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# Iterative Methods Improving Newton's Method by the Decomposition Method

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**Abstract**—In this paper, we present a sequence of iterative methods improving Newton's method for solving nonlinear equations. The Adomian decomposition method is applied to an equivalent coupled system to construct the sequence of the methods whose order of convergence increases as it progresses. The orders of convergence are derived analytically, and then rederived by applying symbolic computation of MAPLE. Some numerical illustrations are given. © 2005 Elsevier Ltd. All rights reserved.

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## 1. INTRODUCTION

One of the oldest and most basic problems in mathematics is that of solving nonlinear equations $f(x) = 0$. To solve these equations, we can use iterative methods such as Newton's method and its variants. Newton's method is one of the most powerful and well-known iterative methods known to converge quadratically. Recently, there has been some progress on iterative methods with higher order of convergence that do require the computation of as lower-order derivatives as possible [1–5]. In that direction, there has been another approach based on the Adomian decomposition method on developing iterative methods to solve the equation $f(x) = 0$ [1,6,7]. The Adomian decomposition method is the method which considers the solution as an infinite series usually converging to an accurate solution, and has been successfully applied to a wide class of functional equations over the last 20 years [8–10]. The convergence of the decomposition series have been investigated by several authors [6,8,11,12]. Abbaoui and Cherruault [8] applied the method to solve the equation $f(x) = 0$ and proved the convergence of the series solution. The adomian method has been modified also so as to construct numerical schemes [1,6].

In this paper, we construct a sequence of higher-order iterative methods based on the Adomian decomposition method. In Section 2, we outline the steps of how the decomposition method is

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applied to an equivalent coupled system to nonlinear equation to obtain a sequence of iterative methods, which is shown in later sections to involve lower-order derivatives but show higher-order of convergence. In Section 3, we give a detailed convergence analysis of one of the proposed methods, and then it is discussed in detail that the orders of convergence and error equations of the proposed methods can be easily rederived by applying symbolic computation of mathematical software package MAPLE and using the Taylor series. Numerical results and comparisons are given in the last section.

## 2. DESCRIPTION OF A SEQUENCE OF ITERATIVE METHODS

Consider the nonlinear equation,

$$f(x) = 0. \tag{1}$$

We assume that $f(x)$ has a simple root at $\alpha$ and $\gamma$ is an initial guess sufficiently close to $\alpha$. Let us convert the nonlinear equation (1) into the following coupled system,

$$f(\gamma) + f'(\gamma)(x - \gamma) + g(x) = 0, \tag{2}$$

$$g(x) = f(x) - f(\gamma) - f'(\gamma)(x - \gamma), \tag{3}$$

where $\gamma$ is the initial approximation for a zero of (1).

The equation (2) can be rewritten in the form,

$$x = c + N(x), \tag{4}$$

where $c = \gamma - f(\gamma)/f'(\gamma)$ and $N(x) = -g(x)/f'(\gamma)$ is a nonlinear function.

We now construct a sequence of higher-order iterative methods by applying the Adomian decomposition method. The Adomian decomposition method consists in looking for a solution having the series form,

$$x = \sum_{n=0}^{\infty} x_n, \tag{5}$$

and the nonlinear function is decomposed as

$$N(x) = \sum_{n=0}^{\infty} A_n, \tag{6}$$

where the $A_n$ are functions called the Adomian's polynomials depending on $x_0, x_1, \dots, x_n$ given by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i x_i\right)\right]_{\lambda=0}, \qquad n = 0, 1, 2, \dots. \tag{7}$$

The first few polynomials are given by

$$A_0 = N(x_0),$$

$$A_1 = x_1 N'(x_0),$$

$$A_2 = x_2 N'(x_0) + \frac{1}{2}x_1^2 N''(x_0).$$

Upon substituting (5) and (6) into (4) yields

$$\sum_{n=0}^{\infty} x_n = c + \sum_{n=0}^{\infty} A_n. \tag{8}$$

It follows from (8) that

$$x_0 = c,$$

$$x_{n+1} = A_n, \qquad n = 0, 1, 2, \dots.$$

An elementary calculation shows that

$$A_0 = N(x_0) = -\frac{g(x_0)}{f'(\gamma)} = -\frac{f(x_0)}{f'(\gamma)}, \qquad N'(x_0) = 1 - \frac{f'(x_0)}{f'(\gamma)},$$

$$A_1 = x_1 N'(x_0) = A_0 N'(x_0) = -\frac{f(x_0)}{f'(\gamma)} + \frac{f(x_0)f'(x_0)}{[f'(\gamma)]^2}, \qquad N''(x_0) = -\frac{f''(x_0)}{f'(\gamma)},$$

$$A_2 = x_2 N'(x_0) + \frac{1}{2}x_1^2 N''(x_0) = A_1 N'(x_0) + \frac{1}{2}A_0^2 N''(x_0)$$

$$= -\frac{f(x_0)}{f'(\gamma)} + 2\frac{f(x_0)f'(x_0)}{[f'(\gamma)]^2} - \frac{1}{2}\frac{f(x_0)\left(f(x_0)f''(x_0) + 2[f'(x_0)]^2\right)}{[f'(\gamma)]^3}.$$

Note that $x$ is approximated by

$$X_m = x_0 + x_1 + \cdots + x_m = x_0 + A_0 + A_1 + \cdots + A_{m-1},$$

where $\lim_{m\to\infty} X_m = x$.

For $m = 0$,

$$x \approx X_0 = x_0 = c = \gamma - \frac{f(\gamma)}{f'(\gamma)},$$

which yields the Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

which is well known to converge quadratically.

For $m = 1$,

$$x \approx X_1 = x_0 + x_1 = c + A_0 = \gamma - \frac{f(\gamma)}{f'(\gamma)} - \frac{f(x_0)}{f'(\gamma)},$$

which produces the iteration scheme,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f(x_{n+1}^*)}{f'(x_n)}, \tag{9}$$

where

$$x_{n+1}^* = x_n - \frac{f(x_n)}{f'(x_n)}.$$

For $m = 2$,

$$x \approx X_2 = x_0 + x_1 + x_2 = c + A_0 + A_1 = \gamma - \frac{f(\gamma)}{f'(\gamma)} - 2\frac{f(x_0)}{f'(\gamma)} + \frac{f(x_0)f'(x_0)}{[f'(\gamma)]^2},$$

which produces the iteration scheme,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - 2\frac{f(x_{n+1}^*)}{f'(x_n)} + \frac{f(x_{n+1}^*)f'(x_{n+1}^*)}{[f'(x_n)]^2}. \tag{10}$$

For $m = 3$,

$$x \approx X_3 = x_0 + x_1 + x_2 + x_3 = c + A_0 + A_1 + A_2$$
$$= \gamma - \frac{f(\gamma)}{f'(\gamma)} - 3\frac{f(x_0)}{f'(\gamma)} + 3\frac{f(x_0)f'(x_0)}{[f'(\gamma)]^2}$$
$$- \frac{1}{2}\frac{f(x_0)\left(f(x_0)f''(x_0) + 2[f'(x_0)]^2\right)}{[f'(\gamma)]^3},$$

which produces the iteration scheme,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - 3\frac{f(x^*_{n+1})}{f'(x_n)} + 3\frac{f(x^*_{n+1})f'(x^*_{n+1})}{[f'(x_n)]^2} - \frac{1}{2}\frac{f(x^*_{n+1})\left(f(x^*_{n+1})f''(x^*_{n+1}) + 2[f'(x^*_{n+1})]^2\right)}{[f'(x_n)]^3}. \tag{11}$$

## 3. CONVERGENCE ANALYSIS

In this section, we will be concerned with the order of convergence of each of the methods defined by (9), (10), and (11). We first show that the method defined by (10) is fourth-order convergent.

THEOREM 3.1. *Let $\alpha \in I$ be a simple zero of a sufficiently differentiable function $f : I \to \mathbf{R}$ for an open interval $I$. Then, the new method defined by (10) has the fourth-order convergence and satisfies the following error equation,*

$$e_{n+1} = 5c_2^3 e_n^4 + O(e_n^5), \tag{12}$$

*where $e_n = x_n - \alpha$ and $c_2 = f^{(2)}(\alpha)/2f'(\alpha)$.*

PROOF. Let $\alpha$ be a simple zero of $f$. By the Taylor expansions,

$$f(x_n) = f'(\alpha)\left[e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + O(e_n^5)\right], \tag{13}$$

and

$$f'(x_n) = f'(\alpha)\left[1 + 2c_2e_n + 3c_3e_n^2 + 4c_4e_n^3 + O(e_n^4)\right], \tag{14}$$

where $c_k = (1/k!)f^{(k)}(\alpha)/f'(\alpha), k = 2, 3, \dots$ and $e_n = x_n - \alpha$ . Dividing (13) by (14) gives us

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2e_n^2 + 2(c_2^2 - c_3)e_n^3 + (7c_2c_3 - 3c_4 - 4c_2^3)e_n^4 + O(e_n^5), \tag{15}$$

and hence,

$$x^*_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = \alpha + c_2e_n^2 + 2(c_3 - c_2^2)e_n^3 + (3c_4 + 4c_2^3 - 7c_2c_3)e_n^4 + O(e_n^5). \tag{16}$$

Again, expanding $f(x^*_{n+1})$ and $f'(x^*_{n+1})$ about $\alpha$ and then using (16),

$$f(x^*_{n+1}) = f'(\alpha)\left[c_2e_n^2 + 2(c_3 - c_2^2)e_n^3 + (5c_2^3 - 7c_2c_3 + 3c_4)e_n^4 + O(e_n^5)\right],$$

and

$$f'(x^*_{n+1}) = f'(\alpha) + (x^*_{n+1} - \alpha)f''(\alpha) + \frac{(x^*_{n+1} - \alpha)^2}{2!}f^{(3)}(\alpha) + \cdots$$
$$= f'(\alpha)\left[1 + 2c_2^2e_n^2 + 4c_2(c_3 - c_2^2)e_n^3 + c_2(8c_2^3 - 11c_2c_3 + 6c_4)e_n^4 + O(e_n^5)\right].$$

After an elementary calculation, we obtain

$$\frac{f\left(x_{n+1}^*\right)}{f'\left(x_n\right)} = c_2e_n^2 + 2\left(c_3 - 2c_2^2\right)e_n^3 + \left(13c_2^3 - 14c_2c_3 + 3c_4\right)e_n^4 + O\left(e_n^5\right), \tag{17}$$

$$\frac{f'\left(x_{n+1}^*\right)}{f'\left(x_n\right)} = 1 - 2c_2e_n + 3\left(2c_2^2 - c_3\right)e_n^2 + O\left(e_n^3\right),$$

$$\frac{f\left(x_{n+1}^*\right)f'\left(x_{n+1}^*\right)}{\left[f'\left(x_n\right)\right]^2} = c_2e_n^2 + \left(2c_3 - 6c_2^2\right)e_n^3 + \left(27c_2^3 - 21c_2c_3 + 3c_4\right)e_n^4 + O\left(e_n^5\right). \tag{18}$$

From equations (15), (17), and (18), we obtain

$$\frac{f(x_n)}{f'(x_n)} + 2\frac{f(x_{n+1}^*)}{f'(x_n)} - \frac{f(x_{n+1}^*)f'(x_{n+1}^*)}{[f'(x_n)]^2} = e_n - 5c_2^3e_n^4 + O(e_n^5).$$

Thus,

$$\begin{aligned} x_{n+1} &= x_n - \left[\frac{f\left(x_n\right)}{f'\left(x_n\right)} + 2\frac{f\left(x_{n+1}^*\right)}{f'\left(x_n\right)} - \frac{f\left(x_{n+1}^*\right)f'\left(x_{n+1}^*\right)}{\left[f'\left(x_n\right)\right]^2}\right], \\ e_{n+1} + \alpha &= e_n + \alpha - \left[e_n - 5c_2^3e_n^4 + O\left(e_n^5\right)\right], \\ e_{n+1} &= 5c_2^3e_n^4 + O\left(e_n^5\right). \end{aligned} \tag{19}$$

Equation (19) establishes the fourth-order convergence of the method defined by (10).

From the error equation (12), we see that the method defined by (10) converges with fourth-order. As it can be observed, the higher $m$ is getting, the more complex the corresponding scheme is getting, so that derivation of the error equations carried out by hand for rather complex iteration formulas will be a really difficult task. Computer algebra systems such as mathematical software package MAPLE can take over the hard manual work, such as deriving the error equations and convergence orders.

In what follows, the use of MAPLE in deriving the convergence orders of (9), (10), and (11) is demonstrated. Then, based on the output of MAPLE the error equations for the corresponding methods are derived for (10) in detail, and for (9),(11) without the detail.

It is known (see [13]) that the fixed-point iteration,

$$x_{n+1} = F(x_n), \qquad \text{for } n \geq 0, \tag{20}$$

is of convergence order $q$ if $F$ is sufficiently many times differentiable on an interval containing $\alpha$, a fixed point of $F$, with bounded first derivative, and satisfies

$$F'(\alpha) = F^{(2)}(\alpha) = \cdots = F^{(q-1)}(\alpha) = 0 \qquad \text{and} \qquad F^{(q)}(\alpha) \neq 0. \tag{21}$$

For the iteration function $F$ of the schema (10) which has a fixed point $\alpha$, we run the following MAPLE statements,

```
> h:=x->x-f(x)/D(f)(x);
```

$$h := x -> \frac{f(x)}{\mathrm{D}(f)(x)}$$

```
> F:=x->x-f(x)/D(f)(x)-2.0*(f@h)(x)/D(f)(x)+(f@h)(x)*(D(f)@h)(x)/D(f)(x)^2;
```

$$F := x -> x - \frac{f(x)}{\mathrm{D}(f)(x)} - \frac{2.0(f@h)(x)}{\mathrm{D}(f)(x)} + \frac{(f@h)(x)((\mathrm{D}(f))@h)(x)}{\mathrm{D}(f)(x)^2}$$

```
> algsubs(f(α)=0, F(α));
```

$$\alpha$$

```
> algsubs(f(α)=0, D(F)(α));
```

$$0.$$

```
> algsubs(f(α)=0,D(D(F))(α));
```

$$0.$$

```
> algsubs(f(α)=0,(D@@3)(F)(α));
```

$$0.$$

```
> algsubs(f(α)=0,(D@@4)(F)(α));
```

$$\frac{15.0(\mathrm{D}^{(2)})(f)(\alpha)^3}{\mathrm{D}(f)(\alpha)^3.}$$

Thus, one reads

$$F(\alpha) = \alpha, \qquad F'(\alpha) = F^{(2)}(\alpha) = F^{(3)}(\alpha) = 0, \qquad F^{(4)}(\alpha) = 15\left(\frac{f''(\alpha)}{f'(\alpha)}\right)^3. \tag{22}$$

But $F^{(4)}(\alpha) \neq 0$ provided that

$$15\left(\frac{f''(\alpha)}{f'(\alpha)}\right)^3 \neq 0,$$

and so, in general, the iterative method (10) is of order four. Based on the result (22) the error equation (12) appears in Theorem 3.1 can be rederived by using Taylor series as follows.

By expanding $F(x_n)$ around $x = \alpha$, we obtain

$$\begin{aligned} x_{n+1} = F(x_n) = F(\alpha) + F'(\alpha)(x_n - \alpha) + \frac{F^{(2)}(\alpha)}{2!}(x_n - \alpha)^2 + \frac{F^{(3)}(\alpha)}{3!}(x_n - \alpha)^3 \\ + \frac{F^{(4)}(\alpha)}{4!}(x_n - \alpha)^4 + O((x_n - \alpha)^5). \end{aligned} \tag{23}$$

From (22) and (23), we get

$$\begin{aligned} x_{n+1} &= \alpha + \frac{F^{(4)}(\alpha)}{4!} e_n^4 + O\left(e_n^5\right) \\ &= \alpha + 5\left(\frac{f^{(2)}(\alpha)}{2f'(\alpha)}\right)^3 e_n^4 + O\left(e_n^5\right) \\ &= \alpha + 5c_2^3 e_n^4 + O\left(e_n^5\right), \end{aligned} \tag{24}$$

where $e_n = x_n - \alpha$ and $c_2 = f^{(2)}(\alpha)/2f'(\alpha)$. Thus,

$$e_{n+1} = 5c_2^3 e_n^4 + O(e_n^5). \tag{25}$$

Equation (25) is the error equation that appears in Theorem 3.1.

Doing the same as just in the above for the method (9), we obtain

$$F(\alpha) = \alpha, \qquad F'(\alpha) = F^{(2)}(\alpha) = 0, \qquad F^{(3)}(\alpha) = 3\left(\frac{f^{(2)}(\alpha)}{f'(\alpha)}\right)^2, \tag{26}$$

and so, in general, it is of order three. Based on the result (26) the error equation is easily found to be

$$e_{n+1} = 2c_2^2 e_n^3 + O(e_n^4).$$

For method (11), it follows that

$$\begin{aligned} F(\alpha) &= \alpha, \\ F^{(k)}(\alpha) &= 0, \qquad \text{for } k = 1, \dots, 4, \\ F^{(5)}(\alpha) &= 105 \left( \frac{f^{(2)}(\alpha)}{f'(\alpha)} \right)^4, \end{aligned} \tag{27}$$

and so, in general, it is of order five. Based on the result (27) the error equation is easily found to be

$$e_{n+1} = 14c_2^4 e_n^5 + O(e_n^6).$$

From the above observations, we conjecture that the $m = n$, $n \geq 0$ case produces an iteration scheme of order $(n+2)$.

## 4. EXAMPLES

The order of convergence $\rho$ can be approximated using the formula,

$$\rho \approx \frac{\ln |(x_{n+1} - \alpha)/(x_n - \alpha)|}{\ln |(x_n - \alpha)/(x_{n-1} - \alpha)|}.$$

All computations were done using MAPLE using 64 digit floating-point arithmetics (Digits:=64). We accept an approximate solution rather than the exact root, depending on the precision ($\epsilon$) of the computer. So, the following stopping criteria is used for computer programs:

(i) $|x_{n+1} - x_n| < \epsilon$,
(ii) $|f(x_{n+1})| < \epsilon$.

We present some numerical test results for various iterative schemes, in Table 1. Compared were the Newton method (NM), the method of Homeier [3] (HM), the method of Abbasbandy [7] (AM) and the methods (10) (CM2) and (11) (CM3) introduced in the present contribution. We used $\epsilon = 10^{-15}$ and the test functions the same as in Weerakoon and Fernando [5],

$$\begin{aligned} f_1(x) &= x^3 + 4x^2 - 10, \\ f_2(x) &= \sin^2 x - x^2 + 1, \\ f_3(x) &= x^2 - e^x - 3x + 2, \\ f_4(x) &= \cos x - x, \\ f_5(x) &= (x-1)^3 - 1, \\ f_6(x) &= x^3 - 10, \\ f_7(x) &= xe^{x^2} - \sin^2 x + 3\cos x + 5, \\ f_8(x) &= e^{x^2+7x-30} - 1. \end{aligned}$$

As convergence criterion, it was required that the distance of two consecutive approximations $\delta$ for the zero was less than $10^{-15}$. Also displayed are the number of iterations to approximate the zero (IT), the approximate zero $x_*$, the value $f(x_*)$, the computational order of convergence (COC).

Table 1. Comparison of various iterative schemes and the Newton method.

| | IT | COC | $x_*$ | $f(x_*)$ | $\delta$ |
|---|---|---|---|---|---|
| $f_1, \gamma = -0.3$ | | | | | |
| NM | 55 | 2 | 1.36523001341409684576080682898 | 1.95e − 60 | 4.92e − 31 |
| AM | 22 | 2.97 | 1.36523001341409684576080682898 | −1.47e − 57 | 1.14e − 19 |
| HM | 87 | 3 | 1.36523001341409684576080682898 | 0 | 3.06e − 46 |
| CM2 | 27 | 4 | 1.36523001341409684576080682898 | 0 | 1.82e − 50 |
| CM3 | 49 | 5 | 1.36523001341409684576080682898 | 0 | 1.04e − 52 |
| $f_2, \gamma = 1$ | | | | | |
| NM | 7 | 2 | 1.40449164821534122603508681779 | −1.04e − 50 | 7.33e − 26 |
| AM | 5 | 2.86 | 1.40449164821534122603508681779 | −5.81e − 55 | 1.39e − 18 |
| HM | 4 | 3.01 | 1.40449164821534122603508681779 | −5.4e − 62 | 7.92e − 21 |
| CM2 | 5 | 3.95 | 1.40449164821534122603508681779 | −2.0e − 63 | 1.31e − 17 |
| CM3 | 4 | 4.92 | 1.40449164821534122603508681779 | −2.0e − 63 | 1.41e − 22 |
| $f_3, \gamma = 2$ | | | | | |
| NM | 6 | 2 | 0.257530285439860760455367304937 | 2.93e − 55 | 9.1e − 28 |
| AM | 5 | 3 | 0.257530285439860760455367304937 | 1.0e − 63 | 1.45e − 26 |
| HM | 5 | 3 | 0.257530285439860760455367304937 | 0 | 9.33e − 43 |
| CM2 | 4 | 3.98 | 0.257530285439860760455367304937 | 1.0e − 63 | 9.46e − 29 |
| CM3 | 4 | 4.99 | 0.257530285439860760455367304937 | 0 | 1.61e − 53 |
| $f_4, \gamma = 1.7$ | | | | | |
| NM | 5 | 2 | 0.739085133215160641655312087674 | −2.03e − 32 | 2.34e − 16 |
| AM | 4 | 3.01 | 0.739085133215160641655312087674 | −7.14e − 47 | 8.6e − 16 |
| HM | 4 | 3 | 0.739085133215160641655312087674 | −5.02e − 59 | 9.64e − 20 |
| CM2 | 4 | 4 | 0.739085133215160641655312087674 | 0 | 1.86e − 53 |
| CM3 | 3 | 4.43 | 0.739085133215160641655312087674 | 0 | 5.49e − 20 |
| $f_5, \gamma = 3.5$ | | | | | |
| NM | 8 | 2 | 2 | 2.06e − 42 | 8.28e − 22 |
| AM | 5 | 2.99 | 2 | 0 | 4.3e − 22 |
| HM | 5 | 3.0 | 2 | 0 | 1.46e − 24 |
| CM2 | 5 | 3.99 | 2 | 0 | 2.74e − 24 |
| CM3 | 5 | 5 | 2 | 0 | 4.76e − 50 |
| $f_6, \gamma = 1.5$ | | | | | |
| NM | 7 | 2 | 2.15443469003188372175929356652 | 2.06e − 54 | 5.64e − 28 |
| AM | 5 | 2.99 | 2.15443469003188372175929356652 | −5.0e − 63 | 1.18e − 25 |
| HM | 4 | 3.0 | 2.15443469003188372175929356652 | −5.0e − 63 | 9.8e − 23 |
| CM2 | 5 | 3.99 | 2.15443469003188372175929356652 | −5.0e − 63 | 1.57e − 22 |
| CM3 | 15 | 4.95 | 2.15443469003188372175929356652 | −5.0e − 63 | 1.2e − 21 |

Table 1. (cont.)

| | IT | COC | $x_*$ | $f(x_*)$ | $\delta$ |
|---|---|---|---|---|---|
| $f_7, \gamma = -2$ | | | | | |
| NM | 9 | 2 | -1.2076478271309189270094167584 | $-2.27e-40$ | $2.73e-21$ |
| AM | 6 | 3 | -1.2076478271309189270094167584 | $-4.0e-63$ | $4.35e-45$ |
| HM | 6 | 3 | -1.2076478271309189270094167584 | $-4.0e-63$ | $2.57e-32$ |
| CM2 | 6 | 4.0 | -1.2076478271309189270094167584 | $-4.0e-63$ | $2.15e-36$ |
| CM3 | 5 | 4.92 | -1.2076478271309189270094167584 | $1.3e-62$ | $1.1e-18$ |
| $f_8, \gamma = 3.5$ | | | | | |
| NM | 13 | 2 | 3 | $1.52e-47$ | $4.2e-25$ |
| AM | 7 | 2.98 | 3 | $-4.33e-48$ | $2.25e-17$ |
| HM | 8 | 3 | 3 | $2.0e-62$ | $2.43e-33$ |
| CM2 | 8 | 3.99 | 3 | $2.0e-62$ | $2.12e-23$ |
| CM3 | 7 | 4.85 | 3 | $-2.0e-62$ | $3.36e-16$ |

The test results in Table 1 show that the computational orders of convergence of the proposed methods (CM2, CM3) are in accordance with the theory developed in the previous sections. The most important characteristic of the method (10) (resp., (11)) is that it does not require the computation of second order (resp., third order) or higher-order derivatives of the function to carry out iterations although it is of the fourth order (resp., fifth order).

## 5. CONCLUSION

In this work a sequence of iterative methods for solving nonlinear equation $f(x) = 0$ with higher-order convergence is developed by applying Adomian decomposition method to a coupled system equivalent to $f(x) = 0$. We have discussed that the convergence orders and error equations of the methods can be derived with the help of MAPLE. The method can be continuously applied to generate an iterative scheme with arbitrarily specified order of convergence, which does not require the computation of that higher-order derivatives compared to most other methods of the same order even though it may be attained at the expense of more terms involved in the resulting scheme.

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