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Double parton correlations versus factorized distributions

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Abstract

Using the generalized Lipatov–Altarelli–Parisi–Dokshitzer equations for the two-parton distribution functions we show numerically that the dynamical correlations contribute to these functions quite a lot in comparison with the factorized components. At the scale of CDF hard process (~ 5 GeV) this contribution to the double gluon–gluon distribution is nearly 10% and increases right up to 30% at the LHC scale (~ 100 GeV) for the longitudinal momentum fractions $x \leqslant 0.1$ accessible to these measurements. For the finite longitudinal momentum fractions $x \sim 0.2$–0.4 the correlations are large right up to 90% in accordance with the predicted QCD asymptotic behaviour.

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Recent CDF measurements [1] of the inclusive cross section for double parton scattering have provided new and complementary information on the structure of the proton and possible parton–parton correlations. Both the absolute rate for the double parton process and any dynamics that correlations may introduce are therefore of interest. The possibility of observing two separate hard collisions has been proposed since long [2,3], and from that has also developed in a number of works [4–11]. The Tevatron and specially LHC allow us to obtain huge data samples of these multiple interactions and to answer to many challenging questions of yet poorly-understood aspects of QCD. A brief review of the current situation and some progress in the modeling account of correlated flavour, colour, longitudinal and transverse momentum distributions can be found in Ref. [12]. Multiple interactions require an ansatz for the structure of the incoming beams, i.e., correlations between the constituent partons. As a simple ansatz, usually, the two-parton distributions are supposed to be the product of two single-parton distributions times a momentum conserving phase space factor. In recent paper [13] it has been shown that this hypothesis is in some contradiction with the leading logarithm approximation of perturbative QCD (in the framework of which a parton model, as a matter of fact, was established in the quantum field theories [14–16]). Namely, the two-

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parton distribution functions being the product of two single distributions at some reference scale become to be dynamically correlated at any different scale of a hard process. The value of these correlations in comparison with the factorized components is the main purpose of this Letter.

In order to be clear and to introduce the denotations let us recall that, for instance, the differential cross section for the four-jet process (due to the simultaneous interaction of two parton pairs) is given by [6,7]

$$d\sigma = \sum_{q/g} \frac{d\sigma_{12}\, d\sigma_{34}}{\sigma_{\rm eff}} D_p(x_1, x_3) D_{\bar p}(x_2, x_4), \tag{1}$$

where $d\sigma_{ij}$ stands for the two-jet cross section. The dimensional factor $\sigma_{\rm eff}$ in the denominator represents the total inelastic cross section which is an estimate of the size of the hadron, $\sigma_{\rm eff} \simeq 2\pi r_p^2$ (the factor 2 is introduced due to the identity of the two parton processes). With the effective cross section measured by CDF, $(\sigma_{\rm eff})_{\rm CDF} = (14.5 \pm 1.7^{+1.7}_{-2.3})$ mb [1], one can estimate the transverse size $r_p \simeq 0.5$ fm, which is too small in comparison with the proton radius R_p extracted from ep elastic scattering experiments. The relatively small value of $(\sigma_{\rm eff})_{\rm CDF}$ with respect to the naive expectation $2\pi R_p^2$ was, in fact, considered [9,10] as evidence of nontrivial correlation effects in transverse space. But, apart from these correlations, the longitudinal momentum correlations can also exist and they were investigated in Ref. [13]. The factorization ansatz is just applied to the two-parton distributions incoming in Eq. (1):

$$D_p(x_i, x_j) = D_p(x_i, Q^2) D_p(x_j, Q^2)(1 - x_i - x_j), \tag{2}$$

where $D_p(x_i, Q^2)$ are the single quark/gluon momentum distributions at the scale Q^2 (determined by a hard process).

However many parton distribution functions satisfy the generalized Lipatov–Altarelli–Parisi–Dokshitzer evolution equations derived for the first time in Refs. [17,18] as well as single parton distributions satisfy more known and cited Altarelli–Parisi equations [15,16,19]. Under certain initial conditions these generalized equations lead to solutions, which are identical with the jet calculus rules proposed originally for multiparton fragmentation functions by Konishi–Ukawa–Veneziano [20] and are in some contradiction with the factorization hypothesis (2). Here one should note that at the parton level this is the strict assertion within the leading logarithm approximation.

After introducing the natural dimensionless variable

$$t = \frac{1}{2\pi b} \ln\left[1 + \frac{g^2(\mu^2)}{4\pi} b \ln\left(\frac{Q^2}{\mu^2}\right)\right] = \frac{1}{2\pi b} \ln\left[\frac{\ln\left(\frac{Q^2}{\Lambda_{\rm QCD}^2}\right)}{\ln\left(\frac{\mu^2}{\Lambda_{\rm QCD}^2}\right)}\right],$$

$$b = \frac{33 - 2n_f}{12\pi} \quad \text{in QCD},$$

where $g(\mu^2)$ is the running coupling constant at the reference scale μ^2, n_f is the number of active flavours, $\Lambda_{\rm QCD}$ is the dimensional QCD parameter, the Altarelli–Parisi equations read [15,16,19]

$$\frac{dD_i^j(x,t)}{dt} = \sum_{j'} \int_x^1 \frac{dx'}{x'} D_i^{j'}(x', t) P_{j' \to j}\left(\frac{x}{x'}\right). \tag{3}$$

They describe the scaling violation of the parton distributions $D_i^j(x,t)$ inside a dressed quark or gluon ($i, j = q/g$).

We will not write the kernels P explicitly and derive the generalized equations for two-parton distributions $D_i^{j_1 j_2}(x_1, x_2, t)$, representing the probability that in a dressed constituent i one finds two bare partons of types j_1 and j_2 with the given longitudinal momentum fractions x_1 and x_2 (referring to [13,15–19] for details), we give only their solutions via the convolution of single distributions [17,18]

$$\begin{aligned} D_i^{j_1 j_2}(x_1, x_2, t) &= \sum_{j' j_1' j_2'} \int_0^t dt' \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \\ &\quad \times D_i^{j'}(z_1 + z_2, t') \frac{1}{z_1 + z_2} P_{j' \to j_1' j_2'}\left(\frac{z_1}{z_1 + z_2}\right) \\ &\quad \times D_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, t - t'\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2}, t - t'\right). \end{aligned} \tag{4}$$

This convolution coincides with the jet calculus rules [20] as mentioned above and is the generalization of the well-known Gribov–Lipatov relation installed for single functions [14,16] (the distribution of bare partons inside a dressed constituent is identical to the distribution of dressed constituents in the fragmentation of a bare parton in the leading logarithm approximation). The solution (4) shows that the distribution of partons is *correlated* in the leading logarithm approximation:

$$D_i^{j_1 j_2}(x_1, x_2, t) \neq D_i^{j_1}(x_1, t) D_i^{j_2}(x_2, t). \tag{5}$$

Of course, it is interesting to find out the phenomenological issue of this parton level consideration. This can be done within the well-known factorization of soft and hard stages (physics of short and long distances) [21]. As a result Eq. (3) describe the evolution of parton distributions in a hadron with $t(Q^2)$, if one replaces the index i by index h only. However, the initial conditions for new equations at $t=0$ ($Q^2=\mu^2$) are unknown a priori and must be introduced phenomenologically or must be extracted from experiments or some models dealing with physics of long distances [at the parton level: $D_i^j(x, t=0) = \delta_{ij}\delta(x-1)$; $D_i^{j_1 j_2}(x_1, x_2, t=0) = 0$]. Nevertheless the solution of the generalized Lipatov–Altarelli–Parisi–Dokshitzer evolution equations with the given initial condition may be written as before via the convolution of single distributions [13,18]

$$\begin{aligned} D_h^{j_1 j_2}(x_1, x_2, t) &= D_{h(\mathrm{QCD})}^{j_1 j_2}(x_1, x_2, t) \\ &\quad + \sum_{j_1' j_2'} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D_h^{j_1' j_2'}(z_1, z_2, 0) \\ &\quad \times D_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, t\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2}, t\right), \end{aligned} \tag{6}$$

where

$$\begin{aligned} D_{h(\mathrm{QCD})}^{j_1 j_2}(x_1, x_2, t) &= \sum_{j' j_1' j_2'} \int_0^t dt' \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \\ &\quad \times D_h^{j'}(z_1+z_2, t') \frac{1}{z_1+z_2} P_{j' \to j_1' j_2'}\left(\frac{z_1}{z_1+z_2}\right) \\ &\quad \times D_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, t-t'\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2}, t-t'\right) \end{aligned} \tag{7}$$

are the dynamically correlated distributions given by perturbative QCD (compare (4) with (7)).

The reckoning for the unsolved confinement problem (physics of long distances) is the unknown nonperturbative two-parton correlation function $D_h^{j_1' j_2'}(z_1, z_2, 0)$ at some scale μ^2. One can suppose that this function is the product of two single-parton distributions times a momentum conserving factor at this scale μ^2:

$$\begin{aligned} &D_h^{j_1 j_2}(z_1, z_2, 0) \\ &\quad = D_h^{j_1}(z_1, 0) D_h^{j_2}(z_2, 0) \theta(1 - z_1 - z_2). \end{aligned} \tag{8}$$

Then

$$\begin{aligned} &D_h^{j_1 j_2}(x_1, x_2, t) \\ &\quad = D_{h(\mathrm{QCD})}^{j_1 j_2}(x_1, x_2, t) + \theta(1 - x_1 - x_2) \\ &\qquad \times \Biggl(D_h^{j_1}(x_1, t) D_h^{j_2}(x_2, t) \\ &\qquad\quad + \sum_{j_1' j_2'} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} D_h^{j_1'}(z_1, 0) D_h^{j_2'}(z_2, 0) \\ &\qquad\quad \times D_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, t\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2}, t\right) \\ &\qquad\quad \times \bigl[\theta(1 - z_1 - z_2) - 1\bigr] \Biggr), \end{aligned} \tag{9}$$

where

$$D_h^j(x, t) = \sum_{j'} \int_x^1 \frac{dz}{z} D_h^{j'}(z, 0) D_{j'}^j\left(\frac{x}{z}, t\right) \tag{10}$$

is the solution of Eq. (3) with the given initial condition $D_h^j(x, 0)$ for parton distributions inside a hadron expressed via distributions at the parton level.

This result (9) shows that if the two-parton distributions are factorized at some scale μ^2, then the evolution violates this factorization *inevitably* at any different scale ($Q^2 \neq \mu^2$), apart from the violation due to the kinematic correlations induced by the momentum conservation (given by θ functions).[1]

[1] This is the analogue of the momentum conserving phase space factor in Eq. (2).

For a practical employment it is interesting to know the degree of this violation. Partly this problem was investigated theoretically in Refs. [18,23] and for the two-particle correlations of fragmentation functions in Ref. [24]. That technique is based on the Mellin transformation of distribution functions as

$$M_h^j(n,t) = \int_0^1 dx\, x^n D_h^j(x,t). \tag{11}$$

After that the integrodifferential equations (3) become systems of ordinary linear-differential equations of first order with constant coefficients and can be solved explicitly [18,23]. In order to obtain the distributions in x representation an inverse Mellin transformation must be performed

$$D_h^j(x,t) = \int \frac{dn}{2\pi i} x^{-n} M_h^j(n,t), \tag{12}$$

where the integration runs along the imaginary axis to the right from all n singularities. This can be done numerically. However the asymptotic behaviour can be estimated. Namely, with the growth of $t(Q^2)$ the first term in Eq. (6) becomes *dominant*[2] for finite x_1 and x_2 [23]. Thus the two-parton distribution functions "forget" the initial conditions unknown a priori and the correlations perturbatively calculated appear.

The asymptotic prediction "teaches" us a tendency only and tells nothing about the values of $x_1, x_2, t(Q^2)$ beginning from which the correlations are significant (the more so since the asymptotic behaviour takes place over the double logarithm dimensionless variable t as a function of Q^2). Naturally numerical estimations can give an answer to this specific question. We do it using the CTEQ fit [22] for single distributions as an input in Eq. (7). The nonperturbative initial conditions $D_h^j(x,0)$ are specified in a parametrized form at a fixed low-energy scale $Q_0 = \mu = 1.3$ GeV. The particular function forms and the value of Q_0 are not crucial for the CTEQ global analysis at the flexible enough parametrization, which reads [25]

$$x D_p^j(x,0) = A_0^j x^{A_1^j}(1-x)^{A_2^j} e^{A_3^j x}\left(1+e^{A_4^j}x\right)^{A_5^j}. \tag{13}$$

The independent parameters A_0^j, A_1^j, A_2^j, A_3^j, A_4^j, A_5^j for parton flavour combinations $u_v \equiv u - \bar{u}$, $d_v \equiv d - \bar{d}$, g and $\bar{u} + \bar{d}$ are given in Appendix A of Ref. [25]. To distinguish the $\bar{u}$ and $\bar{d}$ distributions the ratio $\bar{d}/\bar{u}$ is parametrized as a sum of two terms:

$$\begin{aligned} &D_p^{\bar{d}}(x,0)/D_p^{\bar{u}}(x,0) \\ &\quad = A_0 x^{A_1}(1-x)^{A_2} + (1+A_3 x)(1-x)^{A_4} \end{aligned} \tag{14}$$

with the coefficients A_0, A_1, A_2, A_3, A_4 again from Ref. [25]. The initial conditions for strange quarks are assumed:

$$D_p^{\bar{s}}(x,0) = D_p^s(x,0) = 0.2\left(D_p^{\bar{u}}(x,0) + D_p^{\bar{d}}(x,0)\right).$$

The parton distribution functions $D_p^j(x,t)$ at all higher $Q(t)$ are determined from the input initial conditions $D_p^j(x,0)$ by the Altarelli–Parisi evolution equations. The CTEQ Evolution package [26] was used and adapted by us in order to obtain numerically single distributions $D_i^j(x,t)$ at all t and at the parton level also. We fixed the fundamental parameter of perturbative QCD, $\Lambda_{\mathrm{QCD}} = 0.281$ GeV, that is in accordance with the strong coupling constant, $\alpha_s(M_Z) \simeq 0.2$, at the Z resonance in one-loop approximation. Only the light quarks u, d, s ($n_f = 3$) are taken into account in the evolution equations and are treated as massless, as usual. After that the triple integral (7) was calculated numerically for three values of $Q = 5, 100, 250$ GeV as a function of $x = x_1 = x_2$. To be specific we considered the double gluon–gluon distribution function in the proton. In this case only the kernel $P_{g\to gg}$ can be taken into account as giving the main contribution to the perturbative double gluon–gluon distribution. The remnant terms of sum in Eq. (7) are relatively small and can only increase the effect under consideration because they are positive.

The results of numerical calculations are presented on Fig. 1 for the ratio

$$\begin{aligned} R(x,t) = &\Big(D_{p(\mathrm{QCD})}^{gg}(x_1,x_2,t) \\ &\times \left[D_p^g(x_1,t)D_p^g(x_2,t)(1-x_1-x_2)^2\right]^{-1}\Big)\Big|_{x_1=x_2=x}. \end{aligned} \tag{15}$$

[2] Such domination is the mathematical consequence of the relation between the maximum eigenvalues $\lambda(n)$ in the moments representation (after Mellin transformation): $\lambda(n_1+n_2) > \lambda(n_1) + \lambda(n_2)$ in QCD at the large n_1, n_2 (finite x_1, x_2), because $\lambda(n) \sim -\ln(n), n \gg 1$.

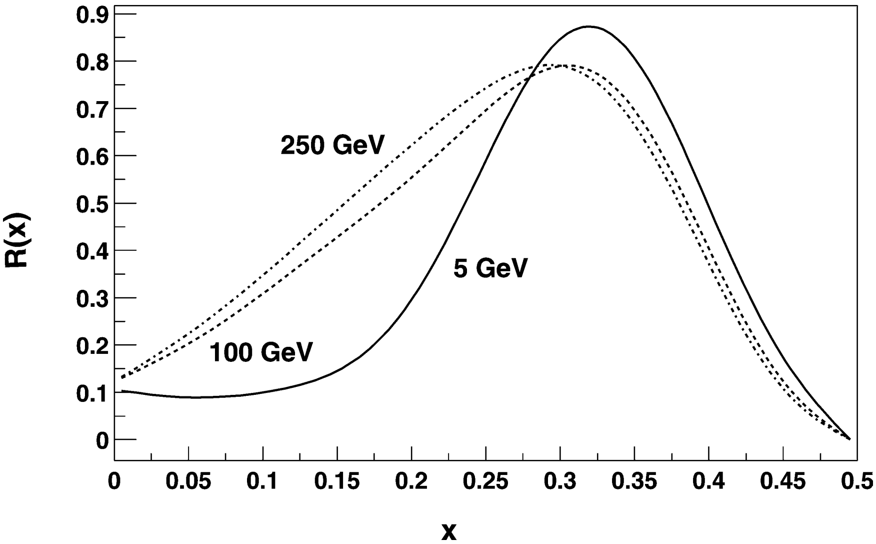

Fig. 1. The ratio of perturbative QCD correlations to the factorized component for the double gluon–gluon distribution in the proton as a function of $x = x_1 = x_2$ for three values of $Q = 5$ (solid), 100 (dashed), 250 (dash-dotted) GeV.

Here one should note that the momentum conserving phase space factor $(1 - x_1 - x_2)^2$ is introduced in Eq. (15) instead of $(1 - x_1 - x_2)$ usually used. The reason is simple: this factor was introduced in Eq. (2), generally speaking, "by hand" in order to "save" the momentum conservation law, i.e., in order to make the product of two single distributions is equal to zero smoothly at $x_1 + x_2 = 1$. However the generalized QCD evolution equations demand higher power of $(1 - x_1 - x_2)$ at $x_1 + x_2 \to 1$: only the phase space integrals in Eqs. (6) and (7) give

$$\int_{x_1}^{1-x_2} dz_1 \int_{x_2}^{1-z_1} dz_2 = (1 - x_1 - x_2)^2/2.$$

In fact this power must depend on t increasing with its growth as this takes place for single distributions at $x \to 1$ [16,27]. The asymptotic behaviour of two-particle fragmentation functions at $x_1 + x_2 \to 1$ was investigated, for instance, in Ref. [28] with the similar result. Our numerical calculations support this assertion also: the power of $(1 - x_1 - x_2)$ for the perturbative QCD gluon–gluon correlations is higher than 2 and increases with $t(Q)$ as one can see from Fig. 1. However the introduced factor $(1 - x_1 - x_2)^2$ has not an influence practically on the ratio under consideration in the region of small x_1, x_2. And namely this region, in which multiple interactions can contribute to the cross section visibly, is interesting from experimental point of view. Fig. 1 shows that at the scale of CDF hard process (~ 5 GeV) the ratio (15) is nearly 10% and increases right up to 30% at the LHC scale (~ 100 GeV) for the longitudinal momentum fractions $x \leqslant 0.1$ accessible to these measurements. For the finite longitudinal momentum fractions $x \sim 0.2$–0.4 the correlations are large right up to 90%. They become important in more and more x region with the growth of t in accordance with the predicted QCD asymptotic behaviour.

The correlation effect is strengthened insignificantly (up to 2%) for the longitudinal momentum fractions $x \leqslant 0.1$ when starting from the slightly lower value $Q_0 = 1$ GeV (early used by CTEQ Collaboration). We conclude also that $R(x,t) \to$ const at $x \to 0$ most likely, calculating this ratio ($\simeq 0.1$) at $x_{\min} = 10^{-4}$.

Seemingly the correction to the double gluon–gluon distributions at the CDF scale can be smoothly absorbed by uncertainties in the σ_{eff} increasing the transverse effective size r_p by a such way. But this augmentation is still not enough to solve a problem of the relatively small value of r_p with respect to the proton radius without nontrivial correlation effects in transverse space [9,10].

In summary, the numerical estimations show that the leading logarithm perturbative QCD correlations are quite comparable with the factorized distributions. With increasing a number of observable multiple collisions (statistic) the more precise calculations of their cross section (beyond the factorization hypothesis) will be needed also. In order to obtain the more delicate their characteristics (distributions over various kinematic variables) it is desirable to implement the QCD evolution of two-parton distribution functions in some Monte Carlo event generator as this was done for single distributions within, for instance, PYTHIA [29].

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