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Asteroidal triples of moplexes

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Abstract

An asteroidal triple is an independent set of vertices such that each pair is joined by a path that avoids the neighborhood of the third, and a moplex is an extension to an arbitrary graph of a simplicial vertex in a triangulated graph. The main result of this paper is that the investigation of the set of moplexes of a graph is sufficient to conclude as to its having an asteroidal triple. Specifically, we show that a graph has an asteroidal triple of vertices if and only if it has an asteroidal triple of moplexes. We also examine the behavior of an asteroidal triple of moplexes in the course of a minimal triangulation process, and give some related properties. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The concept of asteroidal triple of vertices was introduced in 1962 by Lekkerkerker and Boland [15] to characterize the subclass of triangulated graphs which are also interval graphs.

Recently, Corneil et al. [5] extended this concept to non-triangulated graphs, thus defining the class of AT-free graphs (i.e. asteroidal triple-free graphs), which turns out to be a superclass of a variety of well-studied classes, such as interval graphs, permutation graphs, trapezoidal graphs and cocomparability graphs.

AT-free graphs have given rise to a large amount of research in the past few years (see [14,16,17,12,11]), including the discovery of a dominating pair of vertices (i.e. a

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pair xy such that every xy-path is a dominating set in G, see [5]), which can be found with a double pass of LexBFS (see [4]).

One of the developments which is of interest to us in this paper is the relationship which Möhring [16] established between AT-free graphs and their minimal triangulations, namely that if a graph is AT-free, then all its possible minimal triangulations are interval graphs. Parra [17], and, independently Corneil et al. [5] proved the converse, so that this property characterizes AT-free graphs (the reader is referred to [15,7] for precise definitions on interval graphs):

Characterization 1.1. *A graph G is AT-free iff every minimal triangulation of G is an interval graph.*

Our contribution to this is the study of the behavior of the "extremities" of an asteroidal triple-free graph.

In their original paper, Lekkerkerker and Boland, in view of defining the end intervals of the representation of an interval graph, took a close interest in its extremities; in fact, they actually, independently from Dirac [6], defined the concept of simplicial vertex, and showed the not very well-known property that a triangulated graph has an asteroidal triple of vertices if and only if it has an asteroidal triple of *simplicial* vertices.

The idea behind this is that, given one element x of an asteroidal triple of vertices $\{x, y, z\}$, one can always go "farther away" from y and z, until finding an "extremity" x' such that $\{x', y, z\}$ is an asteroidal triple of vertices.

Our previous research had led us to generalize the concept of simplicial vertex in a triangulated graph to that of *moplex* in an arbitrary graph. We used this in [1] to extend Dirac's famous theorem on the presence of simplicial vertices in a triangulated graph to an arbitrary graph [6], and to show that any non-clique graph has at least two moplexes.

In this paper, we likewise extend the property mentioned above, to show that a graph is asteroidal if and only if it has an asteroidal triple of moplexes. We also apply this result to show how to use an asteroidal triple of moplexes to construct a minimal triangulation into a non-interval graph, thus explaining Characterization 1.1 as well as offering an alternate proof for it.

The paper is organized as follows: we first give a few preliminaries on graphs, separation, moplexes and asteroidal triples. In Section 3, we prove our main result and give some observations on special kinds of asteroidal triples. We then go on in Section 4 to explain the mechanisms which govern the preservation of an asteroidal triple of moplexes in the course of a minimal triangulation process. We first give some necessary specialized results on the relationship between triangulation and separation.

2. Preliminaries

We will need some notations and definitions on graphs as well as on separation.

Graphs. All graphs in this work are undirected, connected and finite. A graph is denoted $G=(V,E)$, with $n=|V|$, $m=|E|$. $G(A)$ denotes the subgraph induced by vertex set A. A *clique* is a set of pairwise adjacent vertices; an *independent set* is a set of pairwise non-adjacent vertices. We will say that we *saturate* a set $A\subset V$ if we add all the edges necessary to make A into a clique.

The *neighborhood* of a vertex x is $N(x)=\{y\neq x\,|\,xy\in E\}$; we will say that a vertex x *sees* another vertex y iff $xy\in E$. A vertex is *simplicial* if its neighborhood is a clique. The neighborhood of a set of vertices A is $N(A)=(\bigcup_{x\in A}N(x))-A$. We will say that two disjoint vertex sets A and B *see* each other if there is a vertex a in A which sees some vertex b in B. A *module* is a set A of vertices which share the same external neighborhood $(\forall a_i,a_j\in A,\ N(a_i)\cap N(A)=N(a_j)\cap N(A))$.

A graph is *triangulated* (or *chordal*) if it has no chordless cycle of length greater than 3. A triangulated graph $H=(V,E+F)$ is a *minimal triangulation* of $G=(V,E)$ if there is no proper subset F' of F such that $H=(V,E+F')$ is triangulated (see [19]).

Separation. For $X\subseteq V$, $\mathscr{C}(X)$ denotes the set of connected components of $G(V-X)$ (connected components are vertex sets). $S\subset V$ is called a *separator* if $|\mathscr{C}(S)|\geqslant 2$, an *ab-separator* if a and b are in different connected components of $\mathscr{C}(S)$, a *minimal ab-separator* if S is an *ab*-separator and no proper subset of S is an *ab*-separator, a *minimal separator* if there is some pair $\{a,b\}$ such that S is a minimal *ab*-separator. A component C of $\mathscr{C}(S)$ is called *full* if $N(C)=S$ (i.e. every vertex of S sees C). $\mathscr{S}(G)$ denotes the set of minimal separators of G.

Moplexes. A moplex (see [1]) is defined as a set X of vertices which form a clique and share the same neighborhood, with the additional requirement that $N(X)$ be a minimal separator of the graph. ($X\subset V$ is called a *moplex* iff (1) X is a clique, (2) $\forall x,x'\in X$, $N(x)\cup x=N(x')\cup x'$, and (3) $N(X)$ is a minimal separator of G).

Note that a given vertex belongs to at most one moplex, which is a maximal clique module of the graph. A moplex can be seen as obtained by vertex expansion, behaves as a single vertex, and can be contracted without disturbing the set of paths, cycles and minimal separators. We shall thus extend to moplexes definitions which are basically meant for vertices:

- We will say that two moplexes A and B are *adjacent* if A sees B (note that this is equivalent to saying that $A\subset N(B)$ and also equivalent to $B\subset N(A)$).
- We will say that μ is a path from a moplex X to a moplex Y if μ has one extremity which is a vertex of X and the other which is a vertex of Y.
- We will say that a moplex is *simplicial* if its neighborhood is a clique.

We state the following theorem, which generalizes Dirac's [6] well-known theorem for simplicial vertices in a triangulated graph, which allows us to conclude that a non-clique graph has at least two non-adjacent moplexes:

Theorem 2.1 (Berry and Bordat [1]). *Let G be a non-clique graph, let S be a minimal separator of G; in each component of $\mathscr{C}(S)$, there is at least one moplex.*

Asteroidal triples:

Definition 2.2 (*Lekkerkerker and Boland* [15]). An *asteroidal triple of vertices* is a triple $\{x_1,x_2,x_3\}$ such that between any pair $\{x_i,x_j\}$ of the triple, there is a path that the third vertex x_k does not see. A graph which has an asteroidal triple of vertices is said to be *asteroidal*, and a graph is said to be *AT-free* if it is not asteroidal.

The following property is actually an alternate definition, interesting to us because it is related to separation:

Property 2.3.
An independent set $\{x_1,x_2,x_3\}$ of vertices form an asteroidal triple if for any pair $\{x_i,x_j\}$ of the triple, x_i and x_j belong to the same connected component of $\mathscr{C}(N(x_k))$, where x_k is the third vertex of the triple.

Unbreakable graphs.

Definition 2.4. A separator S is called *a star cutset* if there is some vertex x in S which sees all the other vertices of S. A graph is said to be *unbreakable* if there is no star cutset in G nor in its complement.

3. Asteroidal triples of moplexes

In this section, we use the concept of moplex to extend to an arbitrary graph the following result of Lekkerkerker and Boland [15]: "A graph is triangulated and asteroidal iff it contains an asteroidal triple of simplicial vertices".

3.1. Using moplexes to characterize AT-free graphs

Definition 3.1. We will call *asteroidal triple of moplexes* a triple of pairwise non-adjacent moplexes $\{X_1,X_2,X_3\}$ such that for any pair $\{X_i,X_j\}$ of the triple, there is a path from X_i to X_j which the third moplex X_k does not see.

We will give an example before proceeding.

Example 3.2. A non-triangulated graph (Fig. 1).

Set of minimal separators: $\mathscr{S}(G)=\{\{c,d\},\{c,f\},\{d,f\},\{d,g\},\{d,x\},\{f,x\},$ $\{c,g,z\},\{d,e,y\},\{e,x,y\},\{g,x,z\},\{e,g,y,z\}\}$.

Set of moplexes: $\{\{e,y\},\{c\},\{f\},\{h\},\{a,b\}\}$.

$\{\{x\},\{y\},\{z\}\}$ is an asteroidal triple of vertices, but one can go "farther away" in each direction and find an asteroidal triple of moplexes, for example

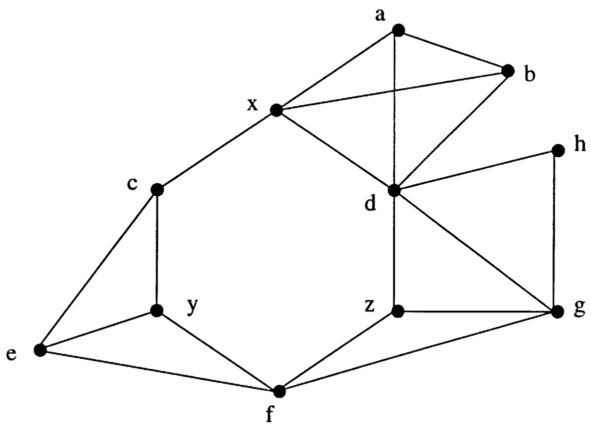

Fig. 1. A non-triangulated graph.

$\{\{a,b\},\{e,y\},\{g,h\}\}$. $\{a,b\}$ is a simplicial moplex, y belongs to moplex $\{b,y\}$, while x does not belong to any moplex. Note that any combination of vertices from $\{a,b\} \times \{e,y\} \times \{g,h\}$ will form an asteroidal triple of vertices.

Main Theorem 3.3. *A graph G has an asteroidal triple of vertices iff it has an asteroidal triple of moplexes.*

Proof. $\Longleftarrow$: Let $\{X,Y,Z\}$ be an asteroidal triple of moplexes of graph G, let x be in X, y be in Y, z be in Z. We claim that $\{x,y,z\}$ is an asteroidal triple of vertices. Let μ be a path from Y to Z which does not see X, let $y' \in Y$ and $z' \in Z$ be the extremities of μ. Since by definition of a moplex y and y' (resp. z and z') are adjacent and share the same neighborhood, we can construct a path $yy' \rightarrow z'z$ which does not see X either, and thus in particular does not see x.

$\Rightarrow$: Let $\{x,y,z\}$ be an asteroidal triple of vertices of graph G. By Property 2.3, y and z are in the same component C of $\mathscr{C}(N(x))$. Let us denote $S_x = N(C)$; clearly, S_x is a minimal separator, separating x from y and z. Let C_x be the component of $\mathscr{C}(S_x)$ which contains x. By Theorem 2.1, C_x contains at least one moplex X. y and z lie in the same component C_{yz} of $\mathscr{C}(N(X))$, and $C \subseteq C_{yz}$, thus if $x \notin X$, there must be a path xX in C_x which neither y nor z can see.

Let us likewise construct moplexes Y and Z corresponding to vertices y and z by way of minimal separators S_y and S_z.

We claim that $\{X,Y,Z\}$ is an asteroidal triple of moplexes. Let us construct a path from Y to Z which does not see X. There is some path μ from y to z in C. As seen before, there must be two paths Yy and zZ which x fails to see; consequently, these paths lie entirely in C. We can thus construct a path $Yy \cup \mu \cup zZ$ from Y to Z in C which X cannot see, since $C \subseteq C_{yz}$. Note that if $x \in X$, $y \in Y$, or $z \in Z$, the proof remains valid. $\square$

Theorem 3.3 can be used to reformulate the characterization for interval graphs given in [15] to: "G is an interval graph iff G is triangulated and has no asteroidal triple of moplexes".

As a triangulated graph may have simplicial vertices which do not belong to any moplex, this strengthens the previous formulation.

In relationship with the notion of extremity in an AT-free graph, let us mention the use made by [4] of a double pass of LexBFS (see [19] for a description of this algorithm) to find a dominating pair of vertices in an AT-free graph. In [1], we show that a pass of LexBFS will define a moplex, so that in fact the double pass of LexBFS defines a dominating pair of *moplexes*. This process unfortunately does not yield a pair belonging to an asteroidal triple of moplexes in an asteroidal graph.

3.2. Compact moplexes

Another interesting concept defined by [15] is that of *strongly simplicial point*, which is a simplicial vertex x such that the removal of its neighborhood leaves a single component other than x itself (i.e. $G(V - x \cup N(x))$ is connected).

Lekkerkerker and Boland [15] claim that in an interval graph, any such vertex is an endinterval in any representation of G. Clearly, this notion is closely related to separation, and can be extended to define a special type of moplex.

Definition 3.4. We will say that a moplex A of G is compact if $G(V - (A \cup N(A)))$ is connected.

The notions of strongly simplicial point and of compact moplex are interesting primarily because there is no equivalence between the presence of an independent set of moplexes and that of an asteroidal set of moplexes, as a graph may well have an independent set of moplexes which is not asteroidal, as for example the three extremities of a K_{13}.

For compact moplexes, however, we have the following result:

Property 3.5. *If a graph G has three pairwise non-adjacent compact moplexes, they form an asteroidal triple of moplexes.*

Proof. Let $\{X, Y, Z\}$ be a triple of pairwise non-adjacent compact moplexes. Since X is compact, Y and Z must lie in the same component of $\mathscr{C}(X)$, so there must be a path from Y to Z which lies entirely in this component, and thus does not intersect $N(X)$. □

We shall illustrate the importance of the notion of compact moplex by a brief digression on AT-free graphs in relation to the proof of the Strong Perfect Graph Conjecture (the reader is referred to [7,8] for details).

In answer to the natural question: "Do some graphs have as many as n moplexes?", we showed (see [2]) that the class of unbreakable graphs is characterized as the class such that every vertex is a compact moplex in the graph as well as in its complement.

In this class, the notion of moplex is thus identical to the notion of vertex, and the notion of independent set is identical to that of asteroidal set, so in a non-clique AT-free unbreakable graph, the maximum size of an independent set is exactly two, which is the basis for the argument used by Maffray (see private communication in [5]) to show that the Strong Perfect Graph Conjecture holds for the class of AT-free graphs.

3.3. Complexity issues

Computing the set of moplexes of a graph can be done in $O(nm)$ time, by first computing the set of maximal clique modules of the graph in $O(m)$ time, for instance with the algorithm based on LexBFS described by Hsu and Ma in [9]. For each maximal clique module, we can check by an $O(m)$ time graph search whether it is a moplex as well as whether it is compact.

Whenever more than two compact moplexes are found, or the number of moplexes is small, one can thus conclude in $O(nm)$ time as to the asteroidality of the graph.

Some classes of graphs have few moplexes. As an example, proper interval graphs have been studied by Roberts [18], who shows that these graphs have exactly two extremities, which are moplexes, as shown in [1]. Actually, we can use Robert's results to characterize proper interval graphs as graphs such that every non-clique connected subgraph has exactly two moplexes.

In the general case, however, as we have seen in the previous subsection, a graph may have as many as n moplexes.

Consequently, Theorem 3.3 fails to yield a better complexity for the recognition of AT-free graphs, which is currently close to $O(n^3)$, a straightforward result established in [15].

4. Preservation of an asteroidal triple of moplexes in the course of a triangulation process

In this section, we study the aspects of asteroidality related to Characterization 1.1. Recent research has shown that minimal triangulation is closely related to minimal separation (see [14,17]); however, the proofs given for Characterization 1.1 do not involve minimal separation: one of our contributions is to establish this relationship.

A graph has, in a general fashion, an exponential number of minimal separators. Triangulated graphs have less than n, and Dirac [6] characterized these graphs as having only minimal separators that are cliques. The process of computing a minimal triangulation simply forces the graph into respecting this characterization, and can be described by the following procedure: repeat, until all the minimal separators of the graph are cliques: "Choose a minimal separator S and saturate it".

Whenever this step is executed, this causes a number of minimal separators to disappear. Thus during the process, the set of minimal separators shrinks until it reaches the terminal size of less-than-n.

The minimal separators which disappear at some step are well defined as the separators which *cross* S, a notion introduced by Kloks et al. [13] and studied also by Parra [17].

Let us now examine what happens to moplexes in the course of a triangulating process.

Whenever a moplex is made simplicial by the saturation of the minimal separator which is its neighborhood, it will be preserved throughout the entire process. The set of moplexes of the triangulated graph obtained is thus exactly the set of moplexes which have been made simplicial.

The investigation of the crossing relation between separators defined by a moplex leads to a very simple result: two moplexes define crossing separators if they are adjacent. In view of this, if a graph is asteroidal, it is easy to construct a minimal triangulation which is also asteroidal, by simply choosing any triple of moplexes forming an asteroidal triple and making them simplicial.

4.1. Preliminaries

We will first give some technical results which we need to complete our proofs.

Definition 4.1 (*Kloks et al.* [14])**.** Let G be a graph, let S and T be in $\mathscr{S}(G)$. S and T are said to be *crossing separators* if $\exists C_1, C_2 \in \mathscr{C}(T)$, $C_1 \neq C_2$ such that $S \cap C_1 \neq \emptyset$ and $S \cap C_2 \neq \emptyset$.

Lemma 4.2 (Parra [17])**.** *Let G be a graph, let S and T be in $\mathscr{S}(G)$. S and T are crossing separators iff $S \cap C \neq \emptyset$ for every full component C of $\mathscr{C}(T)$.*

We will compress the results obtained in [14,10,17,3] into the following:

Property 4.3. *Let G be a graph. Let G' be the graph obtained from G by saturating a set of pairwise non-crossing minimal separators.*

1. *Every minimal triangulation H of G' is a minimal triangulation of G.*
2. *$\mathscr{S}(H) \subseteq \mathscr{S}(G') \subseteq \mathscr{S}(G)$, and $\mathscr{S}(H)$ forms a maximal set of pairwise non-crossing minimal separators of G', and also of G.*
3. *A minimal separator S of H defines the same connected components in H as in G' and as in G.*
4. *Every moplex of G' is a moplex of G, with the same neighborhood and the same connected components in G' as in G.*

4.2. Triangulation and asteroidal triples of moplexes

We will now give some results from which will follow an alternate proof of Characterization 1.1, based on the study of the behavior of the set of moplexes in the course of a triangulating process.

Lemma 4.4. *Let H be a minimal triangulation of a graph G. Any asteroidal triple of moplexes of H is also an asteroidal triple of moplexes of G.*

Proof. Let G be a graph, let H be a minimal triangulation of G. Let $\{X, Y, Z\}$ be an asteroidal triple of moplexes of H. By definition of a moplex, $N(X)$, $N(Y)$ and $N(Z)$ are minimal separators of H. By Definition 3.1, Y and Z lie in the same component of $\mathscr{C}(N(X))$ in H.

By Property 4.3, X, Y and Z are moplexes of G, and their neighborhoods define the same components in G as in H. $\{X, Y, Z\}$ is thus also a asteroidal triple of moplexes of G. □

Lemma 4.5. *Let X, Y be moplexes of a graph G. $N(X)$ and $N(Y)$ are crossing separators iff X and Y are adjacent.*

Proof. ⇒: Let X, Y be moplexes such that $N(X)$ and $N(Y)$ are crossing separators. By Lemma 4.2, $N(X)$ has a vertex in each component of $\mathscr{C}(N(Y))$, and in particular in Y. X and Y are thus adjacent.

⇐: Let X, Y be adjacent moplexes. X is a full component of $\mathscr{C}(N(X))$, and $X \cap N(Y)$ is non-empty, since it contains X. By definition, $N(X)$ and $N(Y)$ are crossing. □

Lemma 4.6. *Let G be a graph, let $\{X, Y, Z\}$ be an asteroidal triple of moplexes of G, let G' be the graph obtained from G by saturating $N(X)$, $N(Y)$ and $N(Z)$. Every minimal triangulation of G' is a minimal triangulation of G, and features $\{X, Y, Z\}$ as asteroidal triple of moplexes.*

Proof. Let G be an asteroidal graph, let $\{X, Y, Z\}$ be an asteroidal triple of moplexes of G. By Lemma 4.5, $N(X)$, $N(Y)$ and $N(Z)$ form a set of pairwise non-crossing separators. Let us saturate $N(X)$, $N(Y)$ and $N(Z)$, thus obtaining graph G', and let H be a minimal triangulation of G'. By Property 4.3, H is a minimal triangulation of G. X, Y and Z are moplexes of H, and $N(X)$, $N(Y)$ and $N(Z)$ define the same components as in G. $\{X, Y, Z\}$ must be an asteroidal triple of moplexes of H. □

Characterization 1.1 follows from Lemmas 4.6 and 4.4, expressed as: '*A graph has an asteroidal triple of moplexes iff some minimal triangulation of G has an asteroidal triple of moplexes*'.

In conclusion for this section, let us point out that, while Lemma 4.6 gives us an easy way of triangulating an asteroidal graph into a non-interval graph, some asteroidal

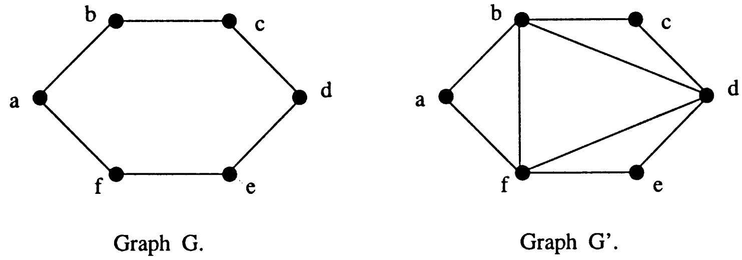

Fig. 2. Triangulating an asteroidal graph into a non-interval graph. Graph G has 6 compact moplexes, is unbreakable and asteroidal. Graph G' is obtained from G by saturating the neighborhoods of moplexes $\{a\}, \{c\}$, and $\{e\}$.

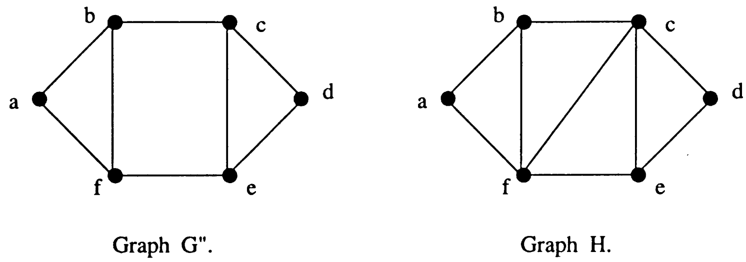

Fig. 3. Triangulating an asteroidal graph into an interval graph. Graph G'', which is AT-free, is obtained from G by making $\{a\}$ and $\{d\}$ simplicial. Graph H is an interval graph, and a minimal triangulation of both G'' and G.

graphs may also have some minimal triangulation into an interval graph. However, it seems that it is not so easy to compute such a minimal triangulation, except on special graph classes, for example those whose moplexes are all compact.

Example 4.7. Graph G is an unbreakable graph, so all its vertices are moplexes. Any independent set of vertices will form an asteroidal triple of moplexes, for example $\{\{a\}, \{c\}, \{e\}\}$ (Fig. 2).

In order to obtain a minimal triangulation into a non-interval graph, we need only to saturate the neighbourhoods $N(a), N(c)$ and $N(e)$. If, on the other hand, we wish to obtain a minimal triangulation into an interval graph, we will choose to saturate the neighborhoods of a maximal independent set of size 2, for example $N(a)$ and $N(d)$.

5. Conclusion

Though we give a new characterization for AT-free graphs, this fails to improve the recognition worst-time complexity for this class. We also leave open the problem of determining in polynomial time whether a given asteroidal graph has at least one minimal triangulatoin into an interval graph (Fig. 3).

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