# Lepton number asymmetry via inflaton decay in a modified radiative seesaw model 

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#### Abstract

We propose a non-thermal scenario for the generation of baryon number asymmetry in a radiative neutrino mass model which is modified to realize inflation at the early Universe. In this scenario, inflaton plays a crucial role in both generation of neutrino masses and lepton number asymmetry. Lepton number asymmetry is firstly generated in the dark matter sector through direct decay of inflaton. It is transferred to the lepton sector via the dark matter annihilation and then converted to the baryon number asymmetry due to the sphaleron interaction. All of the neutrino masses, the baryon number asymmetry and the dark matter are intimately connected to each other through the inflaton.


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## 1. Introduction

Recent experimental and observational data for neutrino masses [1,2] and dark matter (DM) [3-5] suggest that the standard model (SM) should be extended. The radiative neutrino mass model proposed in [6] is such a simple extension of the SM with an inert doublet scalar and right-handed neutrinos. It seems to be a promising candidate which could take the place of the famous canonical seesaw model for neutrino masses [7]. An interesting point of this model is that it could also give the origin of DM [8,9]. A $Z_{2}$ symmetry imposed to forbid the neutrino masses at tree-level could guarantee the stability of the lightest $Z_{2}$ odd field, which could be DM. In this model, DM is an indispensable ingredient for the neutrino mass generation at TeV regions.

Although the model has such interesting aspects, baryon number asymmetry in the Universe [10], which is another crucial problem of the SM, cannot be easily explained in a consistent way with the relic abundance of DM. If we suppose the ordinary thermal leptogenesis [11,12], the sufficient baryon number asymmetry can be generated only in the case where the model has a finely tuned spectrum for the $Z_{2}$ odd fields.

If the lightest right-handed neutrino is assumed to be DM, both its relic abundance and small neutrino masses require $O$ (1) neutrino Yukawa couplings in general ${ }^{1}$ [8]. They can allow to cause

[^0]large $C P$ asymmetry in the decay of right-handed neutrinos even if their masses are of $O(1) \mathrm{TeV}$. However, the same neutrino Yukawa couplings could cause large washout of the generated lepton number asymmetry through the inverse decay and the lepton number violating scattering processes. As a result, the thermal leptogenesis is not easy to generate sufficient lepton number asymmetry in a consistent way with the neutrino oscillation data and the DM abundance at least in the simplest form of the model [13]. On the other hand, if the lightest neutral component of the inert doublet scalar is assumed to be DM [14], the neutrino Yukawa couplings could be small enough to be consistent with both the DM relic abundance and the small neutrino masses. However, the large $C P$ asymmetry in the decay of right-handed neutrinos requires fine mass degeneracy among the right-handed neutrinos [15]. Non-thermal leptogenesis [16,17] might give another consistent scenario for the origin of the baryon number asymmetry in this model or its supersymmetric extension [18].

In this paper, to solve the above mentioned fault for leptogenesis, we propose a simple scenario in the model which is extended so as to incorporate the inflation at the early Universe [19]. The neutrino mass generation is connected with the inflation through the inflaton interaction. The lepton number asymmetry is also produced through the inflaton decay in the inert doublet sector which contains the DM candidate [17,19]. After this lepton number asymmetry is transferred to the lepton sector via lepton number conserving scattering processes, the sphaleron interaction converts a part of it to the baryon number asymmetry.

Remaining parts of the paper are organized as follows. In the next section, we introduce the extended model briefly. In Section 3, we study its phenomenological features. Firstly, we describe the
inflation in the model and also the small neutrino mass generation. After that, we explain the scenario for the generation of the lepton number asymmetry and then estimate the baryon number asymmetry expected to be produced finally. Following this discussion, the consistency of the scenario with DM phenomenology is examined. Relation between the present DM scenario and the asymmetric DM scenario is also remarked. We summarize the paper in Section 4.

## 2. An extension of the radiative seesaw model

Our model considered here is based on the one proposed for the radiative neutrino mass generation [6]. The original model is a simple extension of the SM with an inert doublet scalar $\eta$ and three right-handed neutrinos $N_{R_{i}}$. These new fields are assigned odd parity of an imposed $Z_{2}$ symmetry, although all the SM contents are assumed to have its even parity. Invariant Yukawa couplings and scalar potential which are relevant to these new fields are summarized as

$$
\begin{align*}
-\mathcal{L}_{y}= & h_{i j} \bar{N}_{R_{j}} \eta^{\dagger} \ell_{L_{i}}+h_{i j}^{*} \bar{L}_{L_{i}} \eta N_{R_{j}}+\frac{1}{2}\left(M_{i} \bar{N}_{R_{i}} N_{R_{i}}^{c}+M_{i} \bar{N}_{R_{i}}^{c} N_{R_{i}}\right) \\
& +m_{\phi}^{2} \phi^{\dagger} \phi+m_{\eta}^{2} \eta^{\dagger} \eta+\lambda_{1}\left(\phi^{\dagger} \phi\right)^{2}+\lambda_{2}\left(\eta^{\dagger} \eta\right)^{2} \\
& +\lambda_{3}\left(\phi^{\dagger} \phi\right)\left(\eta^{\dagger} \eta\right)+\lambda_{4}\left(\eta^{\dagger} \phi\right)\left(\phi^{\dagger} \eta\right) \\
& +\frac{\lambda_{5}}{2}\left[\left(\phi^{\dagger} \eta\right)^{2}+\text { h.c. }\right] \tag{1}
\end{align*}
$$

where $\ell_{L_{i}}$ is a left-handed doublet lepton and $\phi$ is an ordinary doublet Higgs scalar. We use the basis for which both matrices for charged lepton Yukawa couplings and right-handed neutrino masses are real and diagonal. Since the $Z_{2}$ is assumed to be the exact symmetry of the model, the new doublet scalar $\eta$ should not have a vacuum expectation value. As its result, neutrino masses are forbidden at tree level and the lightest field with the odd parity is stable to be DM.

In this type of model, the lepton number $L$ is usually assigned to these new fields as $L(\eta)=0$ and $L\left(N_{R_{i}}\right)=1$. In such a case, the neutrino mass generation and leptogenesis have been studied under the assumption that mass terms of the right-handed neutrinos violate the lepton number [13,15]. The DM abundance has also been studied supposing that either the lightest right-handed neutrino or the lightest neutral component of $\eta$ is DM. However, it is useful to note that there could be another assignment of the lepton number such as $L(\eta)=1$ and $L\left(N_{R_{i}}\right)=0$ [17]. In this case, $\lambda_{5}\left(\phi^{\dagger} \eta\right)^{2}$ is forbidden as long as the lepton number is imposed as the exact symmetry. As a result, neutrino masses could not be generated even if the radiative effect is taken into account. Thus, some suitable origin of the lepton number violation should bring about this $\lambda_{5}$ term as an effective interaction at low energy regions. We study such a possibility in the following part.

For this purpose, we consider an extension of the model at high energy regions by introducing canonically normalized complex singlet scalars $S_{\alpha}$ which are assigned odd parity of the $Z_{2}$ symmetry and $L=1$. The potential and interaction terms of $S_{\alpha}$ are assumed to be given by

$$
\begin{aligned}
-\mathcal{L}_{S}= & \sum_{\alpha=1}^{2}\left(\kappa_{1}\left(S_{\alpha}^{\dagger} S_{\alpha}\right)^{2}+\kappa_{2}\left(S_{\alpha}^{\dagger} S_{\alpha}\right)\left(\phi^{\dagger} \phi\right)+\kappa_{3}\left(S_{\alpha}^{\dagger} S_{\alpha}\right)\left(\eta^{\dagger} \eta\right)\right. \\
& +\tilde{m}_{S_{\alpha}}^{2} S_{\alpha}^{\dagger} S_{\alpha}+\frac{1}{2} m_{S_{\alpha}}^{2} S_{\alpha}^{2}+\frac{1}{2} m_{S_{\alpha}}^{2} S_{\alpha}^{\dagger 2} \\
& \left.-\mu_{\alpha} S_{\alpha} \eta^{\dagger} \phi-\mu_{\alpha}^{*} S_{\alpha}^{\dagger} \phi^{\dagger} \eta\right)
\end{aligned}
$$

$$
\begin{align*}
& +c_{1} \frac{\left(S_{1}^{\dagger} S_{1}\right)^{n}}{M_{\mathrm{pl}}^{2 n-4}}\left[1+c_{2}\left\{\left(\frac{S_{1}}{M_{\mathrm{pl}}}\right)^{2 m} \exp \left(i \frac{S_{1}^{\dagger} S_{1}}{\Lambda^{2}}\right)\right.\right. \\
& \left.\left.+\left(\frac{S_{1}^{\dagger}}{M_{\mathrm{pl}}}\right)^{2 m} \exp \left(-i \frac{S_{1}^{\dagger} S_{1}}{\Lambda^{2}}\right)\right\}\right] \tag{2}
\end{align*}
$$

where both $n$ and $m$ in the third line are positive integers and $M_{\mathrm{pl}}$ is the reduced Planck mass. Although the $Z_{2}$ is kept as the symmetry of these terms, the lepton number is violated through the mass terms $m_{S_{\alpha}}^{2} S_{\alpha}^{2}, m_{S_{\alpha}}^{2} S_{\alpha}^{\dagger 2}$ in the second line and also the Planck suppressed $c_{2}$ terms in the third line. The latter one is neglected in the low energy region. On the other hand, the former lepton number violation could be an origin of $\lambda_{5}$ term in eq. (1). In fact, as a simplest case, we might consider the situation where $\tilde{m}_{S_{\alpha}}^{2} \gg m_{S_{\alpha}}^{2}$ is satisfied. In this case, the model defined by eq. (1) can be easily obtained as the effective one with $\lambda_{5}=\sum_{\alpha} \lambda_{5}^{(\alpha)}$, where $\lambda_{5}^{(\alpha)}$ is defined by $\lambda_{5}^{(\alpha)}=\frac{m_{S_{\alpha}}^{2} \mu_{\alpha}^{2}}{\tilde{m}_{S_{\alpha}}^{4}}$. They are induced as the effective interaction terms at low energy regions after the singlet scalars $S_{\alpha}$ are integrated out [17,19].

In the following discussion, we focus our study on the situation such that the terms in the last line in eq. (2) could be a dominant part of the potential at the early Universe. We suppose that $\left|S_{1}\right|$ takes a large but sub-Planckian value in such a period. It could be realized under the condition such $\mathrm{as}^{2}$
$\kappa_{1} \ll c_{1}\left(\frac{\varphi_{1}}{M_{\mathrm{pl}}}\right)^{2 n-4}$,
$\left(\frac{\tilde{m}_{S_{1}}}{\varphi_{1}}\right)^{2},\left(\frac{m_{S_{1}}}{\varphi_{1}}\right)^{2} \ll c_{1}\left(\frac{\varphi_{1}}{M_{\mathrm{pl}}}\right)^{2 n-4}$,
where $\varphi_{1}$ is defined by $S_{1}=\frac{\varphi_{1}}{\sqrt{2}} e^{i \theta_{1}}$ and $\varphi_{1}<M_{\mathrm{pl}}$. If we use the polar coordinate of $S_{1}$ defined here, the last line of eq. (2) can be written as
$V_{S_{1}}=c_{1} \frac{\varphi_{1}^{2 n}}{2^{n} M_{\mathrm{pl}}^{2 n-4}}\left[1+2 c_{2}\left(\frac{\varphi_{1}}{\sqrt{2} M_{\mathrm{pl}}}\right)^{2 m} \cos \left(\frac{\varphi_{1}^{2}}{2 \Lambda^{2}}+2 m \theta_{1}\right)\right]$.

We easily find that $V_{S_{1}}$ has local minima with the potential barrier $V_{b} \simeq \frac{c_{1} c_{2} \varphi_{1}^{2(n+m)}}{2^{n+m-2} M_{\mathrm{pl}}^{2(n+m-2)}}$ in the radial direction, which form a spirallike trajectory. We consider the inflation which is caused by the inflaton evolution along this trajectory.

## 3. Phenomenological features of the model

### 3.1. Inflation

We briefly review the features of the inflation induced by the potential (4). We assume that $\varphi_{1}$ takes a large initial value on a local minimum in the radial direction. In that case, as shown in [19], the model could cause sufficient $e$-foldings through the inflaton evolution along the spiral-like trajectory even for sub-Planckian values of $\varphi_{1}$. An inflaton field $\chi$ could be identified with
$\chi \equiv a_{e}+\frac{\varphi_{1 e}^{3}}{6 m \Lambda^{2}}-a=\frac{\varphi_{1}^{3}}{6 m \Lambda^{2}}$,

[^1]Table 1
Examples of the predicted values for the spectral index $n_{s}$ and the tensor-to-scalar ratio $r$ in this scenario fixed by $n=3$ and $m=1$.

| $\begin{aligned} & c_{1} \\ & \left(\times 10^{-7}\right) \end{aligned}$ | $c_{2}$ | $\frac{\Lambda}{M \mathrm{pl}}$ | $\frac{\varphi_{1}^{*}}{\sqrt{2} M_{\mathrm{pl}}}$ | $\begin{aligned} & H_{*} \\ & \left(\times 10^{14} \mathrm{GeV}\right) \end{aligned}$ | $N_{*}$ | $n_{\text {s }}$ | $r$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9.84 | 1.7 | 0.05 | 0.411 | 5.91 | 60.0 | 0.964 | 0.056 |
| 8.62 | 1.9 | 0.05 | 0.406 | 5.40 | 60.0 | 0.959 | 0.040 |

where the field $a$ is defined as
$d a=\left[\varphi_{1}^{2}+\left(\frac{d \varphi_{1}}{d \theta_{1}}\right)^{2}\right]^{1 / 2} d \theta=\left[1+4 m^{2}\left(\frac{\Lambda}{\varphi_{1}}\right)^{4}\right]^{1 / 2} \varphi_{1} d \theta_{1}$.
Fields with the subscript $e$ stand for the fields at the end of inflation. The number of $e$-foldings caused by $\chi$ is given as
$N=-\frac{1}{M_{\mathrm{pl}}^{2}} \int_{\chi}^{\chi_{e}} d \chi \frac{V_{S_{1}}}{V_{S_{1}}^{\prime}} \equiv N(\chi)-N\left(\chi_{e}\right)$,
where $V_{S_{1}}^{\prime}=\frac{d V_{S_{1}}}{d \chi}$ and $N(\chi)$ is represented by using the hypergeometric function $F$ as

$$
\begin{align*}
& N(\chi) \\
& =\frac{1}{6 m^{2} n}\left(\frac{M_{\mathrm{pl}}}{\Lambda}\right)^{4}\left(\frac{\varphi_{1}}{\sqrt{2} M_{\mathrm{pl}}}\right)^{6}\left[1+\frac{6 c_{2} m}{n(3+m)}\left(\frac{\varphi_{1}}{\sqrt{2} M_{\mathrm{pl}}}\right)^{2 m}\right. \\
& \left.\quad \times F\left(1, \frac{3}{m}+1, \frac{3}{m}+2,2 c_{2}\left(1+\frac{m}{n}\right)\left(\frac{\varphi_{1}}{\sqrt{2} M_{\mathrm{pl}}}\right)^{2 m}\right)\right] \tag{8}
\end{align*}
$$

Here we note that the model could have a different feature from the ordinary inflation scenario such as the chaotic inflation. In eq. (7), $N(\chi) \gg N\left(\chi_{e}\right)$ might not be satisfied generally. In this model, inflation is expected to end at the time when $\frac{1}{2} \dot{\chi}^{2} \simeq V_{b}$ is satisfied. If we apply the slow-roll approximation $3 H \dot{\chi}=-V_{S_{1}}^{\prime}$ to the one of slow-roll parameters $\varepsilon \equiv \frac{M_{\mathrm{pl}}^{2}}{2}\left(\frac{V_{S_{1}}^{\prime}}{V_{S_{1}}}\right)^{2}$ [20], the inflation is found to end at $\varepsilon=\frac{3 V_{b}}{V_{S_{1}}}$. This means that the end of inflation could happen much before the time when $\varepsilon \simeq 1$ is realized since $V_{S_{1}}>V_{b}$ is satisfied. In that case, $N\left(\chi_{e}\right)$ could have a substantial contribution to determine the $e$-foldings $N$ in eq. (7).

The slow-roll parameters $\varepsilon$ and $\eta \equiv M_{\mathrm{pl}}^{2}\left(\frac{V_{S_{1}}^{\prime \prime}}{V_{S_{1}}}\right)$ can be represented by using the model parameters as

$$
\begin{align*}
\varepsilon= & m^{2}\left(\frac{\sqrt{2} M_{\mathrm{pl}}}{\varphi_{1}}\right)^{6}\left(\frac{\Lambda}{M_{\mathrm{pl}}}\right)^{4}\left[\frac{n-2 c_{2}(m+n)\left(\frac{\varphi_{1}}{\sqrt{2} M_{\mathrm{pl}}}\right)^{2 m}}{1-2 c_{2}\left(\frac{\varphi_{1}}{\sqrt{2} M_{\mathrm{pl}}}\right)^{2 m}}\right]^{2}, \\
\eta= & m^{2}\left(\frac{\sqrt{2} M_{\mathrm{pl}}}{\varphi_{1}}\right)^{6}\left(\frac{\Lambda}{M_{\mathrm{pl}}}\right)^{4}, \\
& \times \frac{n(2 n-3)-2 c_{2}(m+n)(2 m+2 n-3)\left(\frac{\varphi_{1}}{\sqrt{2} M_{\mathrm{pl}}}\right)^{2 m}}{1-2 c_{2}\left(\frac{\varphi_{1}}{\sqrt{2} M_{\mathrm{pl}}}\right)^{2 m}} . \tag{9}
\end{align*}
$$

If $c_{2}$ terms are neglected in these formulas, we find very simple formulas for these slow-roll parameters at the period characterized
by the inflaton value $\chi_{*}$. They can be represented by using the $e$-foldings $N_{*}$ defined for $N\left(\chi_{*}\right)$ in eq. (8) as
$\varepsilon \simeq \frac{n}{6\left(N_{*}+N\left(\chi_{e}\right)\right)}, \quad \eta \simeq \frac{2 n-3}{6\left(N_{*}+N\left(\chi_{e}\right)\right)}$.
Thus, the scalar spectral index $n_{s}$ and the tensor-to-scalar ratio $r$ can be derived as [19]
$n_{s}=1-6 \epsilon+2 \eta \simeq 1-\frac{n+3}{3\left(N_{*}+N\left(\chi_{e}\right)\right)}$,
$r=16 \epsilon \simeq \frac{8 n}{3\left(N_{*}+N\left(\chi_{e}\right)\right)}$.
If we focus on the case $n=3$, these formulas reduce to the ones of the $m_{\varphi}^{2} \varphi^{2}$ chaotic inflation scenario [21]. However, as shown in [19], the values of $n_{s}$ and $r$ in this model could deviate from the ones of the $m_{\varphi}^{2} \varphi^{2}$ chaotic inflation due to the non-negligible $c_{2}$ term contribution. Taking account of uncertainty caused by the reheating process and others, $N_{*}$ might be considered to take a value in the range $50-60$. If we estimate both $n_{s}$ and $r$ by fixing the parameters in the potential suitably, they could take consistent values for $N_{*}$ in this range with the ones suggested by a joint analysis of BICEP2, Keck Array and Planck [22,23]. Such examples for $n=3$ are shown in Table 1. The condition (3) requires $\tilde{m}_{S_{1}} \ll 10^{14} \mathrm{GeV}$ in this case. Much better agreement with the observational results for $n_{s}$ and $r$ is found in the case $n=1,2$ [19].

Finally, we note that the polar coordinate cannot be used for $S_{1}$ to rewrite the potential as eq. (4) unless $m_{S_{1}}^{2}=0$ is satisfied. In order to make this inflation scenario possible, $m_{S_{1}}^{2}$ should be generated after the end of inflation at least. It is not difficult to modify the model to satisfy this condition. For example, we may introduce a singlet scalar $\psi$ with $L=-1$. In this case, its potential might be given by
$V_{\psi}=\xi_{1}\left(\psi^{\dagger} \psi\right)^{2}+\left(\xi_{2} S_{1}^{\dagger} S_{1}-m_{\psi}^{2}\right) \psi^{\dagger} \psi+\left(\xi_{3} S_{\alpha}^{2} \psi^{2}+\right.$ h.c. $)$.
If the value of $\left|S_{1}\right|$ becomes smaller than $\sqrt{\frac{m_{\psi}^{2}}{\xi_{2}}}$ after the end of slow-roll inflation, $\psi$ could get the vacuum expectation value which induces the required mass term for $S_{\alpha}$ through the $\xi_{3}$ term. After the generation of these terms in eq. (2) as the effective ones, the mass splitting between the real and imaginary components of $S_{\alpha}$ is brought about. Each mass eigenvalue is expressed as $m_{ \pm \alpha}^{2} \equiv \tilde{m}_{S_{\alpha}}^{2} \pm m_{S_{\alpha}}^{2}$, where + and - signs correspond to the real and imaginary component, respectively. We note that the stability of the vacuum requires $\tilde{m}_{S_{\alpha}}^{2}>m_{S_{\alpha}}^{2}$. The difference of these mass eigenvalues can be a measure of the lepton number violation in the model.

### 3.2. Neutrino masses

The neutrino masses are generated in the similar way to the original model. The one-loop effect which picks up the lepton number violation induced by the mass term $m_{S_{\alpha}}^{2} S_{\alpha}^{2}$ generates the neutrino masses through the electroweak symmetry breaking as shown in the left-hand diagram of Fig. 1. The neutrino mass ma-


Fig. 1. Left: a one-loop diagram contributing to the neutrino mass generation. The dimensionful coupling $\mu_{\alpha}^{( \pm)}$is defined as $\mu_{\alpha}^{(+)}=\frac{\mu_{\alpha}}{\sqrt{2}}$ and $\mu_{\alpha}^{(-)}=\frac{i \mu_{\alpha}}{\sqrt{2}}$ by using $\mu_{\alpha}$ in eq. (2). Right: a one-loop diagram contributing to the lepton flavor violating process $\ell_{i} \rightarrow \ell_{j} \gamma$.
trix obtained in this way can be described by the formula
$\left(\mathcal{M}_{\nu}\right)_{s t}=\sum_{k=1}^{3} \sum_{\alpha=1,2} \sum_{f= \pm} \frac{h_{s k} h_{t k} M_{k} \mu_{\alpha}^{(f) 2}\langle\phi\rangle^{2}}{8 \pi^{2}} I\left(M_{\eta}, M_{k}, m_{f \alpha}\right)$,
where $M_{\eta}^{2}=m_{\eta}^{2}+\left(\lambda_{3}+\lambda_{4}\right)\langle\phi\rangle^{2}$ and $\langle\phi\rangle=174 \mathrm{GeV}$. $\mu_{\alpha}^{(f)}$ stands for $\mu_{\alpha}^{(+)}=\frac{\mu_{\alpha}}{\sqrt{2}}$ and $\mu_{\alpha}^{(-)}=\frac{i \mu_{\alpha}}{\sqrt{2}}$, respectively. The function $I\left(m_{a}, m_{b}, m_{c}\right)$ is defined as

$$
\begin{align*}
& I\left(m_{a}, m_{b}, m_{c}\right) \\
& \qquad \begin{array}{l}
=\frac{\left(m_{a}^{4}-m_{b}^{2} m_{c}^{2}\right) \ln m_{a}^{2}}{\left(m_{b}^{2}-m_{a}^{2}\right)^{2}\left(m_{c}^{2}-m_{a}^{2}\right)^{2}}+\frac{m_{b}^{2} \ln m_{b}^{2}}{\left(m_{c}^{2}-m_{b}^{2}\right)\left(m_{a}^{2}-m_{b}^{2}\right)^{2}} \\
\quad \\
\quad+\frac{m_{c}^{2} \ln m_{c}^{2}}{\left(m_{b}^{2}-m_{c}^{2}\right)\left(m_{a}^{2}-m_{c}^{2}\right)^{2}}-\frac{1}{\left(m_{b}^{2}-m_{a}^{2}\right)\left(m_{c}^{2}-m_{a}^{2}\right)}
\end{array}
\end{align*}
$$

As long as $m_{ \pm \alpha}^{2}, M_{k}^{2} \gg M_{\eta}^{2}$ is satisfied, this formula is found to be reduced to
$\mathcal{M}_{s t}^{v} \simeq \sum_{k=1}^{3} \frac{h_{s k} h_{t k}\langle\phi\rangle^{2}}{16 \pi^{2} M_{k}} \sum_{\alpha=1,2}\left(\frac{\mu_{\alpha}^{2}}{m_{+\alpha}^{2}}-\frac{\mu_{\alpha}^{2}}{m_{-\alpha}^{2}}\right)$,
where we neglect logarithmic factors. If we note that two righthanded neutrinos are enough to explain the neutrino oscillation data, $h_{1}$ could be assumed to be so small that the contribution of $N_{1}$ to the neutrino masses is negligible. We adopt this assumption throughout the following discussion, for simplicity.

If we assume the flavor structure of the neutrino Yukawa couplings discussed in Appendix A, the required mass difference for the atmospheric neutrinos and the solar neutrinos could be explained by the largest mass eigenvalue and the next one in this mass matrix, respectively. ${ }^{3}$ For example, this requirement could be represented as

$$
\begin{align*}
\sum_{\alpha=1,2}\left(\frac{\mu_{\alpha}^{2}}{m_{-\alpha}^{2}}-\frac{\mu_{\alpha}^{2}}{m_{+\alpha}^{2}}\right) & \simeq 10^{-6}\left(\frac{5.1 \times 10^{-2}}{h_{2}}\right)^{2}\left(\frac{M_{2}}{2 \times 10^{4} \mathrm{GeV}}\right) \\
\sum_{\alpha=1,2}\left(\frac{\mu_{\alpha}^{2}}{m_{-\alpha}^{2}}-\frac{\mu_{\alpha}^{2}}{m_{+\alpha}^{2}}\right) & \simeq 10^{-6}\left(\frac{2.7 \times 10^{-2}}{h_{3}}\right)^{2}\left(\frac{M_{3}}{5 \times 10^{4} \mathrm{GeV}}\right) \tag{16}
\end{align*}
$$

where we assume $M_{\eta}=1 \mathrm{TeV}$ and $C P$ phases are neglected in this estimation. It should be noted that the left-hand side of eq. (16) corresponds to the effective coupling $\lambda_{5}$. It plays a crucial role also in the generation of baryon number asymmetry and DM direct search as discussed later.

[^2]It is well-known that these new fields induce the lepton flavor violating processes at one-loop level. The typical one is $\ell_{i} \rightarrow \ell_{j} \gamma$ whose diagram is shown in the right-hand side of Fig. 1. Its branching ratio can be estimated as [24]

$$
\begin{align*}
\operatorname{Br}\left(\ell_{i} \rightarrow \ell_{j} \gamma\right) & =\frac{3 \alpha}{64 \pi\left(G_{F} M_{\eta}^{2}\right)^{2}}\left|\sum_{k=1}^{3} h_{i k} h_{j k} F_{2}\left(\frac{M_{k}}{M_{\eta}}\right)\right|^{2} \\
& \simeq 8 \times 10^{-7}\left|\sum_{k=1}^{3} h_{i k} h_{j k} F_{2}\left(\frac{M_{k}}{M_{\eta}}\right)\right|^{2}, \tag{17}
\end{align*}
$$

where $M_{\eta}=1 \mathrm{TeV}$ is used and $F_{2}(x)$ is given by
$F_{2}(x)=\frac{1-6 x^{2}+3 x^{4}+2 x^{6}-6 x^{4} \ln x^{2}}{6\left(1-x^{2}\right)^{4}}$.
Here we note that $F_{2}(x) \simeq \frac{1}{3 x^{2}}$ for $x \gg 1$ and the present upper bounds for $\operatorname{Br}(\mu \rightarrow e \gamma)$ and $\operatorname{Br}(\tau \rightarrow \mu \gamma)$ are given as $5.7 \times 10^{-13}$ [25] and $4.4 \times 10^{-8}$ [26], respectively. Since $M_{k}>M_{\eta}$ is assumed in the present model, the bounds for these flavor violating processes give no substantial constraint on neutrino Yukawa couplings as found from eqs. (16) and (17).

### 3.3. Baryon number asymmetry

Reheating process should follow the inflation discussed in the previous section. In this scenario, reheating is expected to occur through the decay of $S_{1}$ after the inflaton stops its evolution along the above mentioned spiral-like trajectory and $S_{ \pm 1}$ starts to oscillate around a global minimum of the potential. Although preheating could occur via scalar quartic couplings in the first line of eq. (2), the reheating is expected to be finally completed through the decay of $S_{1}$ [27,28]. Since lepton number asymmetry is not produced through the particle creation in the preheating, we focus our study on the decay of $S_{1}$ here.

The decay of $S_{1}$ is induced by the interaction of $S_{1}$ with $\phi$ and $\eta$ during the oscillation induced by the mass terms which are given in the second line of eq. (2). The reheating temperature may be estimated by using the usual instantaneous thermalization approximation. If we use this approximation, the reheating temperature is determined through the condition $H \simeq \Gamma_{ \pm 1}$. $H$ is the Hubble parameter and $\Gamma_{ \pm 1}$ stands for the decay width of $S_{ \pm 1}$ which is the real and imaginary component of $S_{1}$. Since $\Gamma_{ \pm 1}$ can be approximately estimated as $\Gamma_{ \pm 1} \simeq \frac{1}{16 \pi} \frac{\left|\mu_{1}\right|^{2}}{m_{ \pm 1}}$ where $m_{ \pm \alpha}^{2}=\tilde{m}_{S_{\alpha}}^{2} \pm m_{S_{\alpha}}^{2}$, the decay products of $S_{ \pm 1} \rightarrow \eta \phi^{\dagger}$, $\eta^{\dagger} \phi$ finally make thermal plasma with possible reheating temperature ${ }^{4}$ [28]
$T_{R}^{( \pm)} \simeq 0.35 g_{*}^{-1 / 4}\left|\mu_{1}\right|\left(\frac{M_{\mathrm{pl}}}{m_{ \pm 1}}\right)^{\frac{1}{2}}$,

[^3]

Fig. 2. Feynman diagrams which contribute to the transfer and the washout of the lepton number asymmetry. The left diagrams are lepton number conserving scattering processes whose reaction densities are represented by $\gamma_{a}$ (upper ones) and $\gamma_{b}$ (lower one). The right diagrams are lepton number violating scattering processes whose reaction densities are represented by $\gamma_{x}$ (upper ones) and $\gamma_{y}$ (lower one), respectively.
where we use $g_{*}=116$ as the relativistic degrees of freedom in this model. If we consider a situation such that $S_{ \pm \alpha}$ is not thermally generated through the inverse decay or the scatterings, $m_{ \pm \alpha}>T_{R}^{(+)}$should be satisfied at least. This condition could be expressed as
$\frac{\mu_{1}}{m_{+1}}<1.9 \times 10^{-4}\left(\frac{m_{ \pm \alpha}}{m_{+1}}\right)\left(\frac{m_{+1}}{10^{9} \mathrm{GeV}}\right)^{\frac{1}{2}}$.
In the following part, we confine our study to the case where this condition is satisfied.

The inflaton decay is relevant to the generation of baryon number asymmetry in this model. The lepton number asymmetry could be directly generated through this process non-thermally since this decay violates the lepton number. In fact, if $\mu_{\alpha}$ is complex, the cross term between tree and one-loop diagrams for the decay could bring about the $C P$ asymmetry. The $C P$ asymmetry induced through this decay of $S_{ \pm 1}$ can be estimated as ${ }^{5}$

$$
\begin{align*}
\epsilon_{ \pm} \equiv & \frac{\Gamma\left(S_{ \pm 1} \rightarrow \eta \phi^{\dagger}\right)-\bar{\Gamma}\left(S_{ \pm 1} \rightarrow \eta^{\dagger} \phi\right)}{\Gamma\left(S_{ \pm 1} \rightarrow \eta \phi^{\dagger}\right)+\bar{\Gamma}\left(S_{ \pm 1} \rightarrow \eta^{\dagger} \phi\right)} \\
= & \pm \frac{\left|\mu_{2}\right|^{2} \sin 2\left(\theta_{1}-\theta_{2}\right)}{16 \pi}\left(\frac{1}{m_{ \pm 1}^{2}} \ln \frac{\left(m_{ \pm 1}^{2}+m_{+2}^{2}\right) m_{-2}^{2}}{\left(m_{ \pm 1}^{2}+m_{-2}^{2}\right) m_{+2}^{2}}\right. \\
& +\frac{m_{ \pm 1}^{2}-m_{+2}^{2}}{\left(m_{ \pm 1}^{2}-m_{+2}^{2}\right)^{2}+m_{+2}^{2} \Gamma_{+2}^{2}} \\
& \left.-\frac{m_{ \pm 1}^{2}-m_{-2}^{2}}{\left(m_{ \pm 1}^{2}-m_{-2}^{2}\right)^{2}+m_{-2}^{2} \Gamma_{-2}^{2}}\right) \tag{21}
\end{align*}
$$

where $\theta_{i}=\arg \left(\mu_{i}\right)$ and $\Gamma_{ \pm \alpha}=\frac{\left|\mu_{\alpha}\right|^{2}}{16 \pi m_{ \pm \alpha}}\left(1-\frac{M_{\eta}^{2}}{m_{ \pm \alpha}^{2}}\right)$. As long as the condition (20) is satisfied, the lepton number asymmetry generated through the inflaton decay could be the only source for the baryon number asymmetry since there are no mother particles $S_{ \pm \alpha}$ in the thermal bath.

If both components $S_{ \pm 1}$ have finely degenerate masses $m_{+1}^{2} \simeq m_{-1}^{2}$, their decay occurs almost simultaneously and then $T_{R}^{(+)} \simeq T_{R}^{(-)}$. We also find that $\epsilon_{+} \simeq-\epsilon_{-}$is satisfied. Since the lepton number asymmetry generated in the $\eta$ sector through this decay could be estimated as $\Delta L \simeq \epsilon_{+} n_{S_{+1}}\left(T_{R}^{(+)}\right)+\epsilon_{-} n_{S_{-1}}\left(T_{R}^{(-)}\right)$, $\Delta L$ may not take a large value in this case because of the cancellation due to $\epsilon_{-} n_{S_{-1}}\left(T_{R}^{(-)}\right) \simeq-\epsilon_{+} n_{S_{+1}}\left(T_{R}^{(+)}\right)$. On the other hand, if substantial mass splitting appears between the components $S_{ \pm 1}$ and then $m_{+1}^{2}>m_{-1}^{2}$ is satisfied, the $S_{+1}$ decay is expected to occur later compared with the decay of $S_{-1}$ because of $\Gamma_{-1}>\Gamma_{+1}$. In such a case, a part of lepton number asymmetry generated by the $S_{-1}$ decay could be washed out by the lepton number violating

[^4]processes before the delayed $S_{+1}$ decay. Thus, the lepton number asymmetry expected in the $\eta$ sector after the $S_{+1}$ decay could be estimated as $\Delta L \simeq \epsilon_{+} n_{S_{+1}}\left(T_{R}^{(+)}\right)+\mathcal{K}_{w}\left(T_{R}^{(+)}\right) \epsilon_{-} n_{S_{-1}}\left(T_{R}^{(-)}\right)$where $\mathcal{K}_{w}\left(T_{R}^{(+)}\right)$represents the washout effects from $T_{R}^{(-)}$to $T_{R}^{(+)}$. If the lepton number violating processes decouple and then $\mathcal{K}_{w}=1$ is satisfied in this period, $\Delta L$ is expected to take a substantial value because $\epsilon_{-} n_{S_{-1}}\left(T_{R}^{(-)}\right) \neq-\epsilon_{+} n_{S_{+1}}\left(T_{R}^{(+)}\right)$is satisfied.

The lepton number asymmetry generated in the $\eta$ sector via the $S_{ \pm 1}$ decay cannot be transferred to the SM contents through the decay of $\eta$. We should note that $\eta$ does not have any decay modes to the SM contents because of the $Z_{2}$ symmetry. However, it could be partially transferred to the lepton sector through the lepton number conserving scatterings $\eta \eta \rightarrow \ell \ell$ and $\eta \bar{\ell} \rightarrow \eta^{\dagger} \ell$. These are induced by neutrino Yukawa couplings and their diagrams are given in the left-hand side of Fig. 2. On the other hand, it could also be washed out through the lepton number violating scattering processes $\eta \eta \rightarrow \phi \phi$ and $\eta \phi^{\dagger} \rightarrow \eta^{\dagger} \phi$. These are caused by the $S_{ \pm \alpha}$ exchange due to the $\mu_{\alpha}$ couplings. Their diagrams are also shown in the right-hand side of Fig. 2. In the situation where these processes are competing with each other before reaching the weak scale, the lepton number asymmetry kept in the lepton sector could be converted to the baryon number asymmetry through the sphaleron interaction. We examine this scenario quantitatively by solving relevant Boltzmann equations.

For this purpose, we define the lepton number asymmetry in the co-moving volume as $\Delta Y_{\ell} \equiv \frac{n_{\ell}-n_{\bar{\ell}}}{s}$ in the lepton sector and $\Delta Y_{\eta} \equiv \frac{n_{\eta}-n_{\eta^{\dagger}}}{s}$ in the $\eta$ sector, respectively. The entropy density $s$ is expressed as $s=\frac{2 \pi^{2}}{45} g_{*} T^{3}$. As discussed in the previous part, the lepton number asymmetry in the $\eta$ sector is expected to be fixed through the decay of $S_{ \pm 1}$. Thus, at the reheating temperature $T_{R}^{(+)}$, the lepton number asymmetry in each sector are supposed to be $\Delta Y_{\ell}\left(T_{R}^{(+)}\right)=0$ and $\Delta Y_{\eta}\left(T_{R}^{(+)}\right)=\frac{\epsilon_{+} n_{S_{+1}}\left(T_{R}^{(+)}\right)+\epsilon_{-} n_{S_{-1}}\left(T_{R}^{(-)}\right)}{s_{R}}$ where $s_{R}$ stands for the entropy density at $T_{R}^{(+)}$. If we use $n_{S_{ \pm 1}}\left(T_{R}^{( \pm)}\right)=$ $\frac{\rho_{S_{ \pm 1}}\left(T_{R}^{( \pm)}\right)}{m_{ \pm 1}}$ and $\rho_{S_{ \pm 1}}\left(T_{R}^{( \pm)}\right)=\frac{\pi^{2}}{30} g_{*} T_{R}^{( \pm) 4}$ which are derived by assuming the instantaneous thermalization after the $S_{ \pm 1}$ decay, we find that the latter can be expressed as
$\Delta Y_{\eta}\left(T_{R}^{(+)}\right)=\frac{3}{4} \epsilon_{+} \frac{T_{R}^{(+)}}{m_{+1}}+\frac{3}{4} \epsilon_{-} \frac{T_{R}^{(-)}}{m_{-1}}$.
By taking account of the relevant processes which are explained above, Boltzmann equations which describe the evolution of $\Delta Y_{\eta}$ and $\Delta Y_{\ell}$ are given as ${ }^{6}$

[^5]\[

$$
\begin{align*}
\frac{d \Delta Y_{\eta}}{d z}= & -\frac{z}{s H\left(M_{\eta}\right)}\left[2\left(\gamma_{a}+\gamma_{b}\right)\left(\frac{\Delta Y_{\eta}}{Y_{\eta}^{\mathrm{eq}}}-\frac{\Delta Y_{\ell}}{Y_{\ell}^{\mathrm{eq}}}\right)\right. \\
& \left.+2\left(\gamma_{x}+\gamma_{y}\right) \frac{\Delta Y_{\eta}}{Y_{\eta}^{\mathrm{eq}}}\right], \\
\frac{d \Delta Y_{\ell}}{d z}= & \frac{z}{s H\left(M_{\eta}\right)} 2\left(\gamma_{a}+\gamma_{b}\right)\left(\frac{\Delta Y_{\eta}}{Y_{\eta}^{\mathrm{eq}}}-\frac{\Delta Y_{\ell}}{Y_{\ell}^{\mathrm{eq}}}\right) . \tag{23}
\end{align*}
$$
\]

Since we consider the case where the condition (20) is satisfied, the effect of $S_{ \pm \alpha}$ in the thermal bath can be neglected. Each reaction density $\gamma_{i}$ is explained in the caption of Fig. 2 and their formulas are given in Appendix B. The generated baryon number asymmetry could be estimated as [17]
$Y_{B}=-\frac{7}{19} \Delta Y_{\ell}\left(z_{E W}\right)$
by using the lepton number asymmetry $\Delta Y_{\ell}$ obtained as the solution of these equations at the weak scale.

Although detailed analysis of the generated baryon number asymmetry requires to solve the above Boltzmann equations numerically, we briefly discuss their qualitative aspects before proceeding to it. At first, we note the behavior of the ratio of the reaction rate $\Gamma$ to Hubble parameter $H$ for the relevant scattering processes in the case $m_{ \pm \alpha}>T_{R}^{(+)}$which we consider here. $\Gamma$ and $H$ are expressed as $\Gamma_{a, b} \equiv \frac{\gamma_{a, b}}{n_{\ell}^{\text {eq }}}, \Gamma_{x, y} \equiv \frac{\gamma_{x, y},{ }^{\text {eq }}}{n_{\eta}^{q}}$ where $n_{\ell}^{\text {eq }} \simeq$ $\frac{3.6 M_{\eta}^{3}}{\pi^{2}} z^{-3}, n_{\eta}^{\text {eq }} \simeq \frac{2 M_{\eta}^{3}}{\pi^{2}} z^{-1} K_{2}(z)$ and $H(z) \simeq 0.33 g_{*}^{1 / 2} \frac{M_{\eta}^{2}}{M_{\mathrm{pl}}} z^{-2}$. In the lepton number conserving scattering processes caused by the neutrino Yukawa couplings, $\frac{\Gamma_{a}+\Gamma_{b}}{H}$ is a convex function of $z$ which takes a maximum value around $z_{m} \simeq \frac{M_{\eta}}{M_{k}}$. They freeze out at $z_{f}\left(>z_{m}\right)$ in the case where $\frac{\Gamma_{a}+\Gamma_{b}}{H}>1$ is satisfied at $z_{m}$. It is important to note that $\Delta Y_{\ell}$ follows $\Delta Y_{\eta}$ to be $\Delta Y_{\ell}=\Delta Y_{\eta}$ as long as $\frac{\Gamma_{a}+\Gamma_{b}}{H} \gtrsim 1$ is satisfied. On the other hand, the coupling $\mu_{\alpha}$ which causes the lepton number violating scatterings is dimensionful so that $\frac{\Gamma_{x}+\Gamma_{y}}{H}$ increases monotonically with $z$ throughout the range $\frac{M_{\eta}}{T_{R}^{(+)}}<z<1$. Since these processes are expected to be in the thermal equilibrium at a certain period $z_{e}$ where $\frac{\Gamma_{x}+\Gamma_{y}}{H\left(z_{e}\right)}=1$ is satisfied, $\Delta Y_{\eta}$ is expected to be erased at $z \gtrsim z_{e}$. However, these processes are suppressed at $z \gtrsim 1$ by the Boltzmann factor.

Here we note that both $z_{f}$ and $z_{e}$ are determined by the parameters relevant to the neutrino masses. We could make a rough estimation of favored parameters for the generation of baryon number asymmetry by taking account of it and the above arguments. As seen in eq. (16), the neutrino oscillation data imposes a relation for neutrino Yukawa couplings and a GeV unit $M_{k}$ such that
$\frac{\left(h h^{T}\right)_{k k}}{M_{k}} \sum_{\alpha=1,2}\left(\frac{\mu_{\alpha}^{2}}{m_{-\alpha}^{2}}-\frac{\mu_{\alpha}^{2}}{m_{+\alpha}^{2}}\right) \sim O\left(10^{-14}\right)$,
where we assume $M_{\eta}=1 \mathrm{TeV}$. If we use this condition, both $z_{f}$ and $z_{e}$ can be roughly estimated as

$$
\begin{align*}
z_{f} & \sim O\left(10^{18}\right) \sum_{k} \frac{\left(h h^{T}\right)_{k k}^{2}}{M_{k}^{2}} \\
& \sim O\left(10^{-11}\right)\left[\sum_{\alpha}\left(\frac{\mu_{\alpha}^{2}}{m_{-\alpha}^{2}}-\frac{\mu_{\alpha}^{2}}{m_{+\alpha}^{2}}\right)\right]^{-2} \\
z_{e} & \sim O\left(10^{-13}\right)\left[\sum_{\alpha=1,2}\left(\frac{\mu_{\alpha}^{2}}{m_{-\alpha}^{2}}-\frac{\mu_{\alpha}^{2}}{m_{+\alpha}^{2}}\right)\right]^{-2} \tag{26}
\end{align*}
$$

where the CP phases of neutrino Yukawa couplings are neglected. These results suggest that $z_{f}>z_{e}$ is always satisfied.

The washout factor $\mathcal{K}_{w}(z)$ which we have already introduced in the previous discussion is characterized as a decreasing function at $z \gtrsim z_{e}$ and $\mathcal{K}_{w}(z) \simeq 1$ at $z \lesssim z_{e}$. If we use it, the total lepton number at $z$ might be written as
$\Delta Y_{\ell}(z)+\Delta Y_{\eta}(z)=\mathcal{K}_{w}(z) \Delta Y_{\eta}\left(z_{R}\right)$,
where we use eq. (22) as the initially generated lepton number asymmetry. On the other hand, the lepton number asymmetry in both sectors at $z$ could be related as
$\Delta Y_{\ell}(z)=\mathcal{K}_{t}(z) \Delta Y_{\eta}(z)$,
where $\mathcal{K}_{t}(z)$ stands for the transfer efficiency of the lepton number asymmetry from the $\eta$ sector to the doublet lepton sector. If the lepton number conserving scattering processes are in the thermal equilibrium, $\mathcal{K}_{t}(z)=1$ is satisfied. Using these relations, we could consider two possible cases for the generation of lepton number asymmetry in the lepton sector.
(a) If the lepton number conserving scatterings are in the thermal equilibrium at an early stage and freeze out at $z_{f}$, the lepton number asymmetry in the lepton sector at the weak scale is found to be roughly expressed as
$\Delta Y_{\ell}\left(z_{E W}\right) \simeq \frac{\mathcal{K}_{t}\left(z_{f}\right) \mathcal{K}_{w}\left(z_{f}\right)}{1+\mathcal{K}_{t}\left(z_{f}\right)} \Delta Y_{\eta}\left(z_{R}\right)$.
Although $\mathcal{K}_{w}\left(z_{f}\right)=1$ is satisfied for $z_{f}<z_{e}$, the neutrino mass condition allows only the situation $z_{f}>z_{e}$ as shown in eq. (26). Thus, the required value of $\Delta Y_{\ell}\left(z_{E W}\right)$ could be obtained in the case where $\mathcal{K}_{w}\left(z_{f}\right)$ is not so small. It could be realized only for $M_{k} \gg M_{\eta}$.
(b) If the lepton number conserving scattering processes never reach the thermal equilibrium at $z\left(<z_{e}\right)$ but $\frac{\Gamma_{a}+\Gamma_{b}}{H}$ has nonnegligible values, the situation becomes completely different from the case (a). In this case, a part of $\Delta Y_{\eta}$ could be transferred to the lepton sector. Since $\Delta Y_{\eta}$ steeply decreases at $z \sim z_{e}, \Delta Y_{\ell}$ could take a fixed value which might be roughly estimated as $\Delta Y_{\ell}\left(z_{e}\right)$ independently of the value of $\frac{\Gamma_{a}+\Gamma_{b}}{H}$ at $z\left(>z_{e}\right)$. The transferred lepton number asymmetry $\Delta Y_{\ell}\left(z_{e}\right)$ is kept until the weak scale. Thus, $\Delta Y_{\ell}\left(z_{\mathrm{EW}}\right)$ could be expressed as

$$
\begin{equation*}
\Delta Y_{\ell}\left(z_{\mathrm{EW}}\right) \simeq \mathcal{K}_{t}\left(z_{e}\right) \Delta Y_{\eta}\left(z_{R}\right) \tag{30}
\end{equation*}
$$

where $\mathcal{K}_{t}\left(z_{e}\right) \ll 1$. Thus, the required lepton number asymmetry in the lepton sector could be obtained at the weak scale for a suitable $\mathcal{K}_{t}\left(z_{e}\right)$. Such a situation could happen only in the case $M_{k} \gg T_{R}^{(+)}$.

Now we present results of the numerical analysis of the Boltzmann equations. Model parameters used in this analysis are summarized in Table 2, which are numerically fixed to satisfy the conditions for the neutrino masses. If we take account of the conditions (16) and (20), we find that $\frac{\left|\mu_{1}\right|^{2}}{m_{ \pm 1}} \ll \frac{\left|\mu_{2}\right|^{2}}{m_{ \pm 2}}$ should be satisfied and also their phases can be fixed as $\theta_{1} \neq 0$ and $\theta_{2}=0, \frac{\pi}{2}$. This justifies the estimation in eqs. (16), (25) and (26) and the assumption for the maximum CP phase in eq. (21), which is used in this analysis. It also allows $\eta_{R}$ and $\eta_{I}$ to be the mass eigenstates of the neutral components of $\eta$. This becomes important for the study of DM phenomenology in the next subsection.

Solutions of the Boltzmann equations (23) for these parameter settings are presented in Fig. 3. In the upper panels of this figure, the $\frac{\Gamma}{H}$ for the relevant processes are plotted as functions of $z$. In the lower panels, $\Delta Y_{\eta}$ and $\Delta Y_{\ell}$ are plotted as functions of $z$. The lepton number asymmetry required for the suitable baryon number asymmetry is also shown by the horizontal black dotted lines

Table 2
The $C P$ asymmetry $\epsilon_{+}$and the baryon number asymmetry $\left|Y_{B}\right|$ obtained in the present scenario for typical parameter settings. The dimensionful model parameters are taken to be (a) $M_{2}=2 \times 10^{4}, M_{3}=5 \times 10^{4}$ and $\tilde{m}_{S_{1}}=10^{9}$, (b) $M_{2}=2 \times 10^{8}, M_{3}=5 \times 10^{8}$ and $\tilde{m}_{S_{1}}=10^{9}$ in a GeV unit, respectively. Neutrino Yukawa couplings are numerically determined for $M_{\eta}=1 \mathrm{TeV}$ so as to realize the neutrino mass eigenvalues required from the neutrino oscillation data.

|  | $h_{2}$ | $h_{3}$ | $\frac{m_{S_{1}}}{\tilde{m}_{S_{1}}}$ | $\frac{\left\|\mu_{1}\right\|}{\tilde{m}_{s_{1}}}$ | $\frac{\tilde{m}_{s_{2}}}{\tilde{m}_{s_{1}}}$ | $\frac{m_{S_{2}}}{\tilde{m}_{s_{2}}}$ | $\frac{\left\|\mu_{2}\right\|}{\tilde{m}_{s_{2}}}$ | $\epsilon_{+}$ | $\left\|Y_{B}\right\|$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| (a) | $1.0 \cdot 10^{-2}$ | $4.8 \cdot 10^{-3}$ | 0.5 | $2.0 \cdot 10^{-5}$ | 1.3 | 0.5 | $3.0 \cdot 10^{-3}$ | $1.7 \cdot 10^{-5}$ |  |
| (b) | $1.8 \cdot 10^{-2}$ | $9.5 \cdot 10^{-3}$ | 0.5 | $10^{-6}$ | 1.3 | 0.5 | $8.0 \cdot 10^{-2}$ | $1.2 \cdot 10^{-2}$ |  |



Fig. 3. The left-hand panels show the results for the case (a). The ratio of reaction rate $\Gamma$ to the Hubble parameter $H$ for each relevant process is plotted as functions of $z$ in the upper panel. The solutions $\Delta Y_{\eta}$ and $\Delta Y_{\ell}$ of the Boltzmann equations are shown as functions of $z$ in the lower panel. The lepton number asymmetry required to explain the observational results is shown by the horizontal black line. The right-hand panels show the results for the case (b) in the same way as the case (a).
in these panels. The left and right panels show the results corresponding to the cases (a) and (b) discussed above, respectively. They show that the above discussion describes qualitatively the features of the present scenario well. Although our study here is done only for the limited parameter sets, the results show that the scenario could generate the sufficient baryon number asymmetry for suitable model parameters in each case. Detailed study of this scenario for wider range of the model parameters will be given elsewhere.

We should recall again that the same parameters used here are closely related to several low energy phenomena. Although some of them have been discussed already, there is another one which has not been taken into account still now. We need to check the consistency with it to see whether the model works well or not. It is DM physics and this issue is the subject in the next part.

### 3.4. Dark matter

The DM candidate is built in the model as the lightest $Z_{2}$ odd field. We identify it as the lightest neutral component of $\eta$. We choose $\mu_{2}^{2}$ to be real and $\frac{\left|\mu_{1}\right|^{2}}{m_{ \pm 1}} \ll \frac{\left|\mu_{2}\right|^{2}}{m_{ \pm 2}}$ is supposed to be satisfied. In this case, the real and imaginary parts of the neutral component of $\eta$, which are written as $\eta_{R}$ and $\eta_{I}$, become the mass eigenstates as mentioned before. If $\eta_{R}$ is supposed to be a DM candidate, $\eta_{R}$ could be scattered with nuclei inelastically to $\eta_{I}$. It is mediated by the $Z$ boson exchange. Since it contributes to the DM direct search experiment [29], a strong constraint is imposed on the mass
difference $\delta\left(\equiv M_{\eta_{I}}-M_{\eta_{R}}\right)$ between $\eta_{R}$ and $\eta_{I}{ }^{7}$ [15]. This might give the scenario an interesting chance for giving a prediction in the DM direct search experiments as seen below.

We recall the experimental situation that we have no evidence in the DM direct search experiments [30]. If we apply it to the above mentioned process, we could put a bound for $\delta$. It might be estimated as $\delta>150 \mathrm{keV}$ conservatively. Since this mass difference is expressed in the present model as
$\delta \simeq \frac{\langle\phi\rangle^{2}}{M_{\eta}}\left(\frac{\mu_{2}^{2}}{m_{-2}^{2}}-\frac{\mu_{2}^{2}}{m_{+2}^{2}}\right)$,
the constraint is found to be represented as
$\left(\frac{\mu_{2}^{2}}{m_{-2}^{2}}-\frac{\mu_{2}^{2}}{m_{+2}^{2}}\right) \gtrsim 5 \times 10^{-6}\left(\frac{M_{\eta}}{1 \mathrm{TeV}}\right)$.
As noted in the previous part, the left-hand side of eq. (32) corresponds to the effective coupling $\left|\lambda_{5}\right|$ for the assumed parameters. Although this constraint depends on the DM velocity distribution in our galaxy and other uncertain factors, eq. (32) gives an interesting condition for the present scenario on the origin of the baryon number asymmetry. We find that the model parameters used in the case (b) give $\left|\lambda_{5}\right| \sim 3 \times 10^{-3}$ and then this condition is clearly satisfied. On the other hand, the situation is subtle in the case (a)

[^6]since we find $\left|\lambda_{5}\right| \sim 5 \times 10^{-6}$. This suggests that the DM candidate in this model could be detected through the inelastic scattering in the direct search experiments if this leptogenesis scenario is realized in Nature for this parameter range. It may be worthy to reexamine the direct search results in this mass range in detail.

The above scenario should be also consistent with the DM relic abundance. In the present study, DM is assumed to be $\eta_{R}$. In general, its relics could come from two types of origin such as
$\Omega h^{2}=\Omega_{\mathrm{th}} h^{2}+\Omega_{\mathrm{nonth}} h^{2}$.
The first one is the usual thermal relic, that is, the remnant of $\eta_{R}$ decoupled from the thermal equilibrium distribution. It can be estimated by using the usual formulas [31],
$\Omega_{\mathrm{th}} h^{2}=\frac{1.07 \times 10^{9} z_{D M}}{g_{*}^{1 / 2} m_{\mathrm{pl}}(\mathrm{GeV})\left\langle\sigma_{\eta} v\right\rangle}$,
$z_{D M}=\ln \frac{0.038 g m_{\mathrm{pl}} M_{\eta_{R}}\left\langle\sigma_{\eta} v\right\rangle}{g_{*}^{1 / 2} z_{D M}^{1 / 2}}$,
where $m_{\mathrm{pl}}=\sqrt{8 \pi} M_{\mathrm{pl}}$ and $g$ is internal degrees of freedom of DM. $z_{D M}$ is defined by $z_{D M}=\frac{M_{\eta_{R}}}{T_{f}}$ for the $\eta_{R}$ freeze-out temperature $T_{f}$. The relevant thermally averaged annihilation cross section $\left\langle\sigma_{\eta} v\right\rangle$ including the co-annihilation processes can be found in [14, 15]. Since $\left\langle\sigma_{\eta} v\right\rangle$ has a crucial dependence on the couplings $\lambda_{3,4}$ given in eq. (1) [15], the relic abundance $\Omega_{\text {th }} h^{2}$ could change its value by varying the values of $\lambda_{3,4}$ without affecting other phenomena discussed in this paper. Thus, it is not difficult to realize the suitable relic abundance from this source.

The second one comes from the non-thermal origin, that is, the lepton number asymmetry left in the $\eta$ sector which is produced through the decay of $S_{ \pm 1}$. One may consider that this could play an important role for the DM relic abundance as in the asymmetric DM scenario. In fact, its contribution could be estimated as
$\Omega_{\text {nonth }} h^{2}=2.8 \times 10^{11}\left(\frac{M_{\eta}}{1 \mathrm{TeV}}\right) \Delta Y_{\eta}$,
where $\Delta Y_{\eta}$ is the asymmetry in the present Universe. The nonnegligible contribution to the DM relic abundance is expected in the case $\Delta Y_{\eta}=O\left(10^{-13}\right)$. However, we should note that the relic abundance of $\eta_{R}$ is fixed after the electroweak symmetry breaking. Since the lepton number in the $\eta$ sector is violated through the $\eta_{R}-\eta_{I}$ mass splitting caused by the electroweak symmetry breaking mediated by the effective coupling $\lambda_{5}$, the lepton number asymmetry in the $\eta$ sector disappears completely at this stage. Thus, this non-thermal component cannot contribute to the DM relic abundance in this scenario. The DM relic abundance is completely determined only by the thermal relics as in the same way discussed in the previous studies [15]. This suggests that the leptogenesis scenario presented here can generate sufficient baryon number asymmetry in a consistent way with the generation of the neutrino masses, the DM phenomenology and others. It is notable that they are closely related to each other through the inflaton interaction with the SM Higgs scalar and $\eta$.

## 4. Summary

We have considered an extension of the radiative neutrino mass model with singlet scalars, one of which plays a role of inflaton. The original Ma model can be obtained effectively at low energy regions by integrating out the singlet scalars. In this model, the lepton number violation is prepared as the mass term of inflaton and it plays a crucial role in both the radiative neutrino mass generation and the generation of the lepton number asymmetry. The
lepton number asymmetry is produced by the inflaton decay firstly in the inert doublet sector. It is transferred from the inert doublet sector to the lepton sector through the lepton number conserving scatterings. We have examined this scenario numerically and showed that the sufficient baryon number asymmetry could be generated as long as the model parameters take suitable values. They can be consistent with the neutrino mass generation and the DM phenomenology. The scenario could present a new possibility for the leptogenesis in the framework which makes a close connection between the neutrino mass generation and the inflation of the Universe.

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## Appendix A

In this appendix, we fix the concrete form of the neutrino mass matrix to determine the model parameters based on the neutrino oscillation data. Since it determines the flavor structure of neutrino Yukawa couplings, we can fix the reaction density contained in the Boltzmann equations. As such a typical example, in the present analysis we use
$h_{e i}=0, h_{\mu i}=h_{\tau i} \equiv h_{i} \quad(i=1,2) ; \quad h_{e 3}=h_{\mu 3}=-h_{\tau 3} \equiv h_{3}$,
which could realize the tri-bimaximal neutrino mixing [9]. Although it is not realistic, it could give a good starting point for the purpose of this paper. In this case, three neutrino mass eigenvalues are given as
$m_{\nu_{1}}=0, \quad m_{\nu_{2}}=3 h_{3}^{2} \Lambda_{3}, \quad m_{\nu_{3}}=2\left(h_{1}^{2} \Lambda_{1}+h_{2}^{2} \Lambda_{2}\right)$,
where $\Lambda_{k}$ is defined by
$\Lambda_{k}=\sum_{\alpha=1,2} \sum_{f= \pm} \frac{M_{k} \mu_{\alpha}^{(f) 2}\langle\phi\rangle^{2}}{8 \pi^{2}} I\left(M_{\eta}, M_{k}, m_{f \alpha}\right)$.
Thus, $m_{\nu_{3}}=\sqrt{\Delta m_{\mathrm{atm}}^{2}}$ and $m_{\nu_{2}}=\sqrt{\Delta m_{\text {sol }}^{2}}$ should be satisfied for the normal hierarchy case. We use this relation to fix the values of neutrino Yukawa couplings in the present analysis.

## Appendix B

In this appendix, we give the formulas of the reaction density contributing to the Boltzmann equations for the lepton number asymmetry. In order to give the expression for the reaction density of the relevant processes, we introduce dimensionless variables as
$x=\frac{s}{M_{\eta}^{2}}, \quad a_{j}=\frac{M_{j}^{2}}{M_{\eta}^{2}}, \quad b_{ \pm \alpha}=\frac{m_{ \pm \alpha}^{2}}{M_{\eta}^{2}}, \quad b_{\mu_{\alpha}}=\frac{\left|\mu_{\alpha}\right|^{2}}{M_{\eta}^{2}}$,
where $s$ is the squared center of mass energy.
The reaction density for the scattering process is expressed as
$\gamma(a b \rightarrow i j)=\frac{T}{64 \pi^{4}} \int_{s_{\text {min }}}^{\infty} d s \hat{\sigma}(s) \sqrt{s} K_{1}\left(\frac{\sqrt{s}}{T}\right)$,
where $\hat{\sigma}(s)$ is the reduced cross section and $K_{1}(z)$ is the modified Bessel function of the second kind. The lower bound of integration is defined as $s_{\text {min }}=\max \left[\left(m_{a}+m_{b}\right)^{2},\left(m_{i}+m_{j}\right)^{2}\right]$.

The lepton number conserving scattering processes is induced by the diagrams with $N_{i}$ exchange which are shown in the lefthand side of Fig. 2. In order to give the expression for the reaction density of these processes, we define the following quantities for convenience:
$\frac{1}{D_{i}(x)}=\frac{x-a_{i}}{\left(x-a_{i}\right)^{2}+a_{i}^{2} c_{i}}$,
$c_{i}=\frac{1}{64 \pi^{2}}\left(\sum_{k=e, \mu, \tau}\left|h_{k i}\right|^{2}\right)^{2}\left(1-\frac{1}{a_{i}}\right)^{4}$.
Using these definitions, their reduced cross sections are expressed as

$$
\begin{align*}
\hat{\sigma}_{a}(x)= & \frac{1}{2 \pi}\left[\sum _ { i = 1 } ^ { 3 } ( h h ^ { \dagger } ) _ { i i } ^ { 2 } \left\{\frac{a_{i}\left(x^{2}-4 x\right)^{1 / 2}}{a_{i} x+\left(a_{i}-1\right)^{2}}\right.\right. \\
& \left.+\frac{a_{i}}{x+2 a_{i}-2} \ln \left(\frac{x+\left(x^{2}-4 x\right)^{1 / 2}+2 a_{i}-2}{x-\left(x^{2}-4 x\right)^{1 / 2}+2 a_{i}-2}\right)\right\} \\
& +\sum_{i>j} \frac{\operatorname{Re}\left[\left(h h^{\dagger}\right)_{i j}^{2}\right] \sqrt{a_{i} a_{j}}}{x+a_{i}+a_{j}-2}\left\{\frac{2 x+3 a_{i}+a_{j}-4}{a_{j}-a_{i}}\right. \\
& \times \ln \left(\frac{x+\left(x^{2}-4 x\right)^{1 / 2}+2 a_{i}-2}{x-\left(x^{2}-4 x\right)^{1 / 2}+2 a_{i}-2}\right) \\
& +\frac{2 x+a_{i}+3 a_{j}-4}{a_{i}-a_{j}} \\
& \left.\left.\times \ln \left(\frac{x+\left(x^{2}-4 x\right)^{1 / 2}+2 a_{j}-2}{x-\left(x^{2}-4 x\right)^{1 / 2}+2 a_{j}-2}\right)\right\}\right] \tag{42}
\end{align*}
$$

for $\eta \eta \rightarrow \ell_{\alpha} \ell_{\beta}$ and

$$
\begin{align*}
\hat{\sigma}_{b}(x)= & \frac{1}{2 \pi} \frac{(x-1)^{2}}{x^{2}}\left[\sum _ { i = 1 } ^ { 3 } ( h h ^ { \dagger } ) _ { i i } ^ { 2 } \frac { a _ { i } } { x } \left\{\frac{x^{2}}{x a_{i}-1}+\frac{x}{D_{i}(x)}+\frac{(x-1)^{2}}{2 D_{i}(x)^{2}}\right.\right. \\
& \left.-\frac{x^{2}}{(x-1)^{2}}\left(1+\frac{x+a_{i}-2}{D_{i}(x)}\right) \ln \left(\frac{x\left(x+a_{i}-2\right)}{x a_{i}-1}\right)\right\} \\
& +\sum_{i>j} \operatorname{Re}\left[\left(h h^{\dagger}\right)_{i j}^{2}\right] \frac{\sqrt{a_{i} a_{j}}}{x}\left\{\frac{x}{D_{i}(x)}+\frac{x}{D_{j}(x)}+\frac{(x-1)^{2}}{D_{i}(x) D_{j}(x)}\right. \\
& +\frac{x^{2}}{(x-1)^{2}}\left(\frac{2\left(x+a_{i}-2\right)}{a_{j}-a_{i}}-\frac{x+a_{i}-2}{D_{j}(x)}\right) \\
& \times \ln \frac{x\left(x+a_{i}-2\right)}{x a_{i}-1} \\
& +\frac{x^{2}}{(x-1)^{2}}\left(\frac{2\left(x+a_{j}-2\right)}{a_{i}-a_{j}}-\frac{x+a_{j}-2}{D_{i}(x)}\right) \\
& \left.\left.\times \ln \frac{x\left(x+a_{j}-2\right)}{x a_{j}-1}\right\}\right] \tag{43}
\end{align*}
$$

for $\ell_{\alpha} \eta^{\dagger} \rightarrow \bar{\ell}_{\beta} \eta$.
The lepton number violating scattering processes are brought about by the diagrams with $S_{ \pm \alpha}$ exchange which are shown in the right-hand side of Fig. 2. In order to represent their reduced cross section, we introduce the definition such as

$$
\begin{align*}
& \frac{1}{\tilde{D}_{ \pm \alpha}(x)}=\frac{1}{\left(x-b_{ \pm \alpha}\right)^{2}+b_{ \pm \alpha}^{2} \tilde{c}_{ \pm \alpha}}, \\
& \tilde{c}_{ \pm \alpha}=\frac{1}{128 \pi^{2}}\left(\frac{b_{\mu_{ \pm \alpha}}}{b_{ \pm \alpha}}\right)^{2}\left(1-\frac{1}{b_{ \pm \alpha}}\right), \\
& P_{ \pm \alpha}=\frac{2\left(1-b_{ \pm \alpha}\right)-x}{[x(x-4)]^{1 / 2}}, \quad Q_{ \pm \alpha}=-1+\frac{2\left(1-x b_{ \pm \alpha}\right)}{(x-1)^{2}} . \tag{44}
\end{align*}
$$

Using these quantities, the reduced cross sections are represented as

$$
\begin{align*}
\hat{\sigma}_{x}(x)= & \sum_{\alpha=1,2} \frac{b_{\mu_{\alpha}}^{2}}{4 \pi} \frac{1}{\left(x^{3}(x-4)\right)^{1 / 2}}\left[\frac{2}{P_{+\alpha}^{2}-1}+\frac{2}{P_{-\alpha}^{2}-1}\right. \\
& +\left(\frac{1}{P_{+\alpha}}+\frac{4 P_{-\alpha}}{P_{+\alpha}^{2}-P_{-\alpha}^{2}}\right) \ln \frac{P_{+\alpha}+1}{P_{+\alpha}-1} \\
& \left.+\left(\frac{1}{P_{-\alpha}}-\frac{4 P_{+\alpha}}{P_{+\alpha}^{2}-P_{-\alpha}^{2}}\right) \ln \frac{P_{-\alpha}+1}{P_{-\alpha}-1}\right] \\
& +(\text { cross terms between } \alpha=1 \text { and } 2) \tag{45}
\end{align*}
$$

for $\eta \eta \rightarrow \phi \phi$ and

$$
\begin{aligned}
\hat{\sigma}_{y}(x)= & \sum_{\alpha=1,2} \frac{b_{\mu_{\alpha}}^{2}}{2 \pi}\left[\frac { 1 } { ( x - 1 ) ^ { 2 } } \left\{\frac{1}{Q_{+\alpha}^{2}-1}+\frac{1}{Q_{-\alpha}^{2}-1}\right.\right. \\
& \left.+\frac{1}{Q_{+\alpha}-Q_{-\alpha}}\left(\ln \frac{Q_{+\alpha}+1}{Q_{+\alpha}-1}-\ln \frac{Q_{-\alpha}+1}{Q_{-\alpha}-1}\right)\right\} \\
& +\frac{(x-1)^{2}}{4 x^{2}}\left\{\frac{1}{\tilde{D}_{+\alpha}(x)}+\frac{1}{\tilde{D}_{-\alpha}(x)}\right. \\
& \left.-\frac{2}{b_{+\alpha}-b_{-\alpha}}\left(\frac{x-b_{+\alpha}}{\tilde{D}_{+\alpha}(x)}-\frac{x-b_{-\alpha}}{\tilde{D}_{-\alpha}(x)}\right)\right\} \\
& +\frac{1}{2 x}\left(\frac{x-b_{+\alpha}}{\tilde{D}_{+\alpha}(x)}-\frac{x-b_{-\alpha}}{\tilde{D}_{-\alpha}(x)}\right) \\
& \left.\times\left(\ln \frac{Q_{+\alpha}+1}{Q_{+\alpha}-1}-\ln \frac{Q_{-\alpha}+1}{Q_{-\alpha}-1}\right)\right]
\end{aligned}
$$

$$
\begin{equation*}
+(\text { cross terms between } \alpha=1 \text { and } 2) \tag{46}
\end{equation*}
$$

for $\eta \phi^{\dagger} \rightarrow \eta^{\dagger} \phi$. Since we consider the case $b_{\mu_{2}} \gg b_{\mu_{1}}$, we can neglect contributions relevant to $b_{\mu_{1}}$.

## References

[1] Super-Kamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 81 (1998) 1562;
SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011301; KamLAND Collaboration, K. Eguchi, et al., Phys. Rev. Lett. 90 (2003) 021802; K2K Collaboration, M.H. Ahn, et al., Phys. Rev. Lett. 90 (2003) 041801.
[2] T2K Collaboration, K. Abe, et al., Phys. Rev. Lett. 107 (2011) 041801; Double Chooz Collaboration, Y. Abe, et al., Phys. Rev. Lett. 108 (2012) 131801; RENO Collaboration, J.K. Ahn, et al., Phys. Rev. Lett. 108 (2012) 191802; The Daya Bay Collaboration, F.E. An, et al., Phys. Rev. Lett. 108 (2012) 171803.
[3] WMAP Collaboration, D.N. Spergel, et al., Astrophys. J. 148 (2003) 175; SDSS Collaboration, M. Tegmark, et al., Phys. Rev. D 69 (2004) 103501.
[4] E. Komatsu, et al., Astrophys. J. Suppl. Ser. 180 (2009) 330;
E. Komatsu, et al., Astrophys. J. Suppl. Ser. 192 (2011) 18.
[5] Planck Collaboration, P.A.R. Ade, et al., arXiv:1303.5082 [astro-ph.CO].
[6] E. Ma, Phys. Rev. D 73 (2006) 077301.
[7] P. Minkowski, Phys. Lett. B 67 (1977) 421;
M. Gell-Mann, P. Ramond, R. Slansky, in: D. Freedman, P. Van Nieuwenhuizen (Eds.), Supergravity, North Holland, Amsterdam, 1979, p. 315; T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, Tsukuba, Japan, KEK, 1979, p. 95.
[8] J. Kubo, E. Ma, D. Suematsu, Phys. Lett. B 642 (2006) 18;
J. Kubo, D. Suematsu, Phys. Lett. B 643 (2006) 336;
D. Aristizabal Sierra, J. Kubo, D. Suematsu, D. Restrepo, O. Zapata, Phys. Rev. D 79 (2009) 013011.
[9] D. Suematsu, T. Toma, T. Yoshida, Phys. Rev. D 79 (2009) 093004;
D. Suematsu, T. Toma, T. Yoshida, Phys. Rev. D 82 (2010) 013012.
[10] A. Riotto, M. Trodden, Annu. Rev. Nucl. Part. Sci. 49 (1999) 35;
W. Bernreuther, Lect. Notes Phys. 591 (2002) 237;
M. Dine, A. Kusenko, Rev. Mod. Phys. 76 (2003) 1.
[11] M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45.
[12] M. Plümacher, Nucl. Phys. B 530 (1998) 207;
W. Buchmüller, M. Plümacher, Int. J. Mod. Phys. A 15 (2000) 5047;
W. Buchmüller, P. Di Bari, M. Plümacher, Phys. Lett. B 547 (2002) 128;
W. Buchmüller, P. Di Bari, M. Plümacher, Nucl. Phys. B 643 (2002) 367;
W. Buchmüller, P. Di Bari, M. Plümacher, Nucl. Phys. B 665 (2003) 445;
G.F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Struma, Nucl. Phys. B 685 (2004) 89;
W. Buchmüller, R.D. Peccei, T. Yanagida, Annu. Rev. Nucl. Part. Sci. 55 (2005) 311.
[13] D. Suematsu, Eur. Phys. J. C 56 (2008) 379;
D. Suematsu, Eur. Phys. J. C 72 (2012) 72.
[14] T. Hambye, F.-S. Ling, L.L. Honorez, J. Roche, J. High Energy Phys. 0907 (2009) 090.
[15] S. Kashiwase, D. Suematsu, Phys. Rev. D 86 (2012) 053001;
S. Kashiwase, D. Suematsu, Eur. Phys. J. C 73 (2013) 2484.
[16] H. Higashi, T. Ishima, D. Suematsu, Int. J. Mod. Phys. A 26 (2011) 995.
[17] D. Suematsu, Phys. Rev. D 85 (2012) 073008.
[18] E. Ma, Ann. Fond. Louis de Broglie 31 (2006) 285;
H. Fukuoka, J. Kubo, D. Suematsu, Phys. Lett. B 678 (2009) 401;
H. Fukuoka, D. Suematsu, T. Toma, J. Cosmol. Astropart. Phys. 1107 (2011) 001; D. Suematsu, T. Toma, Nucl. Phys. B 847 (2011) 567.
[19] R.H.S. Budhi, S. Kashiwase, D. Suematsu, Phys. Rev. D 90 (2014) 113013; R.H.S. Budhi, S. Kashiwase, D. Suematsu, arXiv:1505.05955 [hep-ph].
[20] For reviews, D.H. Lyth, A. Riotto, Phys. Rep. 314 (1999) 1; A.R. Liddle, D.H. Lyth, Cosmological Inflation and Large-Scale Structure, Cambridge University Press, 2000.
[21] J. McDonald, J. Cosmol. Astropart. Phys. 09 (2014) 027.
[22] BICEP2/Keck Collaboration, Planck Collaboration, P.A.R. Ade, et al., Phys. Rev. Lett. 114 (2015) 101301.
[23] Planck Collaboration, P.A.R. Ade, et al., arXiv:1502.01589 [astro-ph.CO]; Planck Collaboration, P.A.R. Ade, et al., arXiv:1502.02114 [astro-ph.CO].
[24] E. Ma, M. Raidal, Phys. Rev. Lett. 87 (2001) 011802.
[25] MEG Collaboration, J. Adam, et al., Phys. Rev. Lett. 110 (2013) 201801.
[26] BABAR Collaboration, B. Aubert, et al., Phys. Rev. Lett. 104 (2010) 021802.
[27] L. Kofman, A. Linde, A.A. Starobinsky, Phys. Rev. Lett. 73 (1994) 3195; L. Kofman, A. Linde, A.A. Starobinsky, Phys. Rev. D 56 (1997) 3258.
[28] For a recent review, R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine, A. Mazumdar, arXiv:1001.2600 [hep-th], and see references therein.
[29] Y. Cui, D.E. Marrissey, D. Poland, L. Randall, J. High Energy Phys. 0905 (2009) 076;
C. Arina, F.-S. Ling, M.H.G. Tytgat, J. Cosmol. Astropart. Phys. 0910 (2009) 018.
[30] CDMS Collaboration, Z. Ahmed, et al., Phys. Rev. Lett. 102 (2009) 011301; XENON100 Collaboration, E. Aprile, et al., Phys. Rev. Lett. 105 (2010) 131302; G. Angloher, et al., Astropart. Phys. 31 (2009) 270; V.N. Lebedenko, et al., Phys. Rev. D 80 (2009) 052010.
[31] K. Griest, D. Seckel, Phys. Rev. D 43 (1991) 3191; P. Gondolo, G. Gelmini, Nucl. Phys. B 360 (1991) 145.


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    ${ }^{1}$ This brings about dangerous lepton number violating processes at large rate unless special flavor structure is assumed for the neutrino Yukawa couplings [9].

[^1]:    ${ }^{2}$ When $S_{1}$ plays a role of inflaton, this condition could be relevant to the $\eta$ problem in this inflation scenario. We cannot fix it at this stage unless the UV completion of the model is clarified.

[^2]:    ${ }^{3}$ It should be noted that one of the eigenvalues of this assumed mass matrix is zero. It may be also useful to recall that the cosmological upper bound for the neutrino masses is 0.23 eV [23].

[^3]:    ${ }^{4}$ In this estimation, the oscillation energy of each component is assumed to dominate the total energy density of the Universe.

[^4]:    ${ }^{5}$ In the following study, we assume the maximum $C P$ phase $\left|\sin 2\left(\theta_{1}-\theta_{2}\right)\right|=1$.

[^5]:    ${ }^{6}$ Following the usual convention, we introduce a dimensionless parameter $z$ as $z=\frac{M_{\eta}}{T}$ by using a convenient mass scale $M_{\eta}$, which is defined below eq. (13).

[^6]:    ${ }^{7}$ The mass of $\eta_{R}$ and $\eta_{I}$ can be expressed as $M_{\eta_{R}}^{2}=M_{\eta}^{2}+\lambda_{5}\langle\phi\rangle^{2}$ and $M_{\eta_{I}}^{2}=$ $M_{\eta}^{2}-\lambda_{5}\langle\phi\rangle^{2}$ respectively, by using the effective coupling $\lambda_{5}$.

