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Determining replenishment lot size and shipment policy for an extended EPQ model with delivery and quality assurance issues

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Abstract This paper derives the optimal replenishment lot size and shipment policy for an Economic Production Quantity (EPQ) model with multiple deliveries and rework of random defective items. The classic EPQ model assumes a continuous inventory issuing policy for satisfying demand and perfect quality for all items produced. However, in a real life vendor–buyer integrated system, multi-shipment policy is practically used in lieu of continuous issuing policy and generation of defective items is inevitable. It is assumed that the imperfect quality items fall into two groups: the scrap and the rework-able items. Failure in repair exists, hence additional scrap items generated. The finished items can only be delivered to customers if the whole lot is quality assured at the end of rework. Mathematical modeling is used in this study and the long-run average production–inventory–delivery cost function is derived. Convexity of the cost function is proved by using the Hessian matrix equations. The closed-form optimal replenishment lot size and optimal number of shipments that minimize the long-run average costs for such an EPQ model are derived. Special case is examined, and a numerical example is provided to show its practical usage.

1. Introduction

In manufacturing firms, the Economic Production Quantity (EPQ) model is commonly used for determining optimal replenishment batch size that minimizes total production–inventory costs for items produced in-house [1,2]. The classical EPQ model assumes that all items manufactured are of perfect quality. However, in real-life production systems, due to various controllable and/or uncontrollable factors, the generation of defective items during production run seems to be inevitable [3,4]. Studies have been carried out to enhance EPQ model by addressing the issues of imperfect quality items produced as well as quality assurance in the production [5–17]. Examples of such research are surveyed below. Rosenblatt and Lee [5] examined an EPQ model that deals with imperfect quality. They assumed that at some random point in time the process might shift from an in-control to an out-of-control state, and a fixed percentage of defective items is produced. Approximate solutions for obtaining an optimal lot size were developed in their paper. Yum and McDowell [6] formulated allocation of inspection effort problem for serial system as a 0–1 Mixed Integer Linear Programming (MILP) problem. Their formulation permitted any combination of scrap, rework or repair at each station. Their proposed problem was solved by using the standard MILP software packages. Groenevelt et al. [7] proposed two production control policies to deal with machine breakdowns. The first policy assumes that production will not resume (called the NR policy) after a breakdown. The second policy is immediately resumed after a breakdown if the on-hand inventory is below a certain threshold level (called the AR policy). Both policies assume the repair time is negligible and they study the effects of machine breakdowns and corrective maintenance on the economic lot sizing decisions. Grosfeld-Nir and Gerchak [11] considered multistage production systems where defective units can be reworked repeatedly at every stage. The yield of each

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stage is uncertain, so several production runs may need to be attempted until the quantity of finished products is sufficient. The trade-off at each stage is between using small lots, possibly necessitating repeated rework set-ups and large lots, which may result in costly overproduction. They showed that a multistage system, where only one of the stages requires a set-up, can be reduced to a single-stage system. They also proved that it is best to make the "bottle-neck" the first stage of the system, and they also developed recursive algorithms for solving two- and three-stage systems, where all stages require set-ups, optimally. Chiu et al. [14] examined an EPQ model with imperfect rework and random breakdown under abort/resume policy. They employed mathematical modeling, and derived the integrated long-run average production–inventory cost per unit time. Bounds for the optimal production run times are proposed and proved in their study. A recursive searching algorithm was developed for locating the optimal run time within the bounds that minimizes the expected production–inventory costs.

The "continuous" inventory issuing policy for satisfying product demand is another unrealistic assumption of EPQ model. In real life, vendor–buyer integrated production–inventory–delivery system, multiple or periodic shipments of finished products are often used. Goyal [18] first studied the integrated inventory model for a single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. He gave example to illustrate his proposed method. Many studies have since been carried out to address various aspects of vendor–buyer supply chain optimization issues [19–27]. Golhar and Sarker [20] developed a simple algorithm to compute the optimal batch size for a system where a Just-In-Time (JIT) buyer demands frequent deliveries of small lots of certain products. They found that the generalized total cost function for the proposed model is a piecewise convex function. When production uptime and cycle time are each equal to an integer multiple of the shipment interval, a perfect matching of shipment size occurs, and for such a situation, the generalized model specializes to more traditional inventory models. Economic impact of ordering and setup costs reduction is also investigated. Hill [22] studied a model in which a manufacturing company purchases a raw material, manufactures a product and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time. His objective is to determine a purchasing and production schedule which minimizes total cost of purchasing, manufacturing and stock-holding. Viswanathan [23] examined the integrated vendor–buyer inventory models with two different strategies:

1. Each replenishing quantity delivered to the buyer is identical.
2. At each delivery, all the inventory available with the vendor is supplied to the buyer.

As a result, there is no one strategy that obtains the best solution for all possible problem parameters. He also provided results of a detailed numerical investigation that analyzed the relative performance of the two strategies for various problem parameters. Diponegoro and Sarker [26] determined an ordering policy for raw materials, as well as an economic batch size for finished products that are delivered to customers frequently at a fixed interval of time for a finite planning horizon. The problem was then extended to compensate for the lost sales of finished products. A closed-form solution to the problem was obtained for the minimal total cost. They also developed a lower bound on the optimal solution for problem, with lost sale. Because little attention has been paid to the investigation of joint effect of multiple deliveries and quality assurance issues on the optimal replenishment lot size and shipment policy of the EPQ model, this paper intends to bridge the gap.

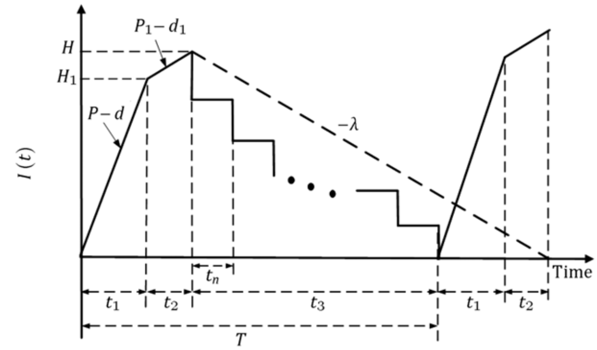

Figure 1: On-hand inventory of perfect quality items in EPQ model with a multi-delivery policy and quality assurance issues.

2. Problem description and mathematical modelling

This paper examines an economic production quantity model with multiple shipments and quality assurance issues. Consider a manufacturing system which has an annual production rate P, and its process may randomly produce x portion of defective items at a production rate d. All items produced are screened, and inspection cost per item is included in the unit production cost C. The imperfect quality items fall into two groups, a θ portion of them is the scrap and the other $(1-\theta)$ portion of them is considered to be rework-able. The rework process starts immediately after the regular production, at a rate of P_1 in each cycle. It is not a perfect process either, a θ_1 portion (where $0 <= \theta_1 <= 1$) of reworked items fails during the rework process and becomes scrap. The annual production rate P is assumed to be larger than the sum of annual demand rate λ and the production rate of defective items d. That is $(P-d-\lambda) > 0$, where the production rate of defective items d can be expressed as $d = Px$. Let d_1 denote production rate of scrap items during the rework process, then d_1 can be expressed as $d_1 = P_1\theta_1$. Unlike classic EPQ model assuming a continuous inventory issuing policy, this study paper considers a multi-delivery policy. It is assumed that the finished items can only be delivered to customers if the whole lot is quality assured at the end of rework. Fixed quantity n installments of the finished batch are delivered by request to customers at a fixed interval of time during production downtime t_3 (see Figure 1). Additional notations used in this paper are given in Appendix A.

The following equations can be obtained directly from Figure 1:

$$T = t_1 + t_2 + t_3, \tag{1}$$

$$H_1 = (P-d)t_1, \tag{2}$$

$$t_1 = \frac{Q}{P} = \frac{H_1}{P-d}, \tag{3}$$

$$t_2 = \frac{xQ(1-\theta)}{P_1}, \tag{4}$$

$$H = H_1 + (P_1 - d_1)t_2, \tag{5}$$

$$t_3 = nt_n = T - (t_1 + t_2) = Q\left(\frac{(1-\varphi x)}{\lambda} - \frac{1}{P} - \frac{x(1-\theta)}{P_1}\right). \tag{6}$$

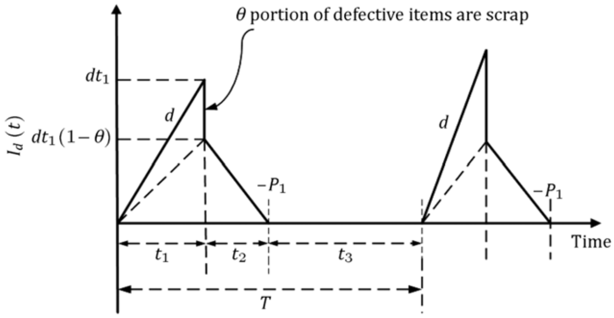

Figure 2: On-hand inventory of defective items in EPQ model with a multi-delivery policy and quality assurance issues.

The on-hand inventory of defective items produced during the production uptime t_1 are as follows (see Figure 2). Among them a θ portion is scrap and the other $(1-\theta)$ portion of defective items is considered to be reworkable.

$$dt_1 = Pxt_1 = xQ. \tag{7}$$

During the rework process, a portion θ_1 of reworked items fails and becomes scrap. Figure 3 depicts the on-hand inventory of scrap items during t_1 and t_2. One notes that maximum level of scrap items φxQ is:

$$\varphi \cdot xQ = [\theta + (1-\theta)\theta_1]\,xQ. \tag{8}$$

During delivery time t_3, n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time. Cost for each delivery is:

$$K_1 + C_T\left(\frac{H}{n}\right), \tag{9}$$

and total delivery costs for n shipments in a cycle are:

$$n\left[K_1 + C_T\left(\frac{H}{n}\right)\right] = nK_1 + C_T H = nK_1 + C_T Q\,(1-\varphi x). \tag{10}$$

Total holding costs of finished products during t_3 at manufacturer's end can be obtained as follows (refer to Appendix B):

$$h\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right) Ht_3 = h\left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right] Ht_3 = h\left(\frac{n-1}{2n}\right) Ht_3. \tag{11}$$

Total holding costs for items kept at customer's end are as follows (see Figure 4 and also refer to Appendix C).

$$\frac{h_2}{2}\left[\frac{Ht_3}{n} + T\,(H - \lambda t_3)\right]. \tag{12}$$

Total production–inventory–delivery cost per cycle $TC(Q,n)$ consists of variable production cost, setup cost, variable rework cost, disposal cost, fixed and variable delivery cost, holding cost at the manufacturer's end during production uptime t_1, reworking time t_2, and delivery time t_3, variable holding cost for items reworked, and holding cost at the customer's end for finished goods during the delivery time t_3. Therefore, the overall production–inventory–delivery cost per cycle $TC(Q,n)$ is:

$$TC(Q,n) = CQ + K + C_R[x(1-\theta)Q] + C_S[x\varphi Q] + nK_1 + C_T[Q(1-\varphi x)]$$
$$+ h\left[\frac{H_1 + dt_1}{2}(t_1) + \frac{H_1 + H}{2}(t_2) + \left(\frac{n-1}{2n}\right)Ht_3\right] + h_1 \cdot \frac{P_1 \cdot t_2}{2} \cdot (t_2) + \frac{h_2}{2}\left[\frac{H}{n}t_3 + T(H - \lambda t_3)\right]. \tag{13}$$

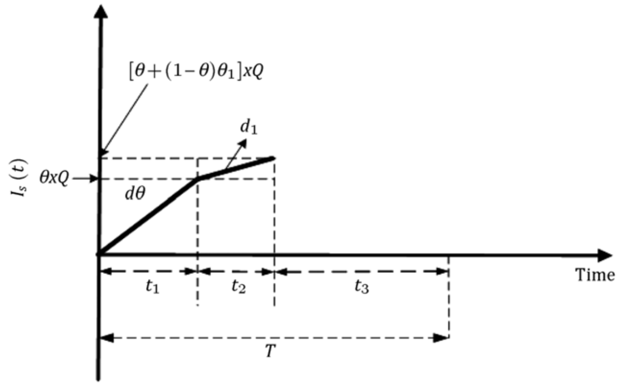

Figure 3: On-hand inventory of scrap items in EPQ model with a multi-delivery policy and quality assurance issues.

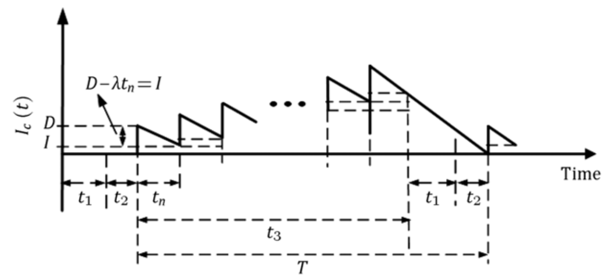

Figure 4: On-hand inventory at the customer's end when n installments of the finished batch are delivered.

Because the proportion x of defective items is assumed to be a random variable with a known probability density function, in order to take the randomness of defective rate into account, the expected values of x can be used in the cost analyses. Substituting all related variables from Eqs. (1) to (12) in Eq. (13), and applying the renewal reward theorem, the expected production–inventory–delivery cost per unit time $E[TCU(Q)]$ can be obtained (see Appendix D for details):

$$\begin{aligned}
E[TCU(Q,n)] &= \frac{E[TC(Q,n)]}{E[T]} = \frac{C\lambda}{1-\varphi E[x]} + \frac{(K+nK_1)\lambda}{Q(1-\varphi E[x])} \\
&+ \frac{C_R E[x](1-\theta)\lambda}{(1-\varphi E[x])} + \frac{C_S E[x]\varphi\lambda}{(1-\varphi E[x])} \\
&+ C_T\lambda + \frac{hQ\lambda}{2P(1-\varphi E[x])} + \frac{hQ\lambda}{2P_1(1-\varphi E[x])}[(2E[x] \\
&- (E[x])^2 - \varphi(E[x])^2)(1-\theta)] + \left(1-\frac{1}{n}\right) \\
&\times \left[\frac{hQ(1-\varphi E[x])}{2} - \frac{hQ\lambda}{2P} - \frac{hQE[x](1-\theta)\lambda}{2P_1}\right] \\
&+ \frac{h_1(E[x])^2 Q\lambda(1-\theta)^2}{2P_1(1-\varphi E[x])} + \left(\frac{1}{n}\right)\frac{h_2 Q}{2}(1-\varphi E[x]) \\
&+ \left(1-\frac{1}{n}\right)\frac{h_2 Q\lambda}{2P} + \frac{h_2 Q}{2}\left[\left(1-\frac{1}{n}\right)\frac{E[x]\lambda(1-\theta)}{P_1}\right].
\end{aligned} \tag{14}$$

3. Derivations of optimal replenishment lot size & shipment policy

For proof of convexity of $E[TCU(Q,n)]$, one could employ Hessian matrix equations [28] and obtains the following derivatives:

$$\frac{\partial E[TCU(Q,n)]}{\partial Q}=\frac{1}{1-\varphi E[x]}\left[-\frac{K\lambda}{Q^2}-\frac{nK_1\lambda}{Q^2}+\frac{h\lambda}{2P}\right.+\frac{h_1(E[x])^2\lambda(1-\theta)^2}{2P_1(1-\varphi E[x])}+\frac{h\lambda(1-\theta)E[x]}{2P_1(1-\varphi E[x])}[2-E[x]-\varphi E[x]]+\left(1-\frac{1}{n}\right)\left[\frac{h(1-\varphi E[x])}{2}-\frac{h\lambda}{2P}-\frac{hE[x](1-\theta)\lambda}{2P_1}\right]+\left(\frac{1}{n}\right)\frac{h_2}{2}(1-\varphi E[x])+\left(1-\frac{1}{n}\right)\frac{h_2\lambda}{2P}+\left(1-\frac{1}{n}\right)\frac{h_2E[x]\lambda(1-\theta)}{2P_1}, \tag{15}$$

$$\frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2}=\frac{2(K+nK_1)\lambda}{Q^3(1-\varphi E[x])}, \tag{16}$$

$$\frac{\partial E[TCU(Q,n)]}{\partial n}=\frac{K_1\lambda}{Q(1-\varphi E[x])}-\frac{1}{n^2}(h_2-h)\times\left[\frac{Q(1-\varphi E[x])}{2}-\frac{Q\lambda}{2P}-\frac{QE[x](1-\theta)\lambda}{2P_1}\right], \tag{17}$$

$$\frac{\partial^2 E[TCU(Q,n)]}{\partial n^2}=\frac{1}{n^3}(h_2-h)\cdot\left[Q(1-\varphi E[x])-\frac{Q\lambda}{P}-\frac{QE[x]\lambda(1-\theta)}{P_1}\right], \tag{18}$$

$$\frac{\partial E[TCU(Q,n)]}{\partial Q\,\partial n}=-\frac{K_1\lambda}{Q^2(1-\varphi E[x])}-\frac{1}{n^2}(h_2-h)\times\left[\frac{(1-\varphi E[x])}{2}-\frac{\lambda}{2P}-\frac{E[x](1-\theta)\lambda}{2P_1}\right]. \tag{19}$$

Substituting Eqs. (15) through (19) in the following Hessian matrix equations [28] and with further derivations, one can obtain:

$$[Q\quad n]\begin{pmatrix}\frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2} & \frac{\partial^2 E[TCU(Q,n)]}{\partial Q\partial n}\\ \frac{\partial^2 E[TCU(Q,n)]}{\partial Q\partial n} & \frac{\partial^2 E[TCU(Q,n)]}{\partial n^2}\end{pmatrix}\begin{bmatrix}Q\\ n\end{bmatrix}=\frac{2(K+nK_1)\lambda}{Q(1-\varphi E[x])}-\frac{2nK_1\lambda}{Q(1-\varphi E[x])}=\frac{2K\lambda}{Q(1-\varphi E[x])}>0. \tag{20}$$

Because K, λ, Q, and $(1-\varphi E[x])$ are all positive, so Eq. (20) is positive. Hence $E[TCU(Q,n)]$ is a strictly convex function for all Q and n different from zero. It follows that for the optimal replenishment lot size Q^* and optimal number of delivery n^*, one can differentiate $E[TCU(Q,n)]$ with respect to Q and n, and solve the linear system of the aforementioned Eqs. (15) and (17), by first setting these partial derivatives equal to zero.

With further derivations, one can obtain the optimal replenishment lot size Q^* and optimal number of delivery n^* as follows:

$$Q^*=\sqrt{\frac{2(K+nK_1)\lambda}{\left\{\begin{array}{l}\frac{h\lambda}{P}+\frac{h\lambda}{P_1}[2E[x]\\ -(E[x])^2-\varphi(E[x])^2](1-\theta)\\ +\left[\left(\frac{n-1}{n}\right)h+\left(\frac{1}{n}\right)h_2\right](1-\varphi E[x])^2\\ +\left(\frac{n-1}{n}\right)(h_2-h)\\ \times\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)(1-\varphi E[x])\\ +\frac{h_1(E[x])^2\lambda(1-\theta)^2}{P_1}\end{array}\right\}}}, \tag{21}$$

$$n^*=\sqrt{\frac{\begin{array}{l}K(h_2-h)(1-\varphi E[x])\\ \left[(1-\varphi E[x])-\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)\right]\end{array}}{K_1\left\{\begin{array}{l}\frac{h\lambda\varphi E[x]}{P}\\ +\frac{h\lambda E[x]}{P_1}(1-E[x])(1-\theta)\\ +h(1-\varphi E[x])^2\\ +\frac{h_1(E[x])^2\lambda(1-\theta)^2}{P_1}\\ +h_2\left(\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right)\\ \times(1-\varphi E[x])\end{array}\right\}}}. \tag{22}$$

3.1. Special case

Suppose all items produced are of perfect quality (i.e. $x=0$), the proposed EPQ model becomes the same as the classic EPQ model with a multi-delivery policy. On-hand inventory of perfect quality item is depicted in Figure 5.

Total production–inventory–delivery cost per cycle $TC1$ (Q,n) is:

$$TC_1(Q,n)=CQ+K+h\left[\frac{H}{2}(t_1)+\left(\frac{n-1}{2n}\right)Ht_2\right]+nK_1+C_TQ+\frac{h_2}{2}\left[\frac{H}{n}t_2+T(H-\lambda t_2)\right]. \tag{23}$$

The expected production–inventory–delivery cost $E[TCU_1$ $(Q,n)]$ for this special model can be derived as follows:

$$E[TCU_1(Q,n)]=C\lambda+\frac{(K+nK_1)\lambda}{Q}+C_T\lambda+\frac{hQ\lambda}{2P}+\left(\frac{n-1}{n}\right)\times\left(\frac{hQ}{2}-\frac{hQ\lambda}{2P}\right)+\left(\frac{1}{n}\right)\frac{h_2Q}{2}+\left(1-\frac{1}{n}\right)\frac{h_2Q\lambda}{2P}. \tag{24}$$

Convexity of $E[TCU_1(Q,n)]$ can be proved as shown in Eq. (25), and optimal lot size Q^* and optimal number of delivery n^* can also be derived accordingly, as shown in Eqs. (26) and (27).

$$[Q\quad n]\begin{pmatrix}\frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2} & \frac{\partial^2 E[TCU(Q,n)]}{\partial Q\partial n}\\ \frac{\partial^2 E[TCU(Q,n)]}{\partial Q\partial n} & \frac{\partial^2 E[TCU(Q,n)]}{\partial n^2}\end{pmatrix}\begin{bmatrix}Q\\ n\end{bmatrix}=\frac{2K\lambda}{Q}>0, \tag{25}$$

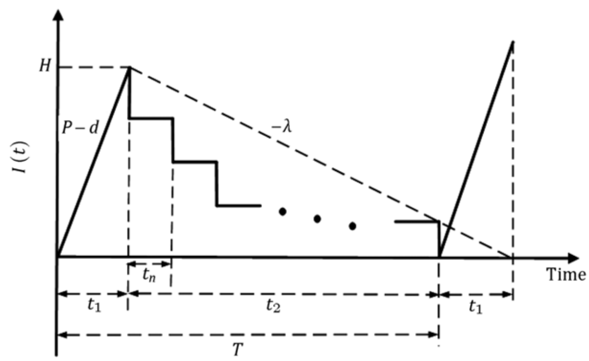

Figure 5: On-hand inventory of perfect quality items in EPQ model with a multi-delivery policy (the special case).

and:

$$Q^* = \sqrt{\frac{2(K + nK_1)\lambda}{\left\{\frac{h\lambda}{P} + \left(\frac{n-1}{n}\right)\left[h + (h_2 - h)\left(\frac{\lambda}{P}\right)\right] + \left(\frac{1}{n}\right)h_2\right\}}}, \tag{26}$$

$$n^* = \sqrt{\frac{K(h_2 - h)[1 - (\lambda/P)]}{K_1[h + h_2(\lambda/P)]}}. \tag{27}$$

4. Numerical example

Assume a product can be manufactured at a rate of 60,000 units per year, and it has experienced a flat demand rate of 3400 units per year. During production uptime, the random defective rate x is assumed and it follows a uniform distribution over the interval [0, 0.3]. Among defective items, a portion $\theta = 0.1$ is considered to be scrap and the other portion is considered to be reworkable with a repair rate $P_1 = 2100$ units per year. During the rework process, a portion $\theta_1 = 0.1$ of reworked items fails and becomes scrap.

It is also assumed that finished items can only be delivered to customers if the whole lot is quality assured at the end of rework. Fixed quantity n installments of the finished batch are delivered by request to customers at a fixed interval of time during the delivery time t_3, as depicted in Figure 1. Additional values of parameters used in this example are given below:

K_1 $2000 per shipment,
C_T $0.1 per item delivered,
C $100 per item,
C_S $20 for each scrap item,
C_R $60 for each item reworked,
K $20,000 per production run,
h $20 per item per year,
h_1 $40 per item reworked per unit time,
h_2 $80 per item kept at the customer's end per unit time.

The optimal number of shipments $n^* = 3$ can be obtained from Eq. (22), then by using Eq. (21), one has the optimal replenishment lot size $Q^* = 1735$. The long-run average cost $E[TCU(Q^*, n^*)] = \$485,541$ can also be obtained from Eq. (14). The optimal replenishment lot size and shipment policies for the special case $Q^* = 2018$ and $n^* = 3$ can also be computed by using Eqs. (26) and (27). Applying Eq. (24), one has the long-run average cost for the special case $E[TCU_1(Q^*, n^*)] = \$427,938$.

4.1. Discussion on practical usage of research results

For practitioners who manage an integrated production-shipment system in real world supply chain environments, both the *optimal production lot size* and *optimal number of deliveries* are important in terms of total production–inventory–delivery cost reduction. In the former, practitioners in factory need to pay extra attention to certain controllable and/or uncontrollable factors that may cause defective items to be generated, and also how these imperfect quality items to be handled in order to minimize the "quality cost" as well as costs related to shortage and safety stocks. In the latter, practitioners need to plan deliveries precisely in order to satisfy customer's demand as well as to minimize the shipping cost and customer's holding costs. The proposed integrated EPQ model incorporates a random defective rate during production process, an imperfect rework process with a scrap rate, production setup cost, inventory holding costs for both manufacturer and customer, and fixed and variable transportation costs. The objective of this study is to determine both the *optimal production lot size* and *optimal number of deliveries* that minimize total production–inventory–delivery costs.

Once practitioners gather all related system parameters form their real world cases, the research results of this paper can be of assistance in their decision-making, as well as sensitive analyses. It may be noted that without an in-depth investigation and robust analysis of such an integrated production-shipment system, the optimal replenishment lot size and distribution policies cannot be obtained.

5. Concluding remarks

Classic EPQ model assumes a continuous inventory issuing policy for satisfying product demand and a perfect quality production for all items produced during the production process. However, in real-life vendor–buyer integrated production–delivery system, the discontinuous issuing policy is often used, and generation of nonconforming items during a production run is inevitable. This paper investigates the aforementioned issues by incorporating a multiple delivery policy and the quality assurance into EPQ with rework and failure in repair. Mathematical modeling is employed here, and the long-run average production–inventory–delivery cost function is derived and proved to be convex. The closed-form solutions in terms of optimal replenishment lot size and optimal number of shipments to the problem are obtained. It may be noted that without an in-depth investigation and robust analysis of such a realistic system, the optimal production-shipment policies cannot be revealed. For future study, interesting topics may be included to examine the effects on the same decisions when shortage with backlogging, or a deviation (e.g. seasonal) of shipment rate to customers are under consideration. Some practical case studies can also be expected to show the effectiveness of the research results.

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Appendix A

Additional notation used in this paper:

- Q manufacturing batch size, to be determined for each cycle
- n number of fixed quantity installments of the finished batch to be delivered by request to customers, to be determined
- T cycle length
- H_1 maximum level of on-hand inventory in units when regular production ends
- H the maximum level of on-hand inventory in units when rework process finishes
- t_1 the production uptime for the proposed EMQ model
- t_2 time required for reworking of defective items
- t_3 time required for delivering all quality assured finished products
- t_n a fixed interval of time between each installment of finished products delivered during production downtime t_3
- φ overall scrap rate per cycle (sum of scrap rates in t_1 and t_2)
- C unit production cost
- K setup cost
- h unit holding cost
- C_R unit rework cost
- C_S disposal cost per scrap item
- h_1 holding cost for each reworked item
- K_1 fixed delivery cost per shipment
- C_T delivery cost per item shipped to customers
- h_2 holding cost for each item kept by customer
- $I_s(t)$ on-hand inventory of scrap items at time t
- $I(t)$ on-hand inventory of perfect quality items at manufacturer's end at time t
- $I_d(t)$ on-hand inventory of defective items at time t
- $I_c(t)$ on-hand inventory of perfect quality items at customer's end at time t
- $TC(Q, n)$ total production–inventory–delivery costs per cycle for the proposed model
- $TC_1(Q, n)$ total production–inventory–delivery per cycle when no defective items produced (i.e. the special case: the classic EPQ model with a multi-delivery policy)
- $E[TCU(Q, n)]$ the long-run average costs per unit time for the proposed model
- $[ETCU_1(Q, n)]$ the long-run average costs per unit time for the special case.

Appendix B

Computations of the holding cost of finished products at manufacturer's end during t_3 are as follows:

1. When $n = 1$, total holding cost in delivery time $= 0$.
2. When $n = 2$, total holding costs in delivery time t_3 become:

$$h\left(\frac{H}{2}\times\frac{t_3}{2}\right) = h\left(\frac{1}{2^2}\right)Ht_3. \tag{B.1}$$

3. When $n = 3$, total holding costs in delivery time t_3 are:

$$h\left(\frac{2H}{3}\times\frac{t_3}{3}+\frac{1H}{3}\times\frac{t_3}{3}\right) = h\left(\frac{2+1}{3^2}\right)Ht_3. \tag{B.2}$$

When $n = 4$, total holding costs in delivery time t_3 become:

$$h\left(\frac{3H}{4}\times\frac{t_3}{4}+\frac{2H}{4}\times\frac{t_3}{4}+\frac{1H}{4}\times\frac{t_3}{4}\right) = h\left(\frac{3+2+1}{3^2}\right)Ht_3. \tag{B.3}$$

Therefore, the following general term for total holding costs during delivery time t_3 can be obtained:

$$h\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)Ht_3 = h\left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]Ht_3 = h\left(\frac{n-1}{2n}\right)Ht_3. \tag{B.4}$$

Appendix C

Computations of the holding cost at the customer's end during t_3 are as follows.

Because n installments (fixed quantity D) of the finished lot are delivered to the customer at a fixed interval of time t_n, one has the following:

$$D = \frac{H}{n}, \tag{C.1}$$

$$t_n = \frac{t_3}{n}. \tag{C.2}$$

At the customer's end, the demand between shipments is (λt_n). If we let I denote number of items that will be left over after satisfying the demand during each fixed interval of time t_n (refer to Figure 4), then:

$$I = D - \lambda t_n. \tag{C.3}$$

From Figure 4, one can calculate the average inventory as follows:

Average inventory

$$= \left[\left(\frac{D+I}{2}\right)t_n\right] + \left(\frac{nI}{2}\right)(t_1+t_2) + \left[\frac{(D+I)+[(D+I)-\lambda t_n]}{2}t_n\right] + \left[\frac{(D+2I)+[(D+2I)-\lambda t_n]}{2}t_n\right] + \cdots + \left[\frac{[D+(n-1)I]+[[D+(n-1)I]-\lambda t_n]}{2}t_n\right]. \tag{C.4}$$

Substituting Eq. (C.3) in Eq. (C.4), the average inventory becomes:

Average inventory

$$= \left(D-\frac{\lambda}{2}t_n\right)t_n + \left(D+I-\frac{\lambda}{2}t_n\right)t_n + \left(D+2I-\frac{\lambda}{2}t_n\right)t_n + \cdots + \left(D+(n-1)I-\frac{\lambda}{2}t_n\right)t_n + \left(\frac{nI}{2}\right)(t_1+t_2) = n\left(D-\frac{\lambda}{2}t_n\right)t_n + \frac{n(n-1)}{2}It_n + \frac{nI}{2}(t_1+t_2). \tag{C.5}$$

Substituting Eqs. (C.1) through (C.3) in Eq. (C.5), the following general term for average inventory at the customer's end can be obtained:

Average inventory

$$\begin{aligned}&= n\left(\frac{H}{n}-\frac{\lambda}{2}t_n\right)t_n+\frac{n(n-1)}{2}\left(\frac{H}{n}-\lambda t_n\right)t_n\\&\quad+\frac{n}{2}\left(\frac{H}{n}-\lambda t_n\right)(t_1+t_2)\\&= Ht_n-\frac{n\lambda}{2}t_n^2+Ht_n\frac{(n-1)}{2}-\frac{n(n-1)}{2}\lambda t_n^2\\&\quad+\frac{H}{2}(t_1+t_2)-\frac{n}{2}(\lambda t_n)(t_1+t_2)\\&= \frac{Ht_3}{n}-\frac{\lambda t_3^2}{2n}+\frac{(n-1)Ht_3}{2n}-\frac{(n-1)\lambda t_3^2}{2n}\\&\quad+\frac{H}{2}(t_1+t_2)-\frac{\lambda t_3}{2}(t_1+t_2)\\&= \frac{1}{2}\left[\frac{Ht_3}{n}+T(H-\lambda t_3)\right].\end{aligned} \tag{C.6}$$

Therefore, total holding cost for items kept at the customer's end is:

$$\frac{h_2}{2}\left[\frac{Ht_3}{n}+T(H-\lambda t_3)\right]. \tag{C.7}$$

Appendix D

Computational procedures of Eq. (14) are as follows. Recall Eq. (13):

$$\begin{aligned}TC(Q,n)&=CQ+K+C_R[x(1-\theta)Q]+C_S[x\varphi Q]\\&\quad+nK_1+C_T[Q(1-\varphi x)]\\&\quad+h\left[\frac{H_1+dt_1}{2}(t_1)+\frac{H_1+H}{2}(t_2)+\left(\frac{n-1}{2n}\right)Ht_3\right]\\&\quad+h_1\cdot\frac{P_1\cdot t_2}{2}\cdot(t_2)+\frac{h_2}{2}\left[\frac{H}{n}t_3+T(H-\lambda t_3)\right].\end{aligned} \tag{D.1}$$

Then:

$$\begin{aligned}TC(Q,n)&=CQ+K+nK_1+C_R[x(1-\theta)Q]\\&\quad+C_S[x\varphi Q]+C_T[Q(1-\varphi x)]+\frac{hQ^2}{2P}\\&\quad+\frac{h_1x^2Q^2(1-\theta)^2}{2P_1}+\frac{hQ^2}{2P_1}[(2x-x^2-\varphi x^2)(1-\theta)]\\&\quad+\left(\frac{n-1}{n}\right)(1-\varphi x)hQ^2\\&\quad\times\left[\frac{(1-\varphi x)}{2\lambda}-\frac{1}{2P}-\frac{x(1-\theta)}{2P_1}\right]\\&\quad+\frac{h_2Q^2}{2n}\left[\frac{(1-\varphi x)^2}{\lambda}-\frac{(1-\varphi x)}{P}-\frac{x(1-\theta)(1-\varphi x)}{P_1}\right]\\&\quad+\frac{h_2Q^2}{2}\left[\frac{(1-\varphi x)}{P}+\frac{x(1-\theta)(1-\varphi x)}{P_1}\right],\end{aligned} \tag{D.2}$$

and because:

$$T=\frac{Q}{\lambda}(1-\varphi\cdot x), \tag{D.3}$$

$$E[TCU(Q,n)]=\frac{E[TC(Q,n)]}{E[T]}, \tag{D.4}$$

then:

$$\begin{aligned}E[TCU(Q,n)]&=\frac{E[TC(Q,n)]}{E[T]}=\frac{C\lambda}{1-\varphi E[x]}\\&\quad+\frac{(K+nK_1)\lambda}{Q(1-\varphi E[x])}+\frac{C_RE[x](1-\theta)\lambda}{(1-\varphi E[x])}+\frac{C_SE[x]\varphi\lambda}{(1-\varphi E[x])}\\&\quad+C_T\lambda+\frac{hQ\lambda}{2P(1-\varphi E[x])}\\&\quad+\frac{hQ\lambda}{2P_1(1-\varphi E[x])}\left[(2E[x]-(E[x])^2-\varphi(E[x])^2)(1-\theta)\right]\\&\quad+\left(1-\frac{1}{n}\right)\left[\frac{hQ(1-\varphi E[x])}{2}-\frac{hQ\lambda}{2P}-\frac{hQE[x](1-\theta)\lambda}{2P_1}\right]\\&\quad+\frac{h_1(E[x])^2Q\lambda(1-\theta)^2}{2P_1(1-\varphi E[x])}+\left(\frac{1}{n}\right)\frac{h_2Q}{2}(1-\varphi E[x])\\&\quad+\left(1-\frac{1}{n}\right)\frac{h_2Q\lambda}{2P}+\frac{h_2Q}{2}\left[\left(1-\frac{1}{n}\right)\frac{E[x]\lambda(1-\theta)}{P_1}\right].\end{aligned} \tag{D.5}$$

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