clearly defined. The pheromone updating is performed following equation (3) with the deposit term $\Delta\phi_{ij}^k = \phi_0 = n_s/OF$ $(k = 1, \ldots, n_a)$, where n_s is the number of design variables, which for a damped spring-mass system is equal to the number of DOF, i.e $n_s = N$, and OF is computed according equation (2) when assumed the structure without damage.

3.1.2 Hybrid Approach (AS+HJ)

To improve the quality of the solution found by the previous AS version, a hybrid method was implemented, where the Hook-Jeeves local search heuristic [13] was used coupled with the AS algorithm. The HJ heuristic aims to intensify the search for better quality solutions in promising regions. For sake of brevity the respective algorithm is not presented here but it is based on the pseudocode presented in [23].

In the proposed hybrid approach, the optimization procedure starts by running the AS algorithm which is paused after a predefined number of iterations is reached. Thereafter, the HJ local search is executed for n_s iterations starting from the best-so-far solution found by the AS algorithm.

For the application of HJ heuristic, the following parameters were set: (i) terminating condition set as 5 algorithm loops; (ii) initial step size $\Delta = 0.01$; and (iii) acceleration factor $\delta = 2$. The first parameter ensures at least $5N$ attempts of improvement of the solution. This value proved to be sufficient to ensure an exploiting search. The second value was set to ensure the local search performing. Both values were defined empirically. The third value was set based on the standard HJ heuristic.

After finishing the HJ heuristic, the AS algorithm is resumed taking into account the final solution provided by the local search (HJ) phase which is used for pheromone updating. Different versions of the hybrid method were evaluated according to the frequency of calling the HJ heuristic.

3.1.3 Rank-based Ant System (RAS)

Another alternative to the AS algorithm was evaluated by a new proposal for the amount of pheromone to be deposited, defined according to the quality of the solution provided by each ant in the current iteration, defined by the value of the objective function. Thus, the best ant in the current iteration deposits the maximum amount of pheromone, while the others contribute with a quantity proportional to its quality. This strategy is similar to the rank-based AS proposed in [3]. The amount of deposited pheromone is defined in equation (5), which replaces the corresponding term in equation (4).

$$\Delta\phi_{ij}^{k} = \phi_0 \, \frac{OF^k}{OF^{best}} \,, \quad k = 1, \ldots, n_a \,, \tag{5}$$

where $\phi_0 = n_s/OF$ as defined in section 3.1.1, OF^k is the objective function value for the $k-th$ ant and OF^{best} is the best objective function value in the current iteration.

3.1.4 Ant System with Heuristic Information (ASH)

Another variation proposed on the original AS algorithm considered the inclusion of a heuristic information η_{ij} to be used together the pheromone trials ϕ_{ij} for computing the p_{ij} probabilities. The probability of choosing a specific arc can be based not only on the pheromone associated to the neighborhood $\mathcal{N}_i$ of an ant when in node i, but also on the neighborhood $\mathcal{N}_j$ of all nodes i that can reach a node j. Therefore, the information associated to the arcs which reach a specific destination node j is proposed as heuristic information, taking place on equation (3) as the term η_{ij}. The relative influence of the pheromone trail and the heuristic information was assumed equal by taking $\alpha = 1$ and $\beta = 1$. Different from the traditional heuristic information used in some combinatorial optimization problems, which can be previously computed, the heuristic information proposed here is a runtime measure, and varies from run to run.

Thus, the probability for the transition from node i to the next node j is still defined by equation (3) however with the parameters η_{ij} defined as

$$\eta_{ij} = \sum_{p \in \mathcal{N}_j^k} \phi_{pj} \,, \quad \text{for} \quad j = 1, \ldots, np \,, \quad k = 1, \ldots, n_a \,, \quad i \in \mathcal{N}_j^k . \tag{6}$$

This variation can be better understood through Figure 2, which shows the parts to be considered in the computing of heuristic information. The next node on the path of an ant placed in the node related to spring K_2 integrity will be determined based not only on the amount of pheromone associated with the arcs that leave it, but also the quantities of pheromone of all the other arcs that reach a node of specific integrity value for spring K_3.

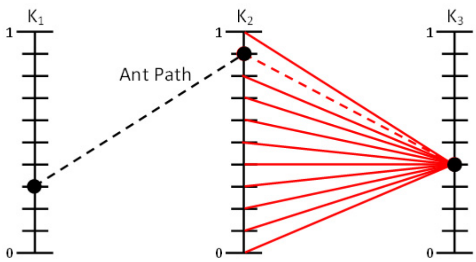

Figure 2: Heuristic information scheme for a 3-DOF damage identification problem.

3.1.5 Elitist Ant System (EAS)

It represents a modification of the original AS method [8]. The main ideia is to provide additional reinforcement to the parts belonging to the best path found since the beginning of the solution procedure. This best path is named best-so-far tour (T^{bs}). In this work, all the ants in the current iteration perform the standard deposit along with the deposit related to the best path found so far since the beginning of the search. In the pheromone updating expression, the term related to the deposit in equation (4) is replaced by the expression in equation (7) if the arc (i, j) belongs to T^{bs} path, otherwise the updating expression remains the same.

$$\phi_{deposit} = \sum_{k=1}^{n_a} \Delta\phi_{ij}^k + e\,\phi_{ij}^{bs}\,, \tag{7}$$

where $\Delta\phi_{ij}^k = n_s/OF$ as defined in section 3.1.1, $\phi_{ij}^{bs} = \phi_0$ is the reinforcement quantity deposited in the arcs that belongs to the best solution found so far, and finally, the symbol e is a user defined parameter to weigh the the best-so-far tour T^{bs}.

4 Numerical Experiments

In this section details about the numerical simulations are presented and the results for the different versions of ACO are discussed. The parameters which define the undamaged configuration of the dampeg spring-mass system, are taken as follows: $M_i = 10.0$ kg, and $K_i = 2 \times 10^5 N/m$, where $i = 1, \ldots, 10$. The following damage configuration with the respective percentage has been considered: $\mathbf{K^d} = [0.1, 0.15, 0.05, 0.2, 0.5, 0.3, 0.1, 0.15, 0.1, 0.2]\%$.

Concerning the ACO metaheuristic, the size of colony was assumed $n_a = 150$ ants and the stopping criteria of 200 iterations when noiseless experimental data is considered. For the simulations where the experimental data are contaminated by measurement noise, the discrepancy criterion [22] was employed to finish the solution process.

Different simulations were performed according to the noise level in the experimental data. Different intensities of noise are assumed to simulate the errors from the measuring apparatus due to, for example, lack of gauge resolution, drift measures or hysteresis effects, among other practical difficulties. The experimental data are generated computationally based on the displacement data obtained from the solution of direct problem, equation (1), and perturbed by a random bias, according to the following expression:

$$\mathbf{x}^{Exp} = \mathbf{x}^{Mod}[1 + \sigma\mathcal{R}]\,; \tag{8}$$

where σ is the standard deviation of measurement errors and $\mathcal{R}$ is a pseudorandom value from the standard normal (Gaussian) distribution.

Since synthetic experimental data were used, the effective error can be computed as:

$$E(\mathbf{K}) = \sum_{s=1}^{N} \left[\frac{K_s - \hat{K}_s}{K_s}\right]^2\;; \tag{9}$$

where K_s is the exact stiffness value of spring s and $\hat{K}_s$ is the respective estimated value.

4.1 The Hybrid Approach - AS+HJ

In the hybrid approach, the AS algorithm is combined with the Hook-Jeeves local search heuristic. In order to identify the best hybrid strategy to achieve more accurate results, different

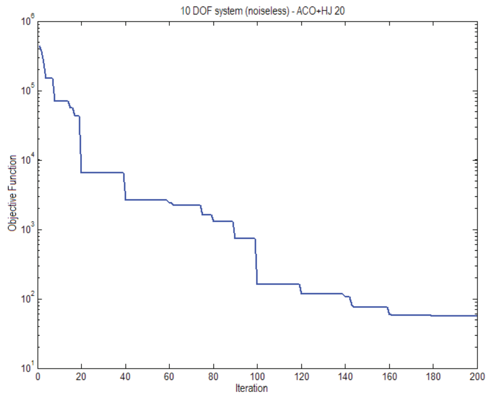

Figure 3: OF evolution for the 10-DOF system, $\sigma = 0\%$ and HJ local search activated after each 20 iterations.

frequencies of calling the local search were evaluated. Also, the hybrid method performance was evaluated for different noise levels.

Based on the results presented in Table 1 it can be seen that there is a suitable routine for the application of local search method. The frequency of calling after every 10 iterations stands out, because better results were achieved for all noise levels, except for $\sigma = 10\%$ for which such frequency provided the second best result. It is also possible to observe that, when experimental data are contaminated by measurement noise, the hybrid method has always provided better results than the ACO applied in isolation.

Method	$\sigma = 0\%$	$\sigma = 1\%$	$\sigma = 5\%$	$\sigma = 10\%$
AS	29.0722	20.8861	25.6597	29.2051
AS+HJ end	18.5038	15.6149	13.7709	12.0315
AS+HJ 50	6.3393	4.7666	6.6205	6.7937
AS+HJ 20	3.7686	4.7666	4.0812	5.7879
AS+HJ 10	2.8920	1.9048	2.3286	4.3357
AS+HJ 5	2.9417	2.7606	5.0644	3.3980

Table 1: Achieved error values for different levels of noise and different frequencies of calling the local search heuristic.

Figure 3 shows the great influence of the local search method for determining the best quality solutions. In the figure, is possible to observe that at every 20 iterations the value of OF has sharp falls. Such behavior demonstrates the effectiveness of local search heuristic coupled with the AS method. In the hybrid approach, the AS algorithm provides a more comprehensive search of the solution space (diversification), while the HJ local search method provides a more intensified search, providing improvements in the final results.

4.2 ACO Variations

The proposed variations of ACO metaheuristic were compared based on the achieved results for the stiffness coefficients identification problem.

Table 2a summarizes the achieved results for each of the cases evaluated. Columns show the average iteration in which the best result is obtained, from a set of 30 runs, the best final objective function value and the error for each of the variations, respectively. The best results were achieved when employed the ASH variation where the rank-based strategy is used together

	Iteration	OF value	Error
AS	107.100	12684.000	41.396
RAS	190.400	2998.000	19.665
ASH	151.400	354.100	2.8373
EAS	153.667	7081.167	29.072

(a)

Error	$\sigma = 0\%$	$\sigma = 1\%$	$\sigma = 5\%$
ASH	2.8373	4.0424	6.5992
ASH+HJ	1.6062	2.7134	1.8253

(b)

Table 2: (a) Achieved results when using different variations of AS algorithm; (b) Achieved errors when using modified AS method.

the heuristic information. It is worth noting that the value of error for the ASH variation is an order of magnitude smaller than the other methods.

Table 2b shows the error values when employed the ASH method solely or coupled with the local search method, applied every 10 iterations, assuming noiseless and noisy experimental data. The hybrid method achieved better results than the original AS, assuming both noiseless and noisy experimental data, reaching lower errors. However, very small amounts of noise compromise the quality of estimation structural stiffness coefficients.

5 Final Remarks

This work presents the application of different variations of the ACO metaheuristic to solve the structural damage detection problem. The study of the variations of the ACO metaheuristic allowed to determine the more suitable strategy to solve de damage identification problem of a damped spring-mass system. The ASH strategy was identified as the one with superior results compared with the others. The reported results showed that also the hybrid method has provided superior results for all the proposed methods, showing that the local search strategy has a key role in intensifying the search for better results. Moreover, the AS algorithm has also proved to be effective in problems with experimental data contaminated by different noise levels. The quality of achieved results suggests the proposed hybrid method as a promising tool in real situations of damage identification. As future works, different structures such as truss and distributed parameters structures should be evaluated as well as experimental data in frequency domain.

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