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Isospin violating effects in e^+e^- vs. τ measurements of the pion form factor $|F_\pi|^2(s)$ [☆]

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Abstract

We study possible so far unaccounted isospin breaking effects in the relation between the pion form factor as determined in e^+e^- experiments and the corresponding quantity obtained after accounting for known isospin breaking effects by an isospin rotation from the τ -decay spectra. In fact the observed 10% discrepancy in the respective pion form factors may be explained by the isospin breaking which is due to the difference between masses and widths of the charged and neutral ρ mesons. Since the hadronic contribution to the muon anomalous magnetic moment can be calculated directly in terms of the e^+e^- -data the corresponding evaluation seems to be more reliable. Our estimate is $a_\mu^{\text{had}(1)} = (694.8 \pm 8.6) \times 10^{-10}$. The τ -data are useful at the presently aimed level of accuracy only after appropriate input from theory.

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1. Introduction

The most precise measurement of the low energy pion form factor in e^+e^- -annihilation experiments is from the CMD-2 Collaboration. The updated results for the process $e^+e^- \rightarrow \rho \rightarrow \pi^+\pi^-$ have just been published [1]. The update appeared necessary due to an overestimate of the integrated luminosity in previous analyses. The latter was published in 2002 [2]. A more progressive error estimate (improving on radiative corrections, in particular) allowed a reduction of the systematic error from 1.4% to 0.6%.

Since 1997 precise τ -spectral functions became available [3–5] which, to the extent that flavor $SU(2)_f$ in the light hadron sector is a symmetry, allows to obtain the iso-vector part of the e^+e^- -cross section [6]. In this way τ data may help to substantially improve our knowledge of $|F_\pi|^2(s)$, which is an important input for the evaluations of the hadronic vacuum polarization contributions to the anomalous magnetic moment of the muon a_μ and of the effective fine structure constant $\alpha_{\text{em}}(M_Z)$ an important input for LEP/SLC precision physics (see, e.g., [7]).

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The idea to use the τ spectral data to improve the evaluation of the hadronic contributions a_μ^{had} and $\Delta\alpha^{\text{had}}$ was pioneered in [8].

With increasing precision of the low energy data it more and more turned out that we are confronted with a serious obstacle to further progress: in the region just above the ω -resonance, the isospin rotated τ -data, corrected for the known isospin violating effects [9], do not agree with the e^+e^- -data at the 10% level [10]. Before the origin of this discrepancy is found it will be hard to make progress in pinning further down theoretical uncertainties in the predictions for a_μ and $\alpha_{\text{em}}(M_Z)$.

In this context isospin breaking effects in the relationship between the τ - and the e^+e^- -data have been extensively investigated in [9]. One point which in our opinion has not been satisfactorily clarified is the role of the isospin breaking effects in the charged vs. neutral ρ line-shape, which must manifest themselves in $m_{\rho^\pm} - m_{\rho^0}$ and $\Gamma_{\rho^\pm} - \Gamma_{\rho^0}$. Looking at the particle data tables [11], there is no established non-zero result as yet. Earlier statements about the problem in [8–10] adopted essentially the PDG estimate $m_{\rho^\pm} - m_{\rho^0} = 0 \pm 1$ MeV. There are theoretical arguments about why this mass difference is expected to be very small: the usual argument assumes $\Delta m_\rho^2 \simeq \Delta m_\pi^2$ via a sum rule, which then yields $m_{\rho^-} - m_{\rho^0} \simeq \frac{1}{2} \frac{\Delta m_\pi^2}{m_{\rho^0}} \sim 0.014$ MeV! This is based, however, on an *assumption* which need not be true for the mass definition adopted in recent ρ -line shape analyses.² In fact a more recent analysis of the CMD-2 (before the last update) and the ALEPH and CLEO data yielded $m_{\rho^\pm} - m_{\rho^0} = 2.6 \pm 0.8$ MeV and $\Gamma_{\rho^\pm} - \Gamma_{\rho^0} = 3.1 \pm 1.7$ MeV [15] where the uncertainty is our estimate. The corresponding isospin corrections may still look too small to account fully for the observed discrepancy in the spectral functions but they clearly point towards a substantial reduction of the problem. Our strategy therefore here is a different one. Our hypothesis is that as a leading effect the discrepancy very likely is due to the isospin breaking by the charged vs. neutral ρ -meson parameters. A similar but subleading contribution is expected to come from possible isospin violations in the respective backgrounds (encoded usually by the ρ' , ρ'' contributions). Since the fit formulae adopted, like the Gounaris–Sakurai formula [16], are far from being based on first principles we should not trust too much in the fitting procedures based on them. E.g., usually just a set of resonances ρ , ω , ρ' , ρ'' is included but we have no idea about the background (continuum) which also should be included somehow. We also would like to advocate that, as the level of accuracy of the present discussion advances, in future one should compile charged and neutral ρ data separately.

Whether the observed discrepancy is an experimental problem, or just a so far underestimated isospin breaking effect will also be settled, hopefully, by new results for hadronic e^+e^- cross sections which are under way from KLOE, BABAR and BELLE. These experiments, running at fixed energies, are able to perform measurements via the radiative return method [17–19]. Results presented recently by KLOE seem to agree very well with the final CMD-2 e^+e^- -data.

2. The τ vs. e^+e^- problem

The iso-vector part of $\sigma(e^+e^- \rightarrow \text{hadrons})$ may be calculated by an isospin rotation from τ -decay spectra, to the extent that the so-called conserved vector current (CVC) would be really conserved (which it is not, see below). The relation may be derived by comparing the relevant lowest order diagrams which for the e^+e^- case translates into

$$\sigma_{\pi\pi}^{(0)} \equiv \sigma_0(e^+e^- \rightarrow \pi^+\pi^-) = \frac{4\pi\alpha^2}{s} v_0(s) \quad (1)$$

² For a more detailed theoretical estimate see [12] and references therein (see also [13,14]). Mass and width of an unstable particle, and in particular of the ρ , depend on the precise definition. In that sense they are pseudo-observables which depend on theoretical input. What we need here is a consistent prescription to extract them from the experimental data (ρ -line shape) and make sure that we compare comparable quantities (with respect to the handling of vacuum polarization effects, final state radiation, energy dependence of width, background, etc.).

and for the τ case into

$$\frac{1}{\Gamma} \frac{d\Gamma}{ds}(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) = \frac{6\pi |V_{ud}|^2 S_{EW}}{m_\tau^2} \frac{B(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)}{B(\tau^- \rightarrow \nu_\tau \pi^- \pi^0)} \left(1 - \frac{s}{m_\tau^2}\right) \left(1 + \frac{2s}{m_\tau^2}\right) v_-(s), \quad (2)$$

where $|V_{ud}| = 0.9752 \pm 0.0007$ [11] denotes the CKM weak mixing matrix element and $S_{EW(\text{new})} = 1.0233 \pm 0.0006$ [$S_{EW(\text{old})} = 1.0194$] accounts for electroweak radiative corrections [9,10,20–23]. The spectral functions are obtained from the corresponding invariant mass distributions. The $B(i)$'s are branching ratios. $SU(2)$ symmetry (CVC) would imply

$$v_-(s) = v_0(s). \quad (3)$$

The spectral functions $v_i(s)$ are related to the pion form factors $F_\pi^i(s)$ by

$$v_i(s) = \frac{\beta_i^3(s)}{12\pi} |F_\pi^i(s)|^2 \quad (i = 0, -), \quad (4)$$

where $\beta_i^3(s)$ is the pion velocity. The difference in phase space of the pion pairs gives rise to the relative factor $\beta_{\pi^-\pi^0}^3/\beta_{\pi^-\pi^+}^3$ [8,24].

It is important to check what precisely the experimental data in each case represent. In CMD-2 e^+e^- measurements we exclude final state radiation (FSR) as well as vacuum polarization effects³ which are not included in the τ data in first place. FSR as far as included by the measurement (virtual and soft real radiation) has been subtracted together with the initial state photon radiation. Hard FSR photons were rejected to a large extent. In our analysis $F_\pi(s)$ obtained from CMD-2 is the undressed (from VP and FSR) pion form-factor (see, e.g., [25]). The available τ decay spectra all include photon radiation (no subtractions were made), which hence has to be subtracted a posteriori (the correction factor G_{EM} below), while photon vacuum polarization effects play no role (i.e., are not included). This is because the τ decay as a charged current (CC) process proceeds by the heavy W exchange, which makes it an effective four-fermion interaction with Fermi constant G_F as a coupling in place of $\alpha(s)$. In contrast to α the Fermi coupling G_F is not running up to LEP energy scales. Electroweak short distance corrections (hadronic relative to leptonic channel) give rise to the correction factor $S_{EW} = 1 + \delta_{EW}$, which is dominated by a leading large logarithm ($1 + (\alpha/\pi) \ln(M_Z/m_\tau)$) which should be resummed using the renormalization group [20]. Note that the overall coupling drops out from the ratios in (2). This also makes it evident that the subtraction of the large and strongly energy dependent vacuum polarization effects (see, e.g., Fig. 1 in [26]) necessary for the e^+e^- -data, which seems to worsen the e^+e^- vs. τ problem, was properly treated in previous analyses.

Before a precise comparison via (3) is possible all kind of isospin breaking effects have to be taken into account. As mentioned earlier, this has been investigated carefully in [9] for the most relevant $\pi\pi$ channel. Accordingly, we may write the corrected version of (3) (see [9] for details) in the form

$$\sigma_{\pi\pi}^{(0)} = \left[\frac{K_\sigma(s)}{K_\Gamma(s)} \right] \frac{d\Gamma_{\pi\pi[\gamma]}}{ds} \times \frac{R_{IB}(s)}{S_{EW}} \quad (5)$$

with

$$K_\Gamma(s) = \frac{G_F^2 |V_{ud}|^2 m_\tau^3}{384\pi^3} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2\frac{s}{m_\tau^2}\right), \quad K_\sigma(s) = \frac{\pi\alpha^2}{3s}, \quad (6)$$

³ Table 1 of [1] lists $|F_\pi|^2$, which includes VP but not FSR, as well as $\sigma_{\pi\pi(\gamma)}^0$ which includes FSR but not VP. VP and FSR are separately known quantities which we may add and subtract according to our needs.

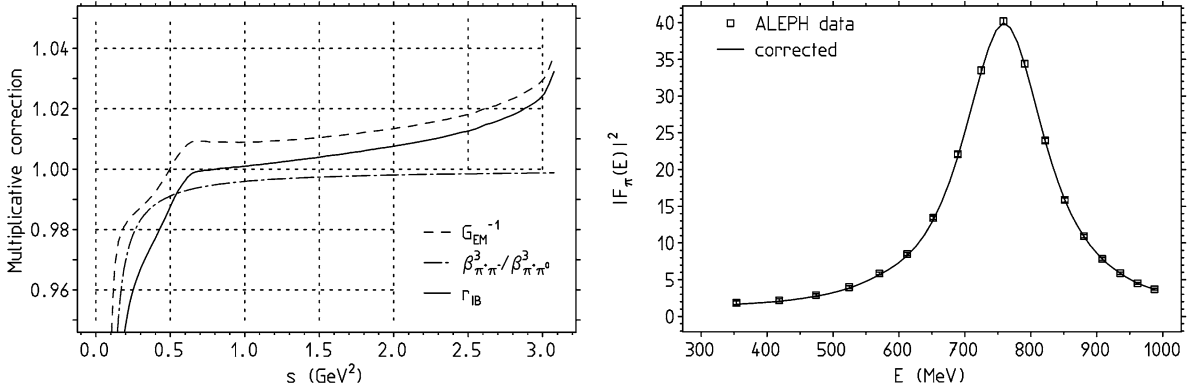


Fig. 1. Isospin corrections applied to the τ data: left the corrections [9], with Γ_{IB} defined in (9), and right the barely visible effect on the ALEPH data.

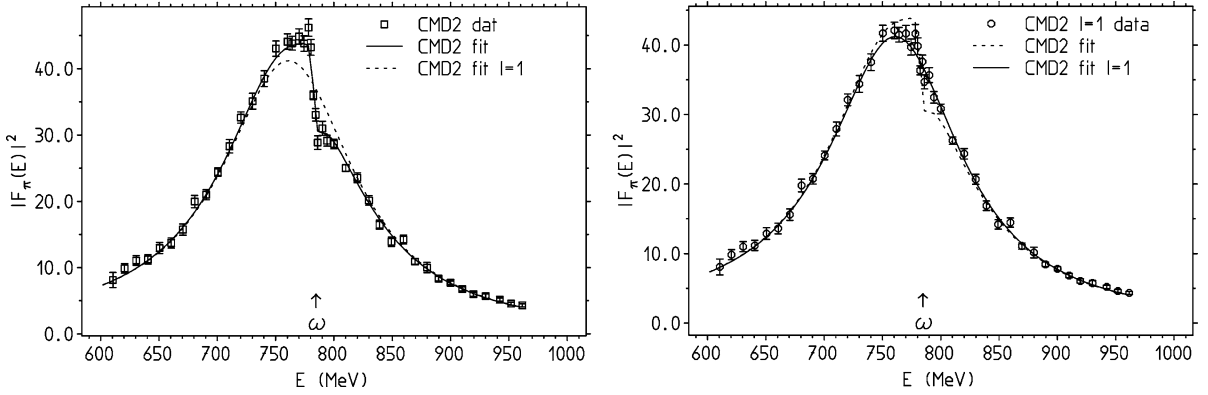


Fig. 2. CMD-2 data for $|F_\pi|^2$ in ρ - ω region together with Gounaris-Sakurai fit. Left: before subtraction, right: after subtraction of the ω .

and the isospin breaking correction

$$R_{IB}(s) = \frac{1}{G_{EM}(s)} \frac{\beta_{\pi^- \pi^+}^3}{\beta_{\pi^- \pi^0}^3} \left| \frac{F_V(s)}{f_+(s)} \right|^2. \quad (7)$$

The factor $G_{EM}(s)$, displayed in Fig. 1, includes the QED corrections to $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ decay with virtual plus real soft and hard (integrated over all phase space) photon radiation calculated in scalar QED, except from the short distance term, which is calculated for the corresponding quark production process and included conventionally in S_{EW} as the leading logarithm, mentioned before. Originating from (4), $\beta_{\pi^- \pi^+}^3 / \beta_{\pi^- \pi^0}^3$ (see Fig. 1) is a phase space correction due to the $\pi^\pm - \pi^0$ mass difference. $F_V(s) = F_\pi^0(s)$ is the NC vector current form factor, which exhibits besides the $I = 1$ part an $I = 0$ contribution. The latter ρ - ω mixing term is due to the $SU(2)$ breaking ($m_d - m_u$ mass difference). Finally, $f_+(s) = F_\pi^-(s)$ is the CC $I = 1$ vector form factor. One of the leading isospin breaking effects is the ρ - ω mixing correction included in $|F_V(s)|^2 \rightarrow |F_V^{(I=1)}(s)|^2 \sim |f_+(s)|^2$ is illustrated in Fig. 2. The form-factor corrections, in principle, also should include the electromagnetic shifts in the masses and the widths of the ρ 's. Up to this last mentioned effect, which was considered to be small, all the corrections were applied in [10] but were not able to eliminate the observed discrepancy between $v_-(s)$ and $v_0(s)$ (see [10] for details and Fig. 3).

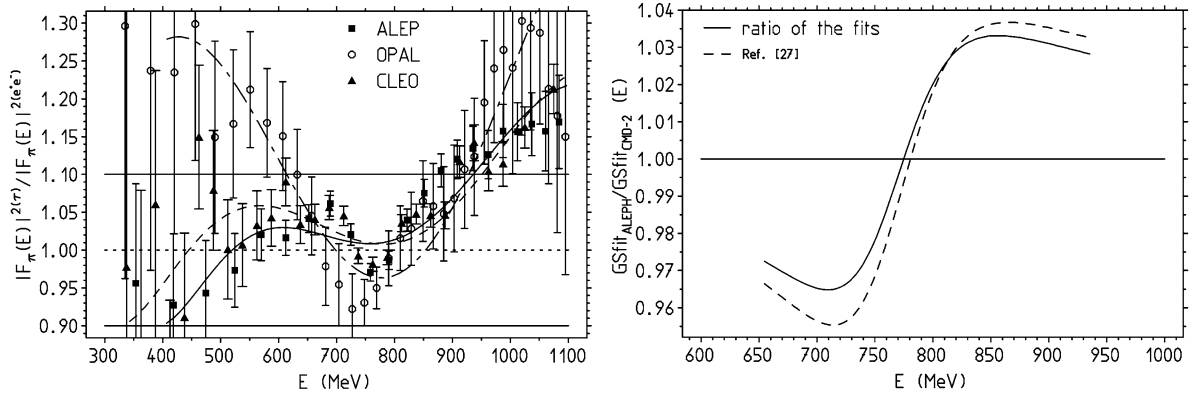


Fig. 3. Left: the ratio between τ -data sets from ALEPH, OPAL and CLEO and the $I = 1$ part of the CMD-2 fit of the e^+e^- -data. The curves which should guide the eye are fits of the ratios using 8th order Tschebycheff polynomials. Right: ratio of the ALEPH vs. CMD-2 fits differing by mass and width only (see Table 1). By the isospin violation correction ($m_{\rho^-}, \Gamma_{\rho^-} \rightarrow m_{\rho^0}, \Gamma_{\rho^0}$) of the τ -data this ratio becomes trivially equal to unity.

In fact, possible isospin breaking corrections due to different electromagnetic shifts of masses and widths of the neutral and charged ρ -mesons (remember that for other quark bound states the π 's we have $m_{\pi^\pm} - m_{\pi^0} = 4.5935 \pm 0.0005$ MeV), respectively, have been mentioned or were very briefly discussed only in [8–10], and they might have been underestimated so far. Such isospin violating mass and width differences are not established experimentally, not even the sign. We therefore ask the question whether applying an isospin breaking correction which accounts for that could resolve the puzzle of the above mentioned discrepancy.

What we do is the following: we take the CMD-2 data and subtract off the VP and the ω -contribution. The latter $I = 0$ part enters via ρ - ω mixing, which is a consequence of isospin violation due to the mass difference $m_u - m_d$ of the light quarks. To this end we may take the Gounaris–Sakurai kind parameterization (see [2,16] for details) (FSR not included; BW = Breit–Wigner; BW^{GS} = Gounaris–Sakurai modified Breit–Wigner)

$$F_\pi(s) = \frac{\text{BW}_{\rho(770)}^{\text{GS}}(s)(1 + \delta \frac{s}{M_\omega^2} \text{BW}_\omega(s)) + \beta \text{BW}_{\rho(1450)}^{\text{GS}}(s) + \gamma \text{BW}_{\rho(1700)}^{\text{GS}}(s)}{1 + \beta + \gamma} \quad (8)$$

of the CMD-2 data and set the mixing parameter $\delta = 0$. In this way we obtain the iso-vector part of the square of the pion form factor $|F_\pi|_{(e^+e^-)}^{2I=1}(s)$ displayed in Fig. 2. To the τ version of the pion form factor, following from (2) and (4), we perform the isospin breaking corrections

$$r_{\text{IB}}(s) = \frac{1}{G_{\text{EM}}(s)} \frac{\beta_{\pi^- \pi^+}^3}{\beta_{\pi^- \pi^0}^3} \frac{S_{\text{EW}(\text{old})}}{S_{\text{EW}(\text{new})}} \quad (9)$$

with $G_{\text{EM}}(s)$ from [9]. The factor $S_{\text{EW}(\text{old})}/S_{\text{EW}(\text{new})}$ corrects for some previously missing corrections [10,23]. The such obtained corrected⁴ pion form factor $|F_\pi|_{(e^+e^-)}^{2I=1}(s)$ is to be compared with $|F_\pi|_{(\tau)}^{2I=1}(s)$. The ratio shows the unexpected large deviations from unity (see Fig. 3). While the ALEPH and CLEO data clearly exhibit the structure as expected from an increase of mass and width of the ρ the OPAL data show a different form of the spectrum. The problem with the OPAL data originates from the fact that in the neighborhood of the ρ peak the cross section apparently is too low. Since the distribution is normalized to the very precisely known total branching fraction, the tails of the resonance get enhanced, which leads to the structure actually seen (the apparent width gets

⁴ The velocity factor correction of course only applies when, as frequently has been done, the wrong velocity was used in (4) in the extraction of the charged channel form factor.

Table 1

Results of fits to the isospin breaking corrected pion form factors squared for τ (ALEPH, CLEO and OPAL) and e^+e^- (CMD-2) data. The Gounaris–Sakurai parameterization of the ρ line shape is utilized. Masses and widths in MeV. $S = \sqrt{\chi^2/(n-1)}$

	τ ALEPH	τ CLEO	τ OPAL	e^+e^- CMD-2
m_{ρ^0}	–	–	–	$772.95 \pm 0.56 \pm 0.12$
Γ_{ρ^0}	–	–	–	$147.93 \pm 0.70^{+0.12}_{-0.13}$
m_{ρ^-}	$775.52 \pm 0.49^{+0.34}_{-0.25}$	$775.01 \pm 0.36^{+0.30}_{-0.29}$	$777.34 \pm 1.21^{+0.29}_{-0.23}$	–
Γ_{ρ^-}	$149.40 \pm 0.68^{+0.10}_{-0.09}$	$149.00 \pm 0.49^{+0.12}_{-0.21}$	$153.91 \pm 1.62^{+1.15}_{-1.50}$	–
S	1.39	1.35	0.62	1.28

enhanced). Of course the data points of the spectrum have imposed strong error correlations via the normalization to the integral rate. In the figure only the diagonal elements of the covariance matrix are visualized.

For a comparison of the e^+e^- with the τ data we have to perform fits in a common energy range. It is important to fix the “ ρ -background” appropriately. The latter is represented in the GS-parametrization by the higher resonances ρ' and ρ'' and in order to fit the corresponding parameters we have to extend the fit range towards higher energies. To this end we are including beyond the CMD-2 limit the available $\pi^+\pi^-$ data from other experiments up to 1.5 GeV above which the τ data get of low quality. We finally decided to adopt the PDG values for $m_{\rho'} = 1465 \pm 25$, $\Gamma_{\rho'} = 310 \pm 60$ and $m_{\rho''} = 1700 \pm 20$, $\Gamma_{\rho''} = 240 \pm 60$ for the masses and widths and fit the complex admixing coefficients β and γ . For $|\beta| = 0.12 \pm 0.005$, $\varphi_\beta = 160 \pm 5$ and $|\gamma| = 0.023 \pm 0.005$, $\varphi_\gamma = 0$ we get reasonable fits for all data sets. In particular the widths depend substantially on the precise values of these parameters. Fortunately, the parameter shifts of interest turn out to be rather stable. In other words, while there are different possible parametrizations of comparable quality for the individual data sets the systematic shift between e^+e^- and τ data is there independently of how we parametrize the data.

We may fit now $|F_\pi|_{(\tau)}^{2I=1}(s)$ with the Gounaris–Sakurai formula (8) with no ω term, i.e., with $\delta = 0$, in order to obtain m_{ρ^\pm} and Γ_{ρ^\pm} . We would like to emphasize that it is important to “zoom-in the ρ ” appropriately in determining $m_{\rho^\pm} - m_{\rho^0}$ and $\Gamma_{\rho^\pm} - \Gamma_{\rho^0}$, i.e., we have to perform the fits at fixed background (i.e., besides m_ρ and Γ_ρ all other parameters in the GS-formula are held fixed) in the ρ dominated region between 610.5 MeV and 961.5 MeV (CMD-2 range).

This simple leading effect analysis yields the results in Table 1.⁵ As can be seen the τ data give consistently larger values for both mass and width of the charged ρ . The evidence is far from impressive, between ALEPH and CMD-2 we have $\Delta m_\rho = 2.6 \pm 0.8$ MeV and $\Delta \Gamma_\rho = 1.5 \pm 1.0$ MeV. The two-parameter fits are not of good quality and a more elaborate analysis would be needed to come to more precise conclusions. An analysis based on more recent preliminary ALEPH data, yields the slightly larger values $\Delta m_\rho = 3.1 \pm 0.9$ MeV and $\Delta \Gamma_\rho = 2.3 \pm 1.6$ MeV [27], which are consistent however with our findings.

Note that our values of m_{ρ^0} and Γ_{ρ^0} differ substantially from the values $775.65 \pm 0.64 \pm 0.5$ and $143.85 \pm 1.33 \pm 0.80$, respectively, given by CMD-2. The latter have been obtained by fitting the physical $\pi\pi$ cross section (before subtracting the VP) and including the ω and the ρ' with mixing parameters $|\delta| = (1.57 \pm 0.16) \times 10^{-3}$, $\varphi_\delta = 13.3^\circ \pm 3.7^\circ$ and $|\beta| = 0.0695 \pm 0.0053$, $\varphi_\beta = 180^\circ$. A ρ'' was not included. The parameters determined by CMD-2, cannot be directly compared with corresponding parameters obtained by fits of the τ data, because the latter do not include VP effects. In order to have a common basis we subtract the VP from the e^+e^- data (of course, alternatively, we could supplement the τ data with the VP (by replacing α in K_σ (6) by $\alpha(s)$) in order to obtain dressed (physical) parameters which are the ones usually listed in the particle data tables). The subtraction of the VP lowers the mass by about 1 MeV and increases the width by about 1.3 MeV. The additional changes are due

⁵ For the e^+e^- channel we fit the data after subtraction of the $I = 0$ part. Including the ω in the fit yields practically the same result.

to the inclusion of the ρ'' and the changes in the other “background parameters”. If we would utilize the CMD-2 background parameters as a common background parametrization we would not be able to get acceptably good fits for the τ data sets.

Now we assume that the systematic deviations seen in the ρ^\pm parameters include electromagnetic isospin breaking which we have to correct for. We now may ask two questions. The first is: how does the test-ratio of Fig. 3 look like if we replace m_{ρ^\pm} and Γ_{ρ^\pm} in the τ -data fit by the more appropriate m_{ρ^0} and Γ_{ρ^0} ? The second is: what mass and width do we get if we fit them in the τ -data parameterization such that the test-ratio comes out to be unity within errors? Not too surprisingly we find them close to the ones given in Table 1 for CMD-2: it makes the central value of the ratio unity within 0.1%! Uncertainties may be obtained from the ones in the parametrizations. Of course keeping the background fixed the result looks pretty trivial. In fact fitting all parameters of the GS formula simultaneously in the much wider range of τ data, as has been performed in [15], yields results which look very similar to ours. The parameters obtained are very strongly correlated and all of them may be affected by some isospin breaking effects. A much more elaborate analysis would be necessary to actually establish tight experimental values for possible isospin breakings in these parameters.

We conclude that the τ vs. e^+e^- discrepancy very likely is an isospin breaking effect which has not been accounted for correctly in previous analyses. This also would establish a significant difference for $m_{\rho^\pm} - m_{\rho^0}$ and $\Gamma_{\rho^\pm} - \Gamma_{\rho^0}$. Of course, what it means is that the τ data cannot be utilized to calculate a_μ^{had} without reference to the e^+e^- data. Also, since now substantially correlated, the inclusion of the τ -data is much less straightforward. The question is how much they still can contribute to reduce the uncertainties in the evaluation of a_μ^{had} . This also makes it very likely that the e^+e^- -data based evaluations are the more trustworthy ones. After the correction in the normalization of the CMD-2 data we get the leading hadronic contribution to the anomalous magnetic moment of the muon. We now obtain

$$a_\mu^{\text{had}(1)} = (694.8 \pm 8.6) \times 10^{-10} \quad [e^+e^- \text{ data based}]. \quad (10)$$

With this estimate we get

$$a_\mu^{\text{the}} = (11659179.4 \pm 8.6_{\text{had}} \pm 3.5_{\text{LBL}} \pm 0.4_{\text{QED+EW}}) \times 10^{-10} \quad (11)$$

which compares to the most recent experimental result [28]

$$a_\mu^{\text{exp}} = (11659203 \pm 8) \times 10^{-10}. \quad (12)$$

The “discrepancy” $|a_\mu^{\text{the}} - a_\mu^{\text{exp}}| = (23.6 \pm 12.3) \times 10^{-10}$ corresponds to a deviation of about 1.9σ . For other recent estimates we refer to [10,29].

3. Summary and conclusion

Since recently we have in each case two reasonably consistent sets of data: the ALEPH and CLEO τ -data sets on the one hand and the CMD-2 and KLOE (still preliminary) e^+e^- -data sets on the other hand. The τ -data samples are about 10% higher than the e^+e^- ones in the tail above the ρ . This can be clearly seen in Fig. 3. Assuming that the experiments are essentially correct we think that a 10% increase in a resonance tail can easily be attributed to a 0.5% increase in the energy scale. Since it is very unlikely a problem of energy calibration, the only explanation remains that the resonance parameters must be different in the charged and the neutral channel. Our estimated shifts in the ρ parameters account for about 6.8% (or 8.1% with the estimates given in [27]) in the cross section (see Fig. 3). The remaining “discrepancy” is likely due to corresponding shifts in the other GS fit-parameters.

Our analysis shows that to a large extent we may understand the e^+e^- vs. τ discrepancy as an isospin breaking effect coming from the fact that mass and width of charged and neutral ρ -mesons, as naively expected, are different and thus that the τ -data must be mapped to the neutral channel parameters before they can be utilized for the

evaluation of a_μ^{had} in addition to the e^+e^- -data. We thus assume that a main part of the problem is due to additional isospin breaking effects and not primarily an experimental one. Of course there are also experimental difficulties which hopefully will be resolved by forthcoming experiments.

The fact that the ρ -mass difference is found to be of size comparable (maybe half of it) to the well-established pion mass difference $m_{\pi^\pm} - m_{\pi^0} \sim 4.6$ MeV seems not so unlikely because the corresponding bound states, apart from the spin orientation, have the same quark content (ρ^\pm vs. π^\pm on the one hand and ρ^0 vs. π^0 on the other hand).⁶ In the charged case the electric potential is repulsive while in the neutral case it is attractive and contributes to lower the mass, irrespective of the spin orientation of the constituents. We do not think that the Goldstone-boson nature of the pions, which derives from the properties of the strong interactions, the spontaneous breaking of the chiral symmetry, necessarily makes such a manifest isospin breaking effects completely different for the vector particles and the scalars.

Since we do not have an independent evaluation of the charged and neutral ρ -meson parameters, the isospin correction needed in order for the τ -data to be useful for the evaluation of the hadronic contribution to the muon anomalous magnetic moment cannot be performed at sufficient precision at the moment. Nevertheless, the τ -data still provide important cross checks and last but not least Ref. [8] triggered a discussion which forced all parties to check more carefully what they have done. One impact was that also the results in the e^+e^- -channel had to be corrected.

The main point is that there must be such effects which were not correctly treated so far. Maybe the available data do not allow us to pin down a solid value for the mass and width differences (all GS parameters in fact must be subject to isospin breaking and the data may not suffice to come to a definite conclusion). We think the main point is that the derivatives of a_μ with respect to m_ρ and Γ_ρ are rather large and hence no stable result can be given if the isospin breaking in these parameters is not known with sufficient precision.

Note that in spite of the fact that the dominating ρ -peak is shifted downwards, due to the correction which we have to apply to the τ -data, the s^{-2} weighted a_μ -integral does not increase. It rather decreases, because the width also substantially decreases by the correction and actually over-compensates the effect of the shift in the mass.

Our conclusion: very likely we are back to *one* prediction for a_μ which is the e^+e^- -based value at about 2σ below the experimental result! Unfortunately, at present, we do not have a precise enough understanding of the isospin violations to be able to utilize the τ -data for the evaluation of the hadronic contribution to $g - 2$ of the muon.

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⁶ Remember that, for example, para- (spin $s = 0$) and ortho- ($s = 1$) positronium have almost the same binding energy. For the pion and the ρ the average distance of the quarks is determined by the strong interactions, and in principle could be different. However, the charge radius of the ρ is of similar size as the one of the pion and thus also the electromagnetic effects are expected to be of similar magnitude.

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