A Multiparameter almost Superadditive Limit Theorem and its Application to Combinatorial Optimization

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Motivated by the properties of a family of random variables which arises in the context of the traveling-salesman problem, we introduce a strong almost superadditivity condition (SASA) for multiparameter families of random variables which is weaker than a certain notion of superadditivity introduced in [1]. We derive an almost sure limit theorem for families of random variables satisfying the SASA, which generalizes an ergodic theorem of [1]. We also indicate how our limit theorem entails results on the growth of matchings, Steiner trees, traveling-salesman processes as well as triangulations in large areas. These applications have been motivated by [2, 3, 4].

References

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Universal Limit Theorems for the Function Indexed Empirical Process

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Let $\mathcal{F}$ be a class of functions on a measurable space $(X, \mathcal{A})$. Using a new combinatorial criteria, we provide probabilistic and combinatorial characterizations of those $\mathcal{F}$ for which the function indexed empirical process $(P_n - P)(f)$, $f \in \mathcal{F}$, satisfies uniform weak and strong laws of large numbers uniformly over all probability measures P on X. It is shown that the criteria, called Rademacher stability, effectively measures the oscillations of arbitrary function classes and plays a unifying role in the study of general universal limit theorems, including central limit theorems. The criteria extends the classic VC criteria to function classes and is closely related to the Talagrand-Fremlin notion of a stable function class; among other things we use it to provide a combinatorial characterization of those $\mathcal{F}$ satisfying the bounded central limit theorem uniformly over all P.