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Potential Velocity Of Water Waves Propagation With Small Bottom Undulation

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Abstract

Basically, when a wave meets a different depth, it will scatter into a transmitted wave and a reflected wave. In this paper, we study the relevance of sinusoidal beds as shoreline protection through the Bragg scattering mechanism. As well known, a relatively small amplitude of sinusoidal beds can reduce the amplitude of incident waves effectivelly, due to Bragg resonance. Bragg resonance will occur if the wavelength of the monochromatic wave is twice the wavelength of sinusoidal beds. We apply the multiple scale asymptotic expansion method to the linear Shallow initial value problem for sinusoidal beds. the effect of Bragg resonance pass through sinusoidal beds has been studied for surface displacement of water waves. We found that a larger amplitude disturbance leads to larger reflected wave amplitude. This result explains that the long shore sandbar indeed can reduce the amplitude of incident wave. Actually, wave's propagation not only derives from surface displacement, but also from potential velocity. At the end, we show that the sinusoidal beds can reduce the potential velocity of incident wave.

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*Keywords:*Bragg Resonance, potential velocity, sinusoidal beds;

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1. Introduction

Beach damage has becoming familiar to us. Many factors can cause beach damage either from human activity or because of natural activity. The natural activity often occurs due to the tidal wave that comes with large amplitude, i.e.: tsunami. Basically, when a wave meets a different depth, it will scatter into a transmitted wave and a reflected wave. Based on these facts, many researchers analyze the breakwater by exploiting the nature of the wave. It aims to reduce the amplitude of incoming wave so it becomes not dangerous when hit the beach.

Breakwater can be in the form of strong and powerful bar with certain size and distance as needed. The optimum bar size can reduce the incoming wave's amplitude optimally[1]. Bottom undulation on seabed can occur naturally and it forms can also split the wave into transmission and reflection. Based on this analysis the author is interested to learn about the effect of the bottom undulation on seabed in minimizing the amplitude of the wave that transmitted to the beach.

Bragg resonance gives a significant influence to reduced wave amplitude. It occurs when a wave propagates through the impermeable sinusoidal beds which wave number is twice the incoming wave numbers[2]. The influences of Bragg resonance and current on the wave propagation over permeable beds have also been studied[3,5]. In another side, LH Wiryanto[7] has also been reviewing propagation on unsteady wave through the permeable beds.

The presence of hard-wall beach on the right of sinusoidal patch will increase the incident wave amplitude that hit the shore much higher, and hence increase the hazard to the shore. The situations will less severe if the shore can absorb the wave partially[4,8].

These researches about the modeling of wave propagation through the bottom undulation on deviation of the water surface. In this paper, we will study the other factors that can affect on waves propagation, specifically on potential velocity of water. The potential velocity is the velocity of water particles; so that the velocity will be different on each position and times. Potential velocity of water is analogous as the first derivative function of the surface displacement of the water. We will analyze whether potential velocity give the significant effect for wave amplitude or not. The method used to obtain approximations solution is multiple scale asymptotic expansion method.

2. Boundary Value Problem of Potential Velocity

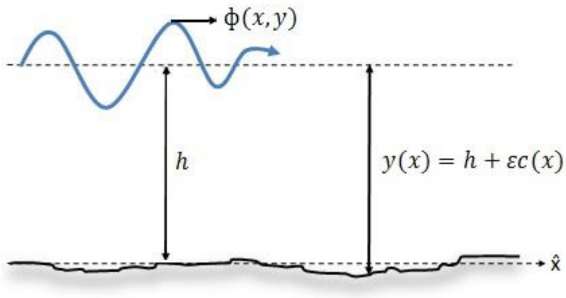

Figure 1. Water waves with small bottom undulation

See figure 1. Consider the depth of water as follows

$$y(x) = h + \varepsilon c(x), \tag{1}$$

where c(x) is a function that representation of bottom topography, h is flat depth, ε is small undimensioneless parameter (ε«1). The parameter ε is used to express that the amplitude of bottom topography is relatively small compared with the flat depth.

Assume that the water is *incompressible* (material density is constant within a fluid parcel) and *irrotational,* so that we use Laplace equation for governing equation as follows

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \qquad -\infty < x < \infty, 0 \leq \mathrm{y} \leq \mathrm{h} + \varepsilon \mathrm{c(x)} \tag{2}$$

$$\frac{\partial \phi}{\partial y} + K\phi = 0, \qquad y = 0 \tag{3}$$

$$\frac{\partial \phi}{\partial n} = 0, \qquad y(x) = h + \varepsilon c(x) \tag{4}$$

where $\phi(x, y)$ is potential velocity of surface wave at position x and time t, $K = \frac{\sigma^2}{g}$ is angular frequency constant of incoming wave with time function $e^{-i\sigma t}$, g is gravitation coefficient , and $\frac{\partial \phi}{\partial n}$ directional derivative at point (x, y).

Incoming wave represented by

$$\phi_0(x, y) = \cosh\ k_0(h - y)e^{ik_0 x} \tag{5}$$

where k_0 is monochromatic wave number, and K declared as

$$K = k \tan \mathrm{h}\, kh \tag{6}$$

When a wave meets a different depth, it will scatter into a transmitted and reflected wave. Let us imagine an incident wave running above a flat bottom with a small undulation. During its evolution, there occur many scattering processes. Assume that the beach on the right of the small undulation beds can absorb wave completely, then the transmitted wave is wave that running to right and reflected wave is the opposite. Therefore, the potential velocity can be written as

$$\phi(x = \begin{cases} \phi_0(x, y) + R\phi_0(-x, y), & x \to -\infty \\ T\phi_0(x, y), & x \to +\infty \end{cases} \tag{7}$$

where R and T are reflected wave and transmitted wave coefficient, respectively.

Substitute equation (5) into (7) to get

$$\phi(x, y) = \begin{cases} (e^{ik_0 x} + Re^{-ik_0 x}) \cosh\ k_0(h - y), & x \to -\infty \\ Te^{ik_0 x} \cosh(h - y), & x \to +\infty \end{cases} \tag{8}$$

When the bottom satisfy the condition $\frac{\partial \phi}{\partial n} = 0$ at $y = h + \varepsilon c(x)$, then these condition can be change to first order equation as

$$\frac{\partial \phi}{\partial y} - \varepsilon \left\{ c(x) \frac{\partial \phi}{\partial x} \right\} = 0, \qquad y = h \tag{9}$$

3. Transmitted And Reflected Coefficient

Using multiple scale expansion, we expand

$$\begin{gathered}\phi = \phi_0 + \varepsilon\phi_1 + O(\varepsilon^2)\\ R = \varepsilon R_1 + O(\varepsilon^2)\\ T = 1 + \varepsilon T_1 + O(\varepsilon^2)\end{gathered} \tag{10}$$

Substitute equation (10) into (2), (3), (8), and (9) so that the boundary value problem for orde ε, as follows

$$\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_1}{\partial y^2} = 0, \qquad -\infty < x < \infty, 0 \le \mathrm{y} \le \mathrm{h} + \varepsilon \mathrm{c(x)} \tag{11}$$

$$\frac{\partial\phi_1}{\partial y} + K\phi_1 = 0, \qquad y = 0 \tag{12}$$

$$\frac{\partial\phi_1}{\partial n} = ik_0\frac{d}{dx}\{c(x)e^{ik_0x}\} \equiv p(x), \qquad y = h \tag{13}$$

$$\phi_1(x,y) \sim \begin{cases} R_1 e^{-ik_0x} cosh\; k_0(h-y), & x \to -\infty \\ T_1 e^{ik_0x} \cosh\; k_0(h-y), & x \to +\infty \end{cases} \tag{14}$$

Assume that the characteristic of potential velocity at order O(ε) and order O(1) are the same, that is periodically monochromatic wave and expressed as a complex function. Because of that, we use Fourier transform to solve boundary value problem (11) – (14) which is infinite domain. After these transformations, we get the reflected and transmitted wave coefficient as follows

$$\mathrm{R}_1 = \frac{-2ik_0^2}{2k_0h + \sinh 2k_0h}\int_{-\infty}^{\infty} c(x)e^{2ik_0x}dx \tag{15}$$

$$\mathrm{T}_1 = \frac{-2ik_0^2}{2k_0h + \sinh 2k_0h}\int_{-\infty}^{\infty} c(x)dx \tag{16}$$

4. Analytic Solution for Sinusoidal Beds

Consider that the bottom undulation is a sinusoidal bed, and then c(x) can be written as

$$c(x) = \begin{cases} a\sin(lx), & L1 \le x \le L2 \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

where a is amplitude of sinusoidal beds and l is wave number of sinusoidal beds.
Substitute equation (17) into (15), we get

$$R1 = \frac{-2ik_0^2}{2k_0h + \sinh 2k_0h}\int_{L1}^{L2} a\sin(lx)e^{2ik_0x}\,dx \tag{18}$$

In equation (18), write the term $\int_{L1}^{L2} a\sin(lx)e^{2ik_0x}dx$ into the form

$$a \int_{L1}^{L2} \sin(lx)\,[\cos 2ik_0x + \mathrm{i}\sin 2ik_0x]dx \tag{19}$$

Using algebra manipulation and integrate the equation (19), we get

$$\frac{1}{2}a\left\{\frac{-1}{l+2k_0}[\cos(l+2k_0)x + i\sin(l+2k_0)\,x] - \frac{1}{l-2k_0}[\cos(l-2k_0)x - i\sin(l-2k_0)\,x]\right\}_{L1}^{L2} \tag{20}$$

That can be changed into exponent form, that is

$$\frac{1}{2}a\left\{\frac{-e^{(l+2k_0)x}i}{l+2k_0} - \frac{e^{-(l-2k_0)x}i}{l-2k_0}\right\}_{L1}^{L2} \tag{21}$$

Substitute (21) into (18), and the choose

$$L_1 = \frac{-n\pi}{l} \qquad \text{and} \qquad L_2 = \frac{m\pi}{l}$$

So we get

$$R1 = \frac{-2ik_0^2a}{2k_0h + \sinh 2k_0h}\,\frac{l}{l^2-(2k_0)^2}[(-1)^n e^{2ik_0L1} - (-1)^m e^{2ik_0L2}] \tag{22}$$

Here, take the value of m and n so that satisfy $\frac{(m+n)}{2} = l$ and $m = n$, then the reflected coefficient represented by

$$R1 = \frac{2k_0a}{2k_0h + \sinh 2k_0h}\,\frac{(-1)^m(\frac{2k_0}{l})}{(\frac{2k_0}{l})^2}\sin(\frac{2k_0m\pi}{l}) \tag{23}$$

5. Simulation

There is no relevant data in Indonesia to this research. Because of that, in this paper, analytic simulation using the data of Martha, Bora, & Chakrabarti, 2009 shown in Table 1. We use these data because these data is relevant and more useful for this research. Our researches are the same topic, but use different method for solving the governing equation and numerical method. At the end, we compare the result of this research with them.

Table 1. Data of analytic simulation

Variable	Value
Amplitude of sinusoidal beds/ flat depth (a/h)	0.1
Wave number of sinusoidal beds × flat depth $(l\,xh)$	1
Wave number of sinusoidal beds (l)	1,3,5

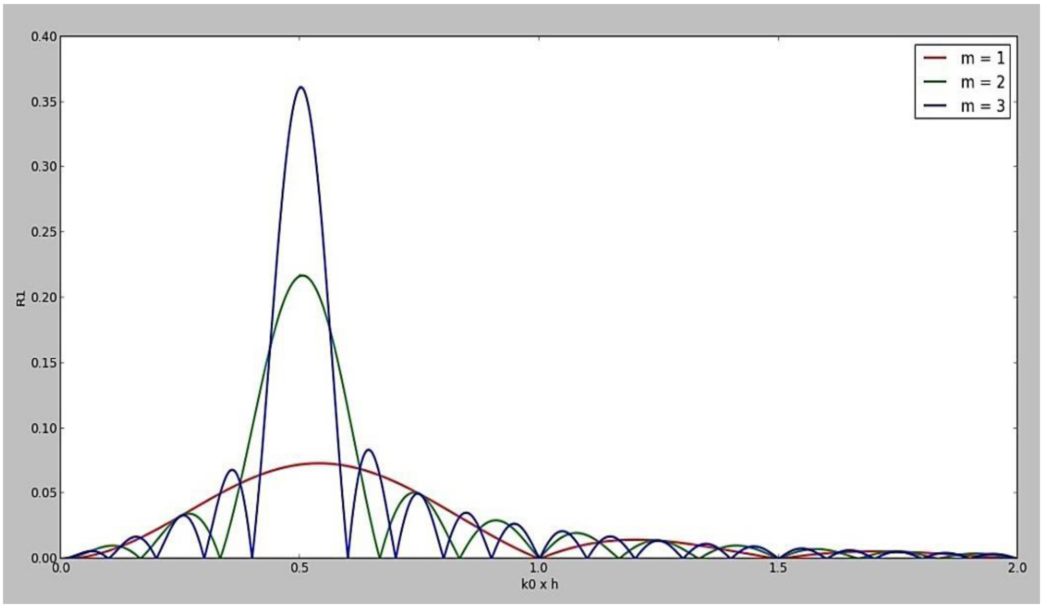

Figure 2. The reflected coefficient (Analytic Solution)

Figure 2 shows the reflected coefficient as a function of incident wave number multiply by flat bottom (k_0h). The reflected coefficient plotting for some cases wave numbers of sinusoidal beds. The reflected coefficient when k_0 = 1, 3, and 5 represented by red, green, and blue line, respectively.

Table 2.The maximum reflection coefficient

Wave number of sinusoidal beds (l)	**Flat depth (h)**	**k_0 x h**	**The maximum reflected coefficient (R1 maximum)**	**Wave number of incident wave (k_0) when R1 Maximum**
1	1	0.5	0.073166	0.5
3	1/3	0.5	0.217668	1.5
5	1/5	0.5	0.361306	2.5

After see Figure 2 and table 2, we know that for same value of a/h, the larger amplitude of sinusoidal beds leads larger amplitude of reflected wave. Besides that, it shows that the maximum reflected coefficient occur when the wave number of sinusoidal beds is twice the wave number of incident wave.

6. Conclusion

Sinusoidal beds give the effect for surface wave propagation. The solution of wave equation over sinusoidal-beds based on potential velocity obtains by perturbation method and Fourier transform. Sinusoidal beds may lead to Bragg resonance. Bragg resonance occurs when wavelength of incident wave is twice of the wavelength of the periodic bottom disturbance. The larger amplitude of sinusoidal beds and potential velocity leads larger amplitude of reflected wave and leads smaller amplitude of transmitted wave. The maximum reflected coefficient occurs when the wave number of sinusoidal beds is twice the wave number of incident wave.

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