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The positive almost periodic solution for Nicholson-type delay systems with linear harvesting terms ☆

Xingguo Liu [a], Junxia Meng [b,*]

[a] College of Business Administration, Hunan University, Changsha, Hunan 410082, PR China
[b] College of Mathematics, Physics and Information Engineering, Jiaxing University, Jiaxing, Zhejiang 314001, PR China

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ABSTRACT

In this paper, we study the existence and exponential convergence of positive almost periodic solutions for a class of Nicholson-type delay system with linear harvesting terms. Under appropriate conditions, we establish some criteria to ensure that the solutions of this system converge locally exponentially to a positive almost periodic solution. Moreover, we give an example to illustrate our main results.

1. Introduction

In [1], to describe the models of Marine Protected Areas and B-cell Chronic Lymphocytic Leukemia dynamics that belong to the Nicholson-type delay differential systems, Berezansky et al. [1] considered the dynamics of the following autonomous Nicholson-type delay systems:

$$\begin{cases} x_1'(t) = -a_1x_1(t) + b_1x_2(t) + c_1x_1(t-\tau)e^{-x_1(t-\tau)}, \\ x_2'(t) = -a_2x_2(t) + b_2x_1(t) + c_2x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \tag{1.1}$$

with initial conditions:

$$x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0], \quad \varphi_i(0) > 0, \tag{1.2}$$

where $\varphi_i \in C([-\tau,0], [0,+\infty))$, a_i, b_i, c_i and τ are nonnegative constants, $i = 1, 2$.

Furthermore, Wang et al. [2] showed the existence and exponential convergence of positive almost periodic solutions for the following non-autonomous Nicholson-type delay systems:

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* Corresponding author. Tel./fax: +86 057383643075.
E-mail address: mengjunxia1968@yahoo.com.cn (J. Meng).

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$$\begin{cases} x_1'(t) = -\alpha_1(t)x_1(t) + \beta_1(t)x_2(t) + \sum_{j=1}^{m} c_{1j}(t)x_1(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)x_1(t-\tau_{1j}(t))}, \\ x_2'(t) = -\alpha_2(t)x_2(t) + \beta_2(t)x_1(t) + \sum_{j=1}^{m} c_{2j}(t)x_2(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)x_2(t-\tau_{2j}(t))}, \end{cases} \tag{1.3}$$

where α_i, β_i, c_{ij}, γ_{ij}, $\tau_{ij}: R^1 \to (0,+\infty)$ are almost periodic functions, and $i = 1, 2$, $j = 1, 2,\ldots,m$.

Recently, assuming that a harvesting function is a function of the delayed estimate of the true population, Berezansky et al. [3] proposed the Nicholson's blowflies model with a linear harvesting term:

$$x'(t) = -\delta x(t) + px(t-\tau)e^{-ax(t-\tau)} - Hx(t-\sigma), \quad \delta, p, \tau, a, H, \ \sigma \in (0,+\infty), \tag{1.4}$$

where $Hx(t-\sigma)$ is a linear harvesting term, $x(t)$ is the size of the population at time t, P is the maximum per capita daily egg production, $\frac{1}{a}$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time. Moreover, Berezansky et al. [3] pointed out an open problem: How about the dynamic behaviors of the Nicholson's blowflies model with a linear harvesting term.

Now, motivated by Berezansky et al. [1], Wang et al. [2], Berezansky et al. [3] a corresponding question arises: How about the existence and convergence of positive almost periodic solutions of Nicholson-type delay differential systems with linear harvesting terms. The main purpose of this paper is to give the conditions to ensure the existence and convergence of positive almost periodic solutions of the following non-autonomous Nicholson-type delay systems with linear harvesting terms:

$$\begin{cases} x_1'(t) = & -\alpha_1(t)x_1(t) + \beta_1(t)x_2(t) + \sum_{j=1}^{m} c_{1j}(t)x_1(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)x_1(t-\tau_{1j}(t))} \\ & -H_1(t)x_1(t-\sigma_1(t)), \\ x_2'(t) = & -\alpha_2(t)x_2(t) + \beta_2(t)x_1(t) + \sum_{j=1}^{m} c_{2j}(t)x_2(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)x_2(t-\tau_{2j}(t))} \\ & -H_2(t)x_2(t-\sigma_2(t)), \end{cases} \tag{1.5}$$

where α_i, β_i, H_i, σ_i, c_{ij}, γ_{ij}, $\tau_{ij}: R^1 \to [0,+\infty)$ are almost periodic functions, and $i = 1, 2$, $j = 1, 2,\ldots,m$.

For convenience, we introduce some notations. Throughout this paper, given a bounded continuous function g defined on R^1, let g^+ and g^- be defined as

$$g^- = \inf_{t\in R} g(t), \quad g^+ = \sup_{t\in R} g(t).$$

It will be assumed that

$$\alpha_i^- > 0, \quad \beta_i^- > 0, \quad c_{ij}^- > 0, \quad r_i = \max\left\{\max_{1\leqslant j\leqslant m}\left\{\tau_{ij}^+\right\}, \sigma_i^+\right\} > 0, \quad i = 1, 2. \tag{1.6}$$

Denote by $R^n(R^n_+)$ the set of all (nonnegative) real vectors. Let

$$C = C([-r_1, 0], R^1) \times C([-r_2, 0], R^1) \quad \text{and } C_+ = C\left([-r_1, 0], R^1_+\right) \times C\left([-r_2, 0], R^1_+\right).$$

If $x_i(t)$ is defined on $[t_0 - r_i, \sigma)$ with $t_0, \sigma \in R^1$ and $i = 1, 2$, then we define $x_t \in C$ as $x_t = (x_t^1, x_t^2)$ where $x_t^i(\theta) = x_i(t+\theta)$ for all $\theta \in [-r_i, 0]$ and $i = 1, 2$. A matrix or vector $A \geqslant 0$ means that all entries of A are greater than or equal to zero. $A > 0$ can be defined similarly. For matrices or vectors A and B, $A \geqslant B$ (resp. $A > B$) means that $A - B \geqslant 0$ (resp. $A - B > 0$). For vector $X = (x_1, x_2) \in R^2$, we let $|X|$ denote the absolute-value vector given by $|X| = (|x_1|, |x_2|)$, and define $\|X\| = \max_{1\leqslant i\leqslant 2}|x_i|$.

The initial conditions associated with system (1.5) are of the form:

$$x_{t_0} = \varphi, \ \varphi = (\varphi_1, \varphi_2) \in C_+ \quad \text{and } \varphi_i(0) > 0, \quad i = 1, 2. \tag{1.7}$$

We write $x_t(t_0, \varphi)(x(t; t_0, \varphi))$ for a solution of the initial value problem (1.5) and (1.7). Also, let $[t_0, \eta(\varphi))$ be the maximal right-interval of existence of $x_t(t_0, \varphi)$.

The remaining part of this paper is organized as follows. In Section 2, we shall give some notations and preliminary results. In Section 3, we shall derive new sufficient conditions for checking the existence, uniqueness and local exponential convergence of the positive almost periodic solution of (1.5). In Section 4, we shall give some example and remark to illustrate our results obtained in the previous sections.

2. Preliminary results

In this section, some lemmas and definitions will be presented, which are of importance in proving our main results in Section 3.

Definition 2.1 (See [4,5]). Let $u(t): R^1 \to R^n$ be continuous in t. $u(t)$ is said to be almost periodic on R^1, if for any $\varepsilon > 0$, the set $T(u,\varepsilon) = \{\delta : |u(t+\delta) - u(t)| < \varepsilon \text{ for all } t \in R^1\}$ is relatively dense, i.e., for any $\varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$, such that for any interval with length $l(\varepsilon)$, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $|u(t+\delta) - u(t)| < \varepsilon$, for all $t \in R^1$.

Definition 2.2 (See [4,5]). Let $x \in R^n$ and $Q(t)$ be a $n \times n$ continuous matrix defined on R^1. The linear system

$$x'(t) = Q(t)x(t) \tag{2.1}$$

is said to admit an exponential dichotomy on R^1 if there exist positive constants k, α, projection P and the fundamental solution matrix $X(t)$ of (2.1) satisfying

$$\|X(t)PX^{-1}(s)\| \leqslant ke^{-\alpha(t-s)} \quad \text{for all } t \geqslant s,$$

$$\|X(t)(I-P)X^{-1}(s)\| \leqslant ke^{-\alpha(s-t)} \quad \text{for all } t \leqslant s.$$

Set

$$B = \{\varphi | \varphi = (\varphi_1(t), \varphi_2(t)) \text{ is an almost periodic function on } R^1\}.$$

For any $\varphi \in B$, we define induced module $\|\varphi\|_B = \sup_{t \in R^1} \|\varphi(t)\|$, then B is a Banach space.

Lemma 2.1 (See [4,5]). *If the linear system (2.1) admits an exponential dichotomy, then almost periodic system*

$$x'(t) = Q(t)x + g(t) \tag{2.2}$$

has a unique almost periodic solution x(t), and

$$x(t) = \int_{-\infty}^{t} X(t)PX^{-1}(s)g(s)ds - \int_{t}^{+\infty} X(t)(I-P)X^{-1}(s)g(s)\,ds. \tag{2.3}$$

Lemma 2.2 (See [4,5]). *Let $c_i(t)$ be an almost periodic function on R^1 and*

$$M[c_i] = \lim_{T \to +\infty} \frac{1}{T} \int_{t}^{t+T} c_i(s)ds > 0, \quad i = 1, 2, \ldots, n.$$

Then the linear system

$$x'(t) = diag\,(-c_1(t), -c_2(t), \ldots, -c_n(t))x(t)$$

admits an exponential dichotomy on R^1.

Lemma 2.3. *Suppose that there exist two positive constants E_{i1} and E_{i2} such that*

$$E_{i1} > E_{i2}, \quad \frac{\beta_1^+ E_{21}}{\alpha_1^-} + \sum_{j=1}^{m} \frac{c_{1j}^+}{\alpha_1^- \gamma_{1j}^-} \frac{1}{e} < E_{11}, \quad \frac{\beta_2^+ E_{11}}{\alpha_2^-} + \sum_{j=1}^{m} \frac{c_{2j}^+}{\alpha_2^- \gamma_{2j}^-} \frac{1}{e} < E_{21}, \tag{2.4}$$

$$\frac{\beta_1^- E_{22}}{\alpha_1^+} + \sum_{j=1}^{m} \frac{c_{1j}^-}{\alpha_1^+} E_{11} e^{-\gamma_{1j}^+ E_{11}} - \frac{H_1^+ E_{11}}{\alpha_1^+} > E_{12} \geqslant \frac{1}{\min\limits_{1 \leqslant j \leqslant m} \gamma_{1j}^-}, \tag{2.5}$$

$$\frac{\beta_2^- E_{12}}{\alpha_2^+} + \sum_{j=1}^{m} \frac{c_{2j}^-}{\alpha_2^+} E_{21} e^{-\gamma_{2j}^+ E_{21}} - \frac{H_2^+ E_{21}}{\alpha_2^+} > E_{22} \geqslant \frac{1}{\min\limits_{1 \leqslant j \leqslant m} \gamma_{2j}^-}, \tag{2.6}$$

where i = 1,2. Let

$$C^0 := \{\varphi | \varphi \in C, E_{i2} < \varphi_i(t) < E_{i1}, \text{ for all } t \in [-r_i, 0], \ i = 1, 2\}.$$

Moreover, assume that $x(t; t_0, \varphi)$ is the solution of (1.5) with $\varphi \in C^0$. Then,

$$E_{i2} < x_i(t; t_0, \varphi) < E_{i1}, \text{ for all } t \in [t_0, \eta(\varphi)), \quad i = 1, 2 \tag{2.7}$$

and $\eta(\varphi) = +\infty$.

Proof. Set $x(t) = x(t;t_0,\varphi)$ for all $t \in [t_0, \eta(\varphi))$. Let $[t_0, T) \subseteq [t_0, \eta(\varphi))$ be an interval such that

$$0 < x_i(t) \text{ for all } t \in [t_0, T), \quad i = 1, 2, \tag{2.8}$$

we claim that

$$0 < x_i(t) < E_{i1} \text{ for all } t \in [t_0, T), \quad i = 1, 2. \tag{2.9}$$

Assume, by way of contradiction, that (2.9) does not hold. Then, one of the following cases must occur.

Case i: There exists $t_1 \in (t_0, T)$ such that

$$x_1(t_1) = E_{11} \text{ and } 0 < x_i(t) < E_{i1} \text{ for all } t \in [t_0 - r_i, t_1), \quad i = 1, 2. \tag{2.10}$$

Case ii: There exists $t_2 \in (t_0, T)$ such that

$$x_2(t_2) = E_{21} \text{ and } 0 < x_i(t) < E_{i1} \text{ for all } t \in [t_0 - r_i, t_2), \quad i = 1, 2. \tag{2.11}$$

If **Case i** holds, calculating the derivative of $x_1(t)$, together with (2.4) and the fact that $\sup_{u \geqslant 0} ue^{-u} = \frac{1}{e}$, (1.5) and (2.10) imply that

$$\begin{aligned}0 \leqslant x_1'(t_1) &= -\alpha_1(t_1)x_1(t_1) + \beta_1(t_1)x_2(t_1) + \sum_{j=1}^{m} c_{1j}(t_1)x_1(t_1 - \tau_{1j}(t_1))e^{-\gamma_{1j}(t_1)x_1(t_1-\tau_{1j}(t_1))} - H_1(t_1)x_1(t_1 - \sigma_1(t_1)) \\ &\leqslant -\alpha_1^- x_1(t_1) + \beta_1^+ E_{21} + \sum_{j=1}^{m} \frac{c_{1j}^+}{\gamma_{1j}^-}\frac{1}{e} = \alpha_1^-\left(-E_{11} + \frac{\beta_1^+ E_{21}}{\alpha_1^-} + \sum_{j=1}^{m} \frac{c_{1j}^+}{\alpha_1^-\gamma_{1j}^-}\frac{1}{e}\right) < 0,\end{aligned}$$

which is a contradiction and implies that (2.9) holds.

If **Case ii** holds, calculating the derivative of $x_2(t)$, together with (2.4) and the fact that $\sup_{u \geqslant 0} ue^{-u} = \frac{1}{e}$, (1.5) and (2.11) imply that

$$\begin{aligned}0 \leqslant x_2'(t_2) &= -\alpha_2(t_2)x_2(t_2) + \beta_2(t_2)x_1(t_2) + \sum_{j=1}^{m} c_{2j}(t_2)x_2(t_2 - \tau_{2j}(t_2))e^{-\gamma_{2j}(t_2)x_2(t_2-\tau_{2j}(t_2))} - H_2(t_2)x_2(t_2 - \sigma_2(t_2)) \\ &\leqslant -\alpha_2^- x_2(t_2) + \beta_2^+ E_{11} + \sum_{j=1}^{m} \frac{c_{2j}^+}{\gamma_{2j}^-}\frac{1}{e} = \alpha_2^-\left(-E_{21} + \frac{\beta_2^+ E_{11}}{\alpha_2^-} + \sum_{j=1}^{m} \frac{c_{2j}^+}{\alpha_2^-\gamma_{2j}^-}\frac{1}{e}\right) < 0,\end{aligned}$$

which is a contradiction and implies that (2.9) holds.

We next show that

$$x_i(t) > E_{i2}, \text{ for all } t \in (t_0, \eta(\varphi)), \quad i = 1, 2. \tag{2.12}$$

Suppose, for the sake of contradiction, that (2.12) does not hold. Then, one of the following cases must occur.

Case I: There exists $t_3 \in (t_0, \eta(\varphi))$ such that

$$x_1(t_3) = E_{12} \quad \text{and } x_i(t) > E_{i2} \text{ for all } t \in [t_0 - r_i, t_3), \quad i = 1, 2. \tag{2.13}$$

Case II: There exists $t_4 \in (t_0, \eta(\varphi))$ such that

$$x_2(t_4) = E_{22} \text{ and } x_i(t) > E_{i2} \text{ for all } t \in [t_0 - r_i, t_4), \quad i = 1, 2. \tag{2.14}$$

If **Case I** holds. Then, from (2.5), (2.6), (2.9) and (2.13), we get

$$E_{i2} < x_i(t) < E_{i1}, \quad \gamma_{ij}^+ x_i(t) \geqslant \gamma_{ij}^+ E_{i2} \geqslant \gamma_{ij}^+ \frac{1}{\min\limits_{1 \leqslant j \leqslant m} \gamma_{ij}^-} \geqslant 1, \tag{2.15}$$

for all $t \in [t_0 - r_i, t_3)$, $i = 1, 2$, $j = 1, 2, \ldots, m$. Calculating the derivative of $x_1(t)$, together with (2.5) and the fact that $\min_{1 \leqslant u \leqslant \kappa} ue^{-u} = \kappa e^{-\kappa}$, (1.5), (2.13) and (2.15) imply that

$$\begin{aligned}0 \geqslant x_1'(t_3) &= -\alpha_1(t_3)x_1(t_3) + \beta_1(t_3)x_2(t_3) + \sum_{j=1}^{m} c_{1j}(t_3)x_1(t_3 - \tau_{1j}(t_3))e^{-\gamma_{1j}(t_3)x_1(t_3-\tau_{1j}(t_3))} - H_1(t_3)x_1(t_3 - \sigma_1(t_3)) \\ &= -\alpha_1(t_3)x_1(t_3) + \beta_1(t_3)x_2(t_3) + \sum_{j=1}^{m} \frac{c_{1j}(t_3)}{\gamma_{1j}^+}\gamma_{1j}^+ x_1(t_3 - \tau_{1j}(t_3))e^{-\gamma_{1j}(t_3)x_1(t_3-\tau_{1j}(t_3))} - H_1(t_3)x_1(t_3 - \sigma_1(t_3))\end{aligned}$$

$$\geqslant -\alpha_1(t_3)x_1(t_3) + \beta_1^- E_{22} + \sum_{j=1}^{m} \frac{c_{1j}^-}{\gamma_{1j}^+} \gamma_{1j}^+ x_1(t_3 - \tau_{1j}(t_3)) e^{-\gamma_{1j}^+ x_1(t_3 - \tau_{1j}(t_3))} - H_1^+ E_{11}$$

$$\geqslant -\alpha_1^+ x_1(t_3) + \beta_1^- E_{22} + \sum_{j=1}^{m} \frac{c_{1j}^-}{\gamma_{1j}^+} \gamma_{1j}^+ E_{11} e^{-\gamma_{1j}^+ E_{11}} - H_1^+ E_{11} = \alpha_1^+ \left(-E_{12} + \frac{\beta_1^- E_{22}}{\alpha_1^+} + \sum_{j=1}^{m} \frac{c_{1j}^-}{\alpha_1^+} E_{11} e^{-\gamma_{1j}^+ E_{11}} - \frac{H_1^+ E_{11}}{\alpha_1^+} \right) > 0,$$

which is a contradiction and implies that (2.12) holds.

If **Case II** holds, we can show that (2.15) holds for all $t \in [t_0 - r_i, t_4)$, $i = 1, 2, j = 1, 2, \ldots, m$. Calculating the derivative of $x_2(t)$, together with (2.6) and the fact that $\min_{1 \leqslant u \leqslant \kappa} ue^{-u} = \kappa e^{-\kappa}$, (1.5), (2.14) and (2.15) imply that

$$0 \geqslant x_2'(t_4) = -\alpha_2(t_4)x_2(t_4) + \beta_2(t_4)x_1(t_4) + \sum_{j=1}^{m} c_{2j}(t_4) x_2(t_4 - \tau_{2j}(t_4)) e^{-\gamma_{2j}(t_4) x_2(t_4 - \tau_{2j}(t_4))} - H_2(t_4) x_2(t_4 - \sigma_2(t_4))$$

$$= -\alpha_2(t_4)x_2(t_4) + \beta_2(t_4)x_1(t_4) + \sum_{j=1}^{m} \frac{c_{2j}(t_4)}{\gamma_{2j}^+} \gamma_{2j}^+ x_2(t_4 - \tau_{2j}(t_4)) e^{-\gamma_{2j}(t_4) x_2(t_4 - \tau_{2j}(t_4))} - H_2(t_4) x_2(t_4 - \sigma_2(t_4))$$

$$\geqslant -\alpha_2(t_4)x_2(t_4) + \beta_2^- E_{12} + \sum_{j=1}^{m} \frac{c_{2j}^-}{\gamma_{2j}^+} \gamma_{2j}^+ x_2(t_4 - \tau_{2j}(t_4)) e^{-\gamma_{2j}^+ x_2(t_4 - \tau_{2j}(t_4))} - H_2^+ E_{21}$$

$$\geqslant -\alpha_2^+ x_2(t_4) + \beta_2^- E_{12} + \sum_{j=1}^{m} \frac{c_{2j}^-}{\gamma_{2j}^+} \gamma_{2j}^+ E_{21} e^{-\gamma_{2j}^+ E_{21}} - H_2^+ E_{21} = \alpha_2^+ \left(-E_{22} + \frac{\beta_2^- E_{12}}{\alpha_2^+} + \sum_{j=1}^{m} \frac{c_{2j}^-}{\alpha_2^+} E_{21} e^{-\gamma_{2j}^+ E_{21}} - \frac{H_2^+ E_{21}}{\alpha_2^+} \right) > 0,$$

which is a contradiction and implies that (2.12) holds.

It follows from (2.9) and (2.12) that (2.7) is true. From Theorem 2.3.1 in [6], we easily obtain $\eta(\varphi) = +\infty$. This ends the proof of Lemma 2.3. □

3. Main results

Theorem 3.1. *Let (2.4)–(2.6) hold. Moreover, suppose that*

$$\max\left\{ \frac{\beta_1^+}{\alpha_1^-} + \sum_{j=1}^{m} \frac{c_{1j}^+}{\alpha_1^- e^2} + \frac{H_1^+}{\alpha_1^-}, \ \frac{\beta_2^+}{\alpha_2^-} + \sum_{j=1}^{m} \frac{c_{2j}^+}{\alpha_2^- e^2} + \frac{H_2^+}{\alpha_2^-} \right\} < 1. \tag{3.1}$$

Then, there exists a unique positive almost periodic solution of system (1.5) in the region $B^* = \{\varphi | \varphi \in B, E_{i2} \leqslant \varphi_i(t) \leqslant E_{i1}$, for all $t \in R^1$, $i = 1, 2\}$.

Proof. For any $\phi \in B$, we consider an auxiliary system

$$\begin{cases} x_1'(t) = & -\alpha_1(t)x_1(t) + \beta_1(t)\phi_2(t) + \sum_{j=1}^{m} c_{1j}(t)\phi_1(t - \tau_{1j}(t)) e^{-\gamma_{1j}(t)\phi_1(t - \tau_{1j}(t))} \\ & -H_1(t)\phi_1(t - \sigma_1(t)), \\ x_2'(t) = & -\alpha_2(t)x_2(t) + \beta_2(t)\phi_1(t) + \sum_{j=1}^{m} c_{2j}(t)\phi_2(t - \tau_{2j}(t)) e^{-\gamma_{2j}(t)\phi(t - \tau_{2j}(t))} \\ & -H_2(t)\phi_2(t - \sigma_2(t)), \end{cases} \tag{3.2}$$

Notice that $M[\alpha_i] > 0 (i = 1,2)$, it follows from Lemma 2.2 that the linear system

$$\begin{cases} x_1'(t) = -\alpha_1(t)x_1(t), \\ x_2'(t) = -\alpha_2(t)x_2(t), \end{cases} \tag{3.3}$$

admits an exponential dichotomy on R. Thus, by Lemma 2.1, we obtain that the system (3.2) has exactly one almost periodic solution:

$$x^{\phi}(t) = \left(x_1^{\phi}(t), x_2^{\phi}(t)\right) = \left(\int_{-\infty}^{t} e^{-\int_s^t \alpha_1(u)du} \left(\beta_1(s)\phi_2(s) + \sum_{j=1}^{m} c_{1j}(s)\phi_1(s - \tau_{1j}(s)) e^{-\gamma_{1j}(s)\phi_1(s - \tau_{1j}(s))} - H_1(s)\phi_1(s - \sigma_1(s)) \right) ds, \right.$$
$$\left. \int_{-\infty}^{t} e^{-\int_s^t \alpha_2(u)du} \left(\beta_2(s)\phi_1(s) + \sum_{j=1}^{m} c_{2j}(s)\phi_2(s - \tau_{2j}(s)) e^{-\gamma_{2j}(s)\phi_2(s - \tau_{2j}(s))} - H_2(s)\phi_2(s - \sigma_2(s)) \right) ds \right). \tag{3.4}$$

Define a mapping $T : B \to B$ by setting

$$T(\phi(t)) = x^{\phi}(t), \quad \forall \phi \in B.$$

Since $B^* = \{\varphi | \varphi \in B, E_{i2} \leqslant \varphi_i(t) \leqslant E_{i1}$, for all $t \in R^1$, $i = 1,2\}$, it is easy to see that B^* is a closed subset of B. For any $\varphi \in B^*$, from (2.4) and (3.4) and the fact that $\sup_{u \geqslant 0} ue^{-u} = \frac{1}{e}$, we have

$$\begin{aligned} x^{\phi}(t) &\leqslant \left(\int_{-\infty}^{t} e^{-\int_s^t \alpha_1(u)du} \left(\beta_1^+ E_{21} + \sum_{j=1}^{m} \frac{1}{\gamma_{1j}(s)e} c_{1j}(s) \right) ds, \int_{-\infty}^{t} e^{-\int_s^t \alpha_2(u)du} \left(\beta_2^+ E_{11} + \sum_{j=1}^{m} \frac{1}{\gamma_{2j}(s)e} c_{2j}(s) \right) ds \right) \\ &\leqslant \left(\frac{\beta_1^+ E_{21}}{\alpha_1^-} + \sum_{j=1}^{m} \frac{c_{1j}^+}{\alpha_1^- \gamma_{1j}^- e}, \frac{\beta_2^+ E_{11}}{\alpha_2^-} + \sum_{j=1}^{m} \frac{c_{2j}^+}{\alpha_2^- \gamma_{2j}^- e} \right) < (E_{11}, E_{21}), \quad \text{for all } t \in R^1. \end{aligned} \tag{3.5}$$

In view of the fact that $\min_{1 \leqslant u \leqslant \kappa} ue^{-u} = \kappa e^{-\kappa}$, from (2.5), (2.6) and (3.4), we obtain

$$\begin{aligned} x^{\phi}(t) &\geqslant \left(\int_{-\infty}^{t} e^{-\int_s^t \alpha_1(u)du} \left(\beta_1^- E_{22} + \sum_{j=1}^{m} c_{1j}(s) \frac{1}{\gamma_{1j}^+} \gamma_{1j}^+ \phi_1(s - \tau_{1j}(s)) e^{-\gamma_{1j}^+ \phi_1(s-\tau_{1j}(s))} - H_1(s)\phi_1(s - \sigma_1(s)) \right) ds, \right. \\ &\quad \left. \int_{-\infty}^{t} e^{-\int_s^t \alpha_2(u)du} \left(\beta_2^- E_{12} + \sum_{j=1}^{m} c_{2j}(s) \frac{1}{\gamma_{2j}^+} \gamma_{2j}^+ \phi_2(s - \tau_{2j}(s)) e^{-\gamma_{2j}^+ \phi_2(s-\tau_{2j}(s))} - H_2(s)\phi_2(s - \sigma_2(s)) \right) ds \right) \\ &\geqslant \left(\frac{\beta_1^- E_{22}}{\alpha_1^+} + \sum_{j=1}^{m} \frac{c_{1j}^-}{\alpha_1^+} E_{11} e^{-\gamma_{1j}^+ E_{11}} - \frac{H_1^+ E_{11}}{\alpha_1^+}, frac\beta_2^- E_{12}\alpha_2^+ + \sum_{j=1}^{m} \frac{c_{2j}^-}{\alpha_2^+} E_{21} e^{-\gamma_{2j}^+ E_{21}} - \frac{H_2^+ E_{21}}{\alpha_2^+} \right) > (E_{12}, E_{22}), \quad \text{for all } t \in R^1. \end{aligned} \tag{3.6}$$

This implies that the mapping T is a self-mapping from B^* to B^*. Now, we prove that the mapping T is a contraction mapping on B^*. In fact, for φ, $\psi \in B^*$, we get

$$\begin{aligned} \left(\sup_{t \in R} |(T(\varphi)(t) - T(\psi)(t))_1|, \sup_{t \in R} |(T(\varphi)(t) - T(\psi)(t))_2| \right) &= \left(\sup_{t \in R} \left| \int_{-\infty}^{t} e^{-\int_s^t \alpha_1(u)du} (\beta_1(s)(\varphi_2(s) - \psi_2(s)) \right. \right. \\ &\quad + \sum_{j=1}^{m} c_{1j}(s)(\varphi_1(s - \tau_{1j}(s)) e^{-\gamma_{1j}(s)\varphi_1(s-\tau_{1j}(s))} \\ &\quad - \psi_1(s - \tau_{1j}(s)) e^{-\gamma_{1j}(s)\psi_1(s-\tau_{1j}(s))}) - H_1(s)(\varphi_1(s - \sigma_1(s)) \\ &\quad - \psi_1(s - \sigma_1(s))))ds|, \sup_{t \in R} \left| \int_{-\infty}^{t} e^{-\int_s^t \alpha_2(u)du} (\beta_2(s)(\varphi_1(s) - \psi_1(s)) \right. \\ &\quad + \sum_{j=1}^{m} c_{2j}(s)(\varphi_2(s - \tau_{2j}(s)) e^{-\gamma_{2j}(s)\varphi_2(s-\tau_{2j}(s))} \\ &\quad - \psi_2(s - \tau_{2j}(s)) e^{-\gamma_{2j}(s)\psi_2(s-\tau_{2j}(s))}) - H_2(s)(\varphi_2(s - \sigma_2(s)) \\ &\quad - \psi_2(s - \sigma_2(s))))ds|) \\ &= \left(\sup_{t \in R} \left| \int_{-\infty}^{t} e^{-\int_s^t \alpha_1(u)du} (\beta_1(s)(\varphi_2(s) - \psi_2(s)) \right. \right. \\ &\quad + \sum_{j=1}^{m} \frac{c_{1j}(s)}{\gamma_{1j}(s)} (\gamma_{1j}(s)\varphi_1(s - \tau_{1j}(s)) e^{-\gamma_{1j}(s)\varphi_1(s-\tau_{1j}(s))} \\ &\quad - \gamma_{1j}(s)\psi_1(s - \tau_{1j}(s)) e^{-\gamma_{1j}(s)\psi_1(s-\tau_{1j}(s))}) - H_1(s)(\varphi_1(s - \sigma_1(s)) \\ &\quad - \psi_1(s - \sigma_1(s))))ds|, sup_{t \in R}| \int_{-\infty}^{t} e^{-\int_s^t \alpha_2(u)du} (\beta_2(s)(\varphi_1(s) \\ &\quad - \psi_1(s)) + \sum_{j=1}^{m} \frac{c_{2j}(s)}{\gamma_{2j}(s)} (\gamma_{2j}(s)\varphi_2(s - \tau_{2j}(s)) e^{-\gamma_{2j}(s)\varphi_2(s-\tau_{2j}(s))} \\ &\quad - \gamma_{2j}(s)\psi_2(s - \tau_{2j}(s)) e^{-\gamma_{2j}(s)\psi_2(s-\tau_{2j}(s))}) - H_2(s)(\varphi_2(s - \sigma_2(s)) \\ &\quad - \psi_2(s - \sigma_2(s))))ds|). \end{aligned} \tag{3.7}$$

In view of (1.5), (2.5), (2.6), (3.5), (3.6) and (3.7), from $\sup_{u \geqslant 1} |\frac{1-u}{e^u}| = \frac{1}{e^2}$ and the inequality

$$|xe^{-x} - ye^{-y}| = \left| \frac{1 - (x + \theta(y - x))}{e^{x+\theta(y-x)}} \right| |x - y| \leqslant \frac{1}{e^2} |x - y| \quad \text{where } x, y \in [1, +\infty), \quad 0 < \theta < 1, \tag{3.8}$$

we have

$$
\begin{aligned}
&\left(\sup_{t\in R}|(T(\varphi)(t)-T(\psi)(t))_1|,\sup_{t\in R}|(T(\varphi)(t)-T(\psi)(t))_2|\right)\\
&\leqslant\Bigg(\frac{\beta_1^+}{\alpha_1^-}\|\varphi-\psi\|_B+\sup_{t\in R}\int_{-\infty}^t e^{-\int_s^t\alpha_1(u)du}\sum_{j=1}^m c_{1j}^+\frac{1}{e^2}|\varphi_1(s-\tau_{1j}(s))-\psi_1(s-\tau_{1j}(s))|ds+\frac{H_1^+}{\alpha_1^-}\|\varphi-\psi\|_B,\frac{\beta_2^+}{\alpha_2^-}\|\varphi-\psi\|_B\\
&\quad+\sup_{t\in R}\int_{-\infty}^t e^{-\int_s^t\alpha_2(u)du}\sum_{j=1}^m c_{2j}^+\frac{1}{e^2}|\varphi_2(s-\tau_{2j}(s))-\psi_2(s-\tau_{2j}(s))|ds+\frac{H_2^+}{\alpha_2^-}\|\varphi-\psi\|_B\Bigg)\\
&\leqslant\left(\left(\frac{\beta_1^+}{\alpha_1^-}+\sum_{j=1}^m\frac{c_{1j}^+}{\alpha_1^-e^2}+\frac{H_1^+}{\alpha_1^-}\right)\|\varphi-\psi\|_B,\left(\frac{\beta_2^+}{\alpha_2^-}+\sum_{j=1}^m\frac{c_{2j}^+}{\alpha_2^-e^2}+\frac{H_2^+}{\alpha_2^-}\right)\|\varphi-\psi\|_B\right).
\end{aligned}\tag{3.9}
$$

Hence

$$
\|T(\varphi)-T(\psi)\|_B\leqslant\max\left\{\frac{\beta_1^+}{\alpha_1^-}+\sum_{j=1}^m\frac{c_{1j}^+}{\alpha_1^-e^2}+\frac{H_1^+}{\alpha_1^-},\frac{\beta_2^+}{\alpha_2^-}+\sum_{j=1}^m\frac{c_{2j}^+}{\alpha_2^-e^2}+\frac{H_2^+}{\alpha_2^-}\right\}\|\varphi-\psi\|_B.
$$

Noting that

$$
\max\left\{\frac{\beta_1^+}{\alpha_1^-}+\sum_{j=1}^m\frac{c_{1j}^+}{\alpha_1^-e^2}+\frac{H_1^+}{\alpha_1^-},\frac{\beta_2^+}{\alpha_2^-}+\sum_{j=1}^m\frac{c_{2j}^+}{\alpha_2^-e^2}+\frac{H_2^+}{\alpha_2^-}\right\}<1,
$$

it is clear that the mapping T is a contraction on B^*. Using Theorem 0.3.1 of [7], we obtain that the mapping T possesses a unique fixed point $\varphi^*\in B^*$, $T\varphi^*=\varphi^*$. By (3.2), φ^* satisfies (1.5). So φ^* is an almost periodic solution of (1.5) in B^*. The proof of Theorem 3.1 is now complete. □

Theorem 3.2. *Let $x^*(t)$ be the positive almost periodic solution of Eq. (1.5) in the region B^*. Suppose that (2.4)–(2.6)* and *(3.1) hold. Then, the solution $x(t;t_0,\varphi)$ of (1.5) with $\varphi\in C^0$ converges exponentially to $x^*(t)$ as $t\to+\infty$.*

Proof. Set $x(t)=x(t;t_0,\varphi)$ and $y_i(t)=x_i(t)-x_i^*(t)$, where $t\in[t_0-r_i,+\infty)$, $i=1,2$. Then

$$
\begin{cases}
y_1'(t)= & -\alpha_1(t)y_1(t)+\beta_1(t)y_2(t)+\sum_{j=1}^m c_{1j}(t)(x_1(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)x_1(t-\tau_{1j}(t))}\\
& -x_1^*(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)x_1^*(t-\tau_{1j}(t))})-H_1(t)y_1(t-\sigma_1(t))\\
y_2'(t)= & -\alpha_2(t)y_2(t)+\beta_2(t)y_1(t)+\sum_{j=1}^m c_{2j}(t)(x_2(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)x_2(t-\tau_{2j}(t))}\\
& -x_2^*(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)x_2^*(t-\tau_{2j}(t))})-H_2(t)y_2(t-\sigma_2(t)),
\end{cases}\tag{3.10}
$$

Set

$$
\Gamma_i(u)=-\left(\alpha_i^--u\right)+\beta_i^++\sum_{j=1}^m c_{ij}^+\frac{1}{e^2}e^{ur_i}+H_i^+e^{ur_i},\quad u\in[0,1],\quad i=1,2.\tag{3.11}
$$

Clearly, $\Gamma_i(u)$, $i=1,2$, are continuous functions on $[0,1]$. In view of (3.1), we obtain

$$
\Gamma_i(0)=-\alpha_i^-+\beta_i^++\sum_{j=1}^m c_{ij}^+\frac{1}{e^2}+H_i^+<0,\quad i=1,2,
$$

we can choose two constants $\eta>0$ and $\lambda\in(0,1]$ such that

$$
\Gamma_i(\lambda)=\left(\lambda-\alpha_i^-\right)+\beta_i^++\sum_{j=1}^m c_{ij}^+\frac{1}{e^2}e^{\lambda r_i}+H_i^+e^{\lambda r_i}<-\eta<0,\quad i=1,2.\tag{3.12}
$$

We consider the Lyapunov functional

$$
V_1(t)=|y_1(t)|e^{\lambda t},\quad V_2(t)=|y_2(t)|e^{\lambda t}.\tag{3.13}
$$

Calculating the upper right derivative of $V_i(t)(i=1,2)$ along the solution $y(t)$ of (3.10), we have

$$
\begin{aligned}
D^+(V_1(t))&\leqslant-\alpha_1(t)|y_1(t)|e^{\lambda t}+\beta_1(t)|y_2(t)|e^{\lambda t}+\sum_{j=1}^m c_{1j}(t)|x_1(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)x_1(t-\tau_{1j}(t))}\\
&\quad-x_1^*(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)x_1^*(t-\tau_{1j}(t))}|e^{\lambda t}+H_1(t)|y_1(t-\sigma_1(t))|e^{\lambda t}+\lambda|y_1(t)|e^{\lambda t}\\
&=\Bigg[(\lambda-\alpha_1(t))|y_1(t)|+\beta_1(t)|y_2(t)|+\sum_{j=1}^m c_{1j}(t)|x_1(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)x_1(t-\tau_{1j}(t))}\\
&\quad-x_1^*(t-\tau_{1j}(t))e^{-\gamma_{1j}(t)x_1^*(t-\tau_{1j}(t))}|+H_1(t)|y_1(t-\sigma_1(t))|\Bigg]e^{\lambda t},\quad\text{for all } t>t_0,
\end{aligned}\tag{3.14}
$$

and

$$D^+(V_2(t)) \leqslant -\alpha_2(t)|y_2(t)|e^{\lambda t} + \beta_2(t)|y_1(t)|e^{\lambda t} + \sum_{j=1}^{m} c_{2j}(t)|x_2(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)x_2(t-\tau_{2j}(t))}$$
$$- x_2^*(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)x_2^*(t-\tau_{2j}(t))}|e^{\lambda t} + H_2(t)|y_2(t-\sigma_2(t))|e^{\lambda t} + \lambda|y_2(t)|e^{\lambda t}$$
$$= \Bigg[(\lambda - \alpha_2(t))|y_2(t)| + \beta_2(t)|y_1(t)| + \sum_{j=1}^{m} c_{2j}(t)|x_2(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)x_2(t-\tau_{2j}(t))}$$
$$-x_2^*(t-\tau_{2j}(t))e^{-\gamma_{2j}(t)x_2^*(t-\tau_{2j}(t))}| + H_2(t)|y_1(t-\sigma_2(t))|\Bigg]e^{\lambda t}, \quad \text{for all } t > t_0. \tag{3.15}$$

Let $\max_{i=1,2}\{e^{\lambda t_0}(\max_{t\in[t_0-r_i,t_0]}|\varphi_i(t) - x_i^*(t)| + 1)\} := M$. We claim that

$$V_i(t) = |y_i(t)|e^{\lambda t} < M \quad \text{for all } t > t_0, \quad i = 1,2. \tag{3.16}$$

Otherwise, one of the following cases must occur.

Case 1: There exists $T_1 > t_0$ such that

$$V_1(T_1) = M \quad \text{and } V_i(t) < M \quad \text{for all } t \in [t_0 - r_i, T_1), \quad i = 1,2. \tag{3.17}$$

Case 2: There exists $T_2 > t_0$ such that

$$V_2(T_2) = M \quad \text{and } V_i(t) < M \quad \text{for all } t \in [t_0 - r_i, T_2), \quad i = 1,2. \tag{3.18}$$

If **Case 1** holds, together with (2.7), (3.8) and (3.14), (3.17) implies that

$$0 \leqslant D^+(V_1(T_1) - M) = D^+(V_1(T_1)) \leqslant [(\lambda - \alpha_1(T_1))|y_1(T_1)| + \beta_1(T_1)|y_2(T_1)|$$
$$+ \sum_{j=1}^{m} c_{1j}(T_1)|x_1(T_1-\tau_{1j}(T_1))e^{-\gamma_{1j}(T_1)x_1(T_1-\tau_{1j}(T_1))} - x_1^*(T_1-\tau_{1j}(T_1))e^{-\gamma_{1j}(T_1)x_1^*(T_1-\tau_{1j}(T_1))}| + H_1(T_1)|y_1(T_1-\sigma_1(T_1))|\Bigg]e^{\lambda T_1}$$
$$= \Bigg[(\lambda - \alpha_1(T_1))|y_1(T_1)| + \beta_1(T_1)|y_2(T_1)| + \sum_{j=1}^{m} \frac{c_{1j}(T_1)}{\gamma_{1j}(T_1)}|\gamma_{1j}(T_1)x_1(T_1-\tau_{1j}(T_1))e^{-\gamma_{1j}(T_1)x(T_1-\tau_{1j}(T_1))}$$
$$-\gamma_{1j}(T_1)x_1^*(T_1-\tau_{1j}(T_1))e^{-\gamma_{1j}(T_1)x^*(T_1-\tau_{1j}(T_1))}| + H_1(T_1)|y_1(T_1-\sigma_1(T_1))|\Bigg]e^{\lambda T_1}$$
$$\leqslant (\lambda - \alpha_1(T_1))|y_1(T_1)|e^{\lambda T_1} + \beta_1(T_1)|y_2(T_1)|e^{\lambda T_1} + \sum_{j=1}^{m} c_{1j}(T_1)\frac{1}{e^2}|y_1(T_1-\tau_{1j}(T_1))|e^{\lambda(T_1-\tau_{1j}(T_1))}e^{\lambda\tau_{1j}(T_1)}$$
$$+H_1(T_1)|y_1(T_1-\sigma_1(T_1))|e^{\lambda(T_1-\sigma_1(T_1))}e^{\lambda\sigma_1(T_1)} \leqslant \Bigg[(\lambda - \alpha_1^-) + \beta_1^+ + \sum_{j=1}^{m} c_{1j}^+\frac{1}{e^2}e^{\lambda r_1} + H_1^+e^{\lambda r_1}\Bigg]M. \tag{3.19}$$

Thus,

$$0 \leqslant (\lambda - \alpha_1^-) + \beta_1^+ + \sum_{j=1}^{m} c_{1j}^+\frac{1}{e^2}e^{\lambda r_1} + H_1^+e^{\lambda r_1},$$

which contradicts with (3.12). Hence, (3.16) holds.

If **Case 2** holds, together with (2.7), (3.8) and (3.15), (3.18) implies that

$$0 \leqslant D^+(V_2(T_2) - M) = D^+(V_2(T_2)) \leqslant [(\lambda - \alpha_2(T_2))|y_2(T_2)| + \beta_2(T_2)|y_1(T_2)|$$
$$+ \sum_{j=1}^{m} c_{2j}(T_2)|x_2(T_2-\tau_{2j}(T_2))e^{-\gamma_{2j}(T_2)x_2(T_2-\tau_{2j}(T_2))} - x_2^*(T_2-\tau_{2j}(T_2))e^{-\gamma_{2j}(T_2)x^*(T_2-\tau_{2j}(T_2))}| + H_2(T_2)|y_2(T_2-\sigma_2(T_2))|\Bigg]e^{\lambda T_2}$$
$$= \Bigg[(\lambda - \alpha_2(T_2))|y_2(T_2)| + \beta_2(T_2)|y_1(T_2)| + \sum_{j=1}^{m} \frac{c_{2j}(T_2)}{\gamma_{2j}(T_2)}|\gamma_{2j}(T_2)x_2(T_2-\tau_{2j}(T_2))e^{-\gamma_{2j}(T_2)x_2(T_2-\tau_{2j}(T_2))}$$
$$-\gamma_{2j}(T_2)x_2^*(T_2-\tau_{2j}(T_2))e^{-\gamma_{2j}(T_2)x_2^*(T_2-\tau_{2j}(T_2))}| + H_2(T_2)|y_2(T_2-\sigma_2(T_2))|\Bigg]e^{\lambda T_2}$$
$$\leqslant (\lambda - \alpha_2(T_2))|y_2(T_2)|e^{\lambda T_2} + \beta_2(T_2)|y_1(T_2)|e^{\lambda T_2} + \sum_{j=1}^{m} c_{2j}(T_2)\frac{1}{e^2}|y_2(T_2-\tau_{2j}(T_2))|e^{\lambda(T_2-\tau_{2j}(T_2))}e^{\lambda\tau_{2j}(T_2)}$$
$$+H_2(T_2)|y_2(T_2-\sigma_2(T_2))|e^{\lambda(T_2-\sigma_2(T_2))}e^{\lambda\sigma_2(T_2)} \leqslant \Bigg[(\lambda - \alpha_2^-) + \beta_2^+ + \sum_{j=1}^{m} c_{2j}^+\frac{1}{e^2}e^{\lambda r_2} + H_2^+e^{\lambda r_2}\Bigg]M. \tag{3.20}$$

Thus,

$$0 \leqslant \left(\lambda - \alpha_2^-\right) + \beta_2^+ + \sum_{j=1}^{m} c_{2j}^+ \frac{1}{e^2} e^{\lambda r_2} + H_2^+ e^{\lambda r_2},$$

which contradicts with (3.12). Hence, (3.16) holds. It follows that

$$|y_i(t)| < Me^{-\lambda t} \quad \text{for all } t > t_0,\ i = 1, 2. \tag{3.21}$$

This completes the proof. □

4. Example and remark

In this section, we give an example to demonstrate the results obtained in previous sections.

Example 4.1. Consider the following Nicholson-type delay system with linear harvesting terms:

$$\begin{cases} x_1'(t) = & -(18 + \cos^2 t)x_1(t) + \left(0.00001 + 0.000005 \sin^2 t\right) e^{e-3} x_2(t) \\ & + e^{e-1}\left(9.5 + 0.005|\sin\sqrt{2}t|\right) x_1\left(t - e^{2|\sin t|}\right) e^{-x_1\left(t - e^{2|\sin t|}\right)}, \\ & + e^{e-1}\left(9.5 + 0.005|\sin\sqrt{5}t|\right) x_1\left(t - e^{2|\cos\sqrt{3}t|}\right) e^{-x_1\left(t - e^{2|\cos\sqrt{3}t|}\right)} \\ & - (0.000001 \cos^2 t) e^{e-3} x_1\left(t - e^{2|\cos\sqrt{3}t|}\right) \\ x_2'(t) = & -\left(18 + \sin^2 t\right) x_2(t) + (0.00001 + 0.000005 \cos^2 t) e^{e-3} x_1(t) \\ & + e^{e-1}\left(9.5 + 0.005|\cos\sqrt{2}t|\right) x_2\left(t - e^{2|\cos t|}\right) e^{-x_2\left(t - e^{2|\cos t|}\right)}, \\ & + e^{e-1}\left(9.5 + 0.005|\sin\sqrt{6}t|\right) x_2\left(t - e^{2|\cos\sqrt{7}t|}\right) e^{-x_2\left(t - e^{2|\cos\sqrt{7}t|}\right)} \\ & - (0.000001 \cos^4 t) e^{e-3} x_2\left(t - e^{2|\cos\sqrt{3}t|}\right), \end{cases} \tag{4.1}$$

Obviously, $\alpha_i^- = 18,\ \alpha_i^+ = 19,\ \gamma_{ij}^+ = \gamma_{ij}^- = 1, \beta_i^- = 0.00001e^{e-3},\ \beta_i^+ = 0.000015e^{e-3},\ c_{ij}^- = 9.5e^{e-1},\ c_{ij}^+ = 9.505e^{e-1}, H_i^+ = 0.000001e^{e-3},\ r_i = \max\left\{\max_{1 \leqslant j \leqslant m}\left\{\tau_{ij}^+\right\},\ \sigma_i^+\right\} = e^2,$

$$\frac{\beta_i^-}{\alpha_i^+} + \sum_{j=1}^{2} \frac{c_{ij}^- e^{1-\gamma_{ij}^+ e}}{\alpha_i^+} - \frac{H_i^+ e}{\alpha_i^+} = \frac{19 + 0.00001e^{e-3} - 0.000001e^{e-2}}{19} > 1, \tag{4.2}$$

$$\frac{\beta_i^+ e}{\alpha_i^-} + \sum_{j=1}^{2} \frac{c_{ij}^+}{\alpha_i^- \gamma_{ij}^-} \frac{1}{e} = \frac{\beta_i^+ e}{\alpha_i^-} + \sum_{j=1}^{2} \frac{c_{ij}^+}{\alpha_i^-} \frac{1}{e} = \frac{0.000015e^{e-2} + 19.01e^{e-2}}{18} < e \tag{4.3}$$

and

$$\max\left\{\frac{\beta_1^+}{\alpha_1^-} + \sum_{j=1}^{m} \frac{c_{1j}^+}{\alpha_1^- e^2} + \frac{H_1^+}{\alpha_1^-}, \frac{\beta_2^+}{\alpha_2^-} + \sum_{j=1}^{m} \frac{c_{2j}^+}{\alpha_2^- e^2} + \frac{H_2^+}{\alpha_2^-}\right\} = \frac{0.000016e^{e-2} + 19.01e^{e-2}}{18e} < 1. \tag{4.4}$$

where $i, j = 1, 2$. Let $E_{i1} = e$ and $E_{i2} = 1$ for $i = 1, 2$. Then, (4.2)–(4.4) imply that the Nicholson-type delay differential system (4.1) satisfies (2.4)–(2.6) and (3.1). Hence, from Theorems 3.1 and 3.2, system (4.1) has a positive almost periodic solution

$$x^*(t) \in B^* = \{\varphi | \varphi \in B, 1 \leqslant \varphi_i(t) \leqslant e, \quad \text{for all} \quad t \in R,\ i = 1, 2\}.$$

Moreover, if $\varphi \in C^0 = \{\varphi | \varphi \in C,\ 1 < \varphi_i(t) < e,\ \text{for all}\ t \in [-e^2, 0],\ i = 1, 2\}$, then $x(t; t_0, \varphi)$ converges exponentially to $x^*(t)$ as $t \to +\infty$.

Remark 4.1. To the best of our knowledge, few authors have considered the problems of positive almost periodic solution of Nicholson-type delay system with linear harvesting terms. Therefore, all the results in [1–3,8] and the references therein cannot be applicable to prove that all the solutions of (4.1) with initial value $\varphi \in C^0$ converge exponentially to the positive almost periodic solution. Moreover, if $H_1(t) \equiv H_2(t) \equiv 0$, we can find that the main results of [2] are special ones of Theorems 3.2 with $E_{i1} = e$ and $E_{i2} = 1$ for $i = 1, 2$. This implies that the results of this paper are new and they complement previously known results.

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