The Cabibbo angle in a supersymmetric D_{14} model

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Abstract

We construct a supersymmetric model with the flavor symmetry D_{14} in which the CKM matrix element $|V_{ud}|$ can take the value $|V_{ud}| = \cos(\frac{\pi}{14}) \approx 0.97493$ implying that the Cabibbo angle θ_C is $\sin(\theta_C) \approx |V_{us}| \approx \sin(\frac{\pi}{14}) \approx 0.2225$. These values are very close to those observed in experiments. The value of $|V_{ud}|$ (θ_C) is based on the fact that different Z_2 subgroups of D_{14} are conserved in the up and down quark sector. In order to achieve this, D_{14} is accompanied by a Z_3 symmetry. The spontaneous breaking of D_{14} is induced by flavons, which are scalar gauge singlets. The quark mass hierarchy is partly due to the flavor group D_{14} and partly due to a Froggatt–Nielsen symmetry $U(1)_{FN}$ under which only the right-handed quarks transform. The model is natural in the sense that the hierarchies among the quark masses and mixing angles are generated with the help of symmetries. The issue of the vacuum alignment of the flavons is solved up to a small number of degeneracies, leaving four different possible values for $|V_{ud}|$. Out of these, only one of them leads to a phenomenological viable model. A study of the Z_2 subgroup breaking terms shows that the results achieved in the symmetry limit are only slightly perturbed. At the same time they allow $|V_{ud}|$ (θ_C) to be well inside the small experimental error bars.
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1. Introduction

The explanation of the hierarchy among the charged fermion masses and of the peculiar fermion mixings, especially in the lepton sector, is one of the main issues in the field of model building. The prime candidate for the origin of fermion mass hierarchies and mixing patterns

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seems to be a flavor symmetry under which the three generations of Standard Model (SM) particles transform in a certain way. Unlike the majority of studies which concentrate on the leptonic sector we propose a dihedral group,[1] D_{14}, as flavor symmetry to predict the CKM matrix element $|V_{ud}|$ or equivalently the Cabibbo angle θ_C. The crucial aspect in this model is the fact that $|V_{ud}|$ is given in terms of group theoretical quantities, like the index n of the dihedral group D_n, the index j of the representation $\underline{2}_j$ under which two of the generations of the (left-handed) quarks transform and the indices $k_{u,d}$ of the subgroups which remain unbroken in the up, $Z_2 = \langle BA^{k_u} \rangle$, and the down quark sector, $Z_2 = \langle BA^{k_d} \rangle$. Thereby, A and B are the two generators of the dihedral group. The general formula for $|V_{ud}|$ is [2–4]

$$|V_{ud}| = \left| \cos\left(\frac{\pi(k_u - k_d)j}{n} \right) \right|. \tag{1}$$

In particular, the Cabibbo angle neither depends on arbitrarily tunable numbers, nor is it connected to the quark masses as is the case for the Gatto–Sartori–Tonin (GST) relation [5], $\sin(\theta_C) \approx |V_{us}| \approx \sqrt{m_d/m_s}$. The only dependence arises through the fact that the ordering of the mass eigenvalues determines which element in the CKM mixing matrix is fixed by the group theoretical quantities. However, since the hierarchy among the quark masses is also naturally accommodated in our model, partly by the flavor group D_{14} itself and partly by an additional Froggatt–Nielsen (FN) symmetry $U(1)_{FN}$ [6], this sort of arbitrariness in the determination of the Cabibbo angle is avoided.[2]

In this paper we consider as framework the Minimal Supersymmetric SM (MSSM). The construction used in our model is in several aspects analogous to the one used in [7] to generate tri-bimaximal mixing in the lepton sector with the help of the group A_4. The flavor group is broken at high energies through vacuum expectation values (VEVs) of gauge singlets, the flavons. The prediction of the actual value of the mixing angle originates from the fact that different subgroups of the flavor symmetry are conserved in different sectors (up and down quark sector) of the theory. The separation of these sectors can be maintained by an additional cyclic symmetry, which is Z_3 in our case. The other crucial aspect for preserving different subgroups is the achievement of a certain vacuum alignment. As in [7], an appropriate flavon superpotential can be constructed by introducing a $U(1)_R$ symmetry and adding a specific set of scalar fields, the driving fields, whose F-terms are responsible for aligning the flavon VEVs. As we show, the vacuum can be aligned such that in the up quark sector a Z_2 symmetry with an even index k_u is preserved, whereas in the down quark sector the residual Z_2 symmetry is generated by BA^{k_d} with k_d being an odd integer. Thus, two different Z_2 groups are maintained in the sectors. We can set $k_u = 0$ without loss of generality. However, we are unable to predict the exact value of k_d such that our model leads to four possible scenarios with four different possible values of $|V_{ud}|$. Out of these scenarios only one, namely $k_d = 1$ or $k_d = 13$, results in a phenomenologically viable value of $|V_{ud}|$ (and θ_C)[3]

$$|V_{ud}| = \cos\left(\frac{\pi}{14} \right) \approx 0.97493 \quad \text{and} \quad \sin(\theta_C) \approx |V_{us}| \approx \sin\left(\frac{\pi}{14} \right) \approx 0.2225. \tag{2}$$

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[1]   Dihedral symmetries have already been frequently used as flavor symmetries, see [1].

[2]   This cannot, for example, be avoided in the A_4 models [7], which successfully predict tri-bimaximal mixing in the lepton sector, since the hierarchy among the light neutrinos, which determines the ordering of the columns in the lepton mixing matrix, is very mild. Actually, a certain fine-tuning is necessary to achieve that the atmospheric mass squared difference is larger than the solar one.

[3]   Note that the result $\theta_C = \frac{\pi}{14}$ has been derived in another context in [8].

Similar to [7], subleading corrections to masses and mixings arise from higher-dimensional operators. In general they are at most of relative order $\epsilon \approx \lambda^2 \approx 0.04$, so that Eq. (2) holds within ± 0.04.

The model presented here surpasses the non-supersymmetric one constructed in [4] in several ways. Since the flavor symmetry is broken only spontaneously at the electroweak scale in the latter model, it contains several copies of the SM Higgs doublet. In contrast to this, the model which we discuss in the following possesses the two MSSM Higgs doublets h_u and h_d, which are neutral under the flavor group, and gauge singlets, the flavons and driving fields, which transform under flavor. The flavons are responsible for breaking the flavor symmetry. As a consequence, none of the problems usually present in models with an extended Higgs doublet sector, such as too low Higgs masses and large flavor changing neutral currents, is encountered here. Additionally, the problem of the vacuum alignment, which determines the value of the Cabibbo angle, is solved, up to a small number of degeneracies. This is impossible in the case of a multi-Higgs doublet model due to the large number of quartic couplings. Only a numerical fit can show that (at least) one set of parameters exists which leads to the desired vacuum structure. Finally, the breaking of the flavor group at high energies is also advantageous, because then domain walls generated through this breaking [9] might well be diluted in an inflationary era.

In the class of models [7] which extends the flavor group A_4, being successful in predicting tri-bimaximal mixing for the leptons, to the quark sector one usually observes that the Cabibbo angle $\theta_C \equiv \lambda \approx 0.22$ produced is generically only of the order of $\epsilon \approx \lambda^2$ and thus too small by a factor of four to five.[4] This observation might indicate that it is not possible to treat the Cabibbo angle only as a small perturbation in this class of models.

The paper is organized as follows: in Section 2 we repeat the necessary group theory of D_{14} and the properties of the subgroups relevant here. Section 3 contains an outline of the model in which the transformation properties of all particles under the flavor group are given. The quark masses and mixings, in the limit of conserved Z_2 subgroups in up and down quark sector, are presented in Section 4. In Section 5, corrections to the quark mass matrices are studied in detail and the results of Section 4 are shown to be only slightly changed. The flavon superpotential is discussed in Section 6. We summarize our results and give a short outlook in Section 7. Details of the group theory of D_{14} such as Kronecker products and Clebsch–Gordan coefficients can be found in Appendix A. In Appendix B the corrections to the flavon superpotential and the shifts of the flavon VEVs are given.

2. Group theory of D_{14}

In this section we briefly review the basic features of the dihedral group D_{14}. Its order is 28, and it has four one-dimensional irreducible representations which we denote as $\mathbf{1}_i$, $i = 1, \ldots, 4$ and six two-dimensional ones called $\mathbf{2}_j$, $j = 1, \ldots, 6$. All of them are real and the representations $\mathbf{2}_j$ with an odd index j are faithful. The group is generated by the two elements A and B which fulfill the relations [11]

$$A^{14} = \mathbb{1}, \qquad B^2 = \mathbb{1}, \qquad ABA = B. \tag{3}$$

The generators A and B of the one-dimensional representations read

$$\mathbf{1}_1: \quad A = 1, \ B = 1, \tag{4a}$$

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[4] This also happens in a recently proposed model using the flavor group S_4 [10].

$$\underline{1}_2: \quad A = 1, \ B = -1, \tag{4b}$$

$$\underline{1}_3: \quad A = -1, \ B = 1, \tag{4c}$$

$$\underline{1}_4: \quad A = -1, \ B = -1. \tag{4d}$$

For the representation $\underline{2}_j$ they are two-by-two matrices of the form

$$A = \begin{pmatrix} e^{(\frac{\pi i}{7})j} & 0 \\ 0 & e^{-(\frac{\pi i}{7})j} \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{5}$$

Note that we have chosen A to be complex, although all representations of D_{14} are real. Due to this, we find for $(a_1, a_2)^T$ forming the doublet $\underline{2}_j$ that the combination $(a_2^\star, a_1^\star)^T$ transforms as $\underline{2}_j$ rather than $(a_1^\star, a_2^\star)^T$. The Kronecker products and Clebsch–Gordan coefficients can be found in Appendix A and can also be deduced from the general formulae given in [2,12].

Since we derive the value of the element $|V_{ud}|$ (the Cabibbo angle θ_C), through a non-trivial breaking of D_{14} in the up and down quark sector, we briefly comment on the relevant type of Z_2 subgroups of D_{14}. These Z_2 groups are generated by an element of the form BA^k for k being an integer between 0 and 13. With the help of Eq. (3) one easily sees that $(BA^k)^2 = BA^k BA^k = BA^{k-1}BA^{k-1} = \cdots = B^2 = \mathbb{1}$. For k being even, singlets transforming as $\underline{1}_3$ are allowed to have a non-vanishing VEV, whereas k being odd only allows a non-trivial VEV for singlets which transform as $\underline{1}_4$ under D_{14}. Clearly, all singlets transforming in the trivial representation $\underline{1}_1$ of D_{14} are allowed to have a non-vanishing VEV. Note that however the fields in the representation $\underline{1}_2$ are not allowed a non-vanishing VEV, since $BA^k = -1$ for all possible values of k. In the case of two fields $\varphi_{1,2}$ which form a doublet $\underline{2}_j$ a Z_2 group generated by BA^k is preserved, if

$$\begin{pmatrix} \langle \varphi_1 \rangle \\ \langle \varphi_2 \rangle \end{pmatrix} \propto \begin{pmatrix} e^{-\frac{\pi i j k}{7}} \\ 1 \end{pmatrix}. \tag{6}$$

In order to see this note that the vector given in Eq. (6) is an eigenvector of the two-by-two matrix BA^k to the eigenvalue $+1$. Due to the fact that singlets transforming as $\underline{1}_3$ can only preserve Z_2 subgroups generated by BA^k with k even and singlets in $\underline{1}_4$ only those with k odd, it is possible to ensure that the Z_2 subgroup conserved in the up quark is different from the one in the down quark sector. Note that for this purpose the dihedral group has to have an even index, since only then the representations $\underline{1}_{3,4}$ are present [2]. So, it is not possible to choose D_7 as flavor symmetry, as it has been done in [3,4], to predict θ_C, if distinct values of k in the up quark and down quark sector are supposed to be guaranteed by the choice of representations. One can check that the subgroup preserved by VEVs of the form given in Eq. (6) cannot be larger than Z_2, if the index j of the representation $\underline{2}_j$ is odd, i.e. the representation is faithful. For an even index j the subgroup is a D_2 group generated by the two elements A^7 and BA^k with k being an integer between 0 and 6.[5] Obviously, in the case that only flavons residing in representations $\underline{1}_i$, $i = 1, \ldots, 4$, acquire a VEV the conserved subgroup is also generally larger than only Z_2.

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[5] In general, for fields in representations $\underline{2}_j$, whose index j has a greatest common divisor with the group index n larger than one, the preserved subgroup is larger than a Z_2 symmetry. In the case under consideration, namely $n = 14$, this statement is equivalent to the statement that the preserved subgroup is larger than Z_2, if the index j of the representation $\underline{2}_j$ is even. We note that there is a mistake in the first version of [2] concerning this aspect.

3. Outline of the model

In our model the left-handed quarks Q_1 and Q_2 are unified into the D_{14} doublet $\underline{2}_1$, denoted by Q_D, while the third generation of left-handed quarks Q_3, the right-handed up-type quark t^c, and the right-handed down-type quark s^c, transform trivially under D_{14}, i.e. as $\underline{1}_1$. The remaining two generations of right-handed fields, i.e. c^c and u^c in the up quark and d^c and b^c in the down quark sector, are assigned to the one-dimensional representations $\underline{1}_3$ and $\underline{1}_4$.[6] The MSSM Higgs doublets h_u and h_d do not transform under D_{14}. Therefore, we need to introduce gauge singlets, flavons, to form D_{14}-invariant Yukawa couplings. These flavons transform according to the singlets $\underline{1}_1$, $\underline{1}_3$, $\underline{1}_4$ and the doublets $\underline{2}_1$, $\underline{2}_2$ and $\underline{2}_4$. All Yukawa operators involving flavon fields are non-renormalizable and suppressed by (powers of) the cutoff scale Λ which is expected to be of the order of the scale of grand unification or the Planck scale. Additionally, we have to introduce a symmetry which allows us to separate the up and down quark sector. The minimal choice of such a symmetry in this setup is a Z_3 group. We assign a trivial Z_3 charge to left-handed quarks, right-handed up quarks and to the flavon fields $\psi^u_{1,2}$, $\chi^u_{1,2}$, $\xi^u_{1,2}$ and η^u, which ought to couple dominantly to up quarks. The right-handed down quarks transform as ω^2 under Z_3 with $\omega = e^{\frac{2\pi i}{3}}$. The flavon fields $\psi^d_{1,2}$, $\chi^d_{1,2}$, $\xi^d_{1,2}$, η^d and σ, mainly responsible for down quark masses, acquire a phase ω under Z_3. The MSSM Higgs fields transform trivially also under the Z_3 symmetry. Since the right-handed down quarks have charge ω^2 under Z_3, whereas Q_D, Q_3 and h_d are neutral, the bottom quark does not acquire a mass at the renormalizable level, unlike the top quark. As a result, the hierarchy between the top and bottom quark is explained without large $\tan\beta = \langle h_u \rangle / \langle h_d \rangle$. The hierarchy between the charm and top quark mass, $m_c/m_t \sim \mathcal{O}(\epsilon^2)$ with $\epsilon \approx \lambda^2 \approx 0.04$, is naturally accommodated in our model. To achieve the correct ratio between strange and bottom quark mass, $m_s/m_b \sim \mathcal{O}(\epsilon)$, we apply the FN mechanism. We add the FN field θ to our model which is only charged under $U(1)_{FN}$. Without loss of generality we can assume that its charge is -1. Note that we distinguish in our discussion between the FN field θ and the flavon fields $\psi^u_{1,2}$, $\chi^u_{1,2}$, $\xi^u_{1,2}$, η^u, $\psi^d_{1,2}$, $\chi^d_{1,2}$, $\xi^d_{1,2}$, η^d and σ which transform non-trivially under $D_{14} \times Z_3$. If we assign a $U(1)_{FN}$ charge $+1$ to the right-handed down-type quark s^c, we arrive at $m_s/m_b \sim \mathcal{O}(\epsilon)$. Finally, to reproduce the hierarchy between the first generation and the third one, $m_u/m_t \sim \mathcal{O}(\epsilon^4)$ and $m_d/m_b \sim \mathcal{O}(\epsilon^2)$, also the right-handed quarks, u^c and d^c, have to have a non-vanishing $U(1)_{FN}$ charge. The transformation properties of the quarks and flavons under $D_{14} \times Z_3 \times U(1)_{FN}$ are summarized in Table 1. Given these we can write down the superpotential w which consists of two parts

$$w = w_q + w_f. \tag{7}$$

w_q contains the Yukawa couplings of the quarks and w_f the flavon superpotential responsible for the vacuum alignment of the flavons. The mass matrices arising from w_q are discussed in Sections 4 and 5, while w_f is studied in Section 6.

As already explained in the introduction, the prediction of the CKM matrix element $|V_{ud}|$ or equivalently the Cabibbo angle θ_C is based on the fact that the VEVs of the flavons $\{\psi^u_{1,2}, \chi^u_{1,2}, \xi^u_{1,2}, \eta^u\}$ preserve a Z_2 subgroup of D_{14} which is generated by the element BA^{k_u},

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[6] The fact that the transformation properties of the right-handed down quark fields are permuted compared to those of the right-handed up quark fields is merely due to the desire to arrive at a down quark mass matrix $\mathcal{M}_d$ which has a large (33) entry, see Eq. (18). However, since this is just a permutation of the right-handed fields it is neither relevant for quark masses nor for mixings.

Table 1

Particle content of the model. Here we display the transformation properties of fermions and scalars under the flavor group $D_{14} \times Z_3 \times U(1)_{\mathrm{FN}}$. The symmetry Z_3 separates the up and down quark sector. The left-handed quark doublets are denoted by $Q_D = (Q_1, Q_2)^T$, $Q_1 = (u, d)^T$, $Q_2 = (c, s)^T$, $Q_3 = (t, b)^T$ and the right-handed quarks by u^c, c^c, t^c and d^c, s^c, b^c. The flavon fields indexed by a u give masses to the up quarks only, at lowest order. Similarly, the fields which carry an index d (including the field σ) couple only to down quarks at this order. We assume the existence of a field θ which is a gauge singlet transforming trivially under $D_{14} \times Z_3$. It is responsible for the breaking of the $U(1)_{\mathrm{FN}}$ symmetry. Without loss of generality its charge under $U(1)_{\mathrm{FN}}$ can be chosen as -1. Note that ω is the third root of unity, i.e. $\omega = \mathrm{e}^{\frac{2\pi i}{3}}$.

Field	Q_D	Q_3	u^c	c^c	t^c	d^c	s^c	b^c	$h_{u,d}$	$\psi^u_{1,2}$	$\chi^u_{1,2}$	$\xi^u_{1,2}$	η^u	$\psi^d_{1,2}$	$\chi^d_{1,2}$	$\xi^d_{1,2}$	η^d	σ
D_{14}	$\mathbf{2}_1$	$\mathbf{1}_1$	$\mathbf{1}_4$	$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{1}_4$	$\mathbf{1}_1$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_4$	$\mathbf{1}_3$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_4$	$\mathbf{1}_4$	$\mathbf{1}_1$
Z_3	1	1	1	1	1	ω^2	ω^2	ω^2	1	1	1	1	1	ω	ω	ω	ω	ω
$U(1)_{\mathrm{FN}}$	0	0	2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0

whereas the VEVs of $\{\psi^d_{1,2}, \chi^d_{1,2}, \xi^d_{1,2}, \eta^d, \sigma\}$ coupling dominantly to down quarks keep a Z_2 group originating from the element BA^{k_d} conserved with $k_u \neq k_d$. Due to the fact that η^u transforms as $\mathbf{1}_3$ and η^d as $\mathbf{1}_4$ under D_{14} k_u has to be an even integer between 0 and 12 and k_d an odd integer between 1 and 13, implying the non-equality of k_u and k_d. Since $k_u \neq k_d$, it is also evident that D_{14} is completely broken in the whole theory. As mentioned, the separation of the two symmetry-breaking sectors is maintained by the Z_3 symmetry. However, in terms with more than one flavon in the down and more than two flavons in the up quark sector the fields $\{\psi^u_{1,2}, \chi^u_{1,2}, \xi^u_{1,2}, \eta^u\}$ couple to down quarks and $\{\psi^d_{1,2}, \chi^d_{1,2}, \xi^d_{1,2}, \eta^d, \sigma\}$ to up quarks so that this separation of the two sectors is not rigid anymore. Similarly, non-renormalizable operators in the flavon superpotential mix the two different sectors inducing shifts in the aligned flavon VEVs. This fact is explained in more detail in Sections 5 and 6.2. To elucidate the origin of the prediction of $|V_{ud}|$ (θ_C) we first consider in Section 4 the mass matrices arising in the case that the two different Z_2 subgroups remain unbroken in up and down quark sector. Then we turn in Section 5 to the discussion of the mass matrix structures including the subgroup non-preserving corrections from multi-flavon insertions and VEV shifts and show that the results achieved in the limit of unbroken Z_2 subgroups in both sectors still hold, especially the prediction of $|V_{ud}|$ (θ_C) is valid up to $\mathcal{O}(\epsilon)$ corrections.

4. Quark masses and mixings in the subgroup conserving case

As mentioned above, all Yukawa terms containing up to two flavons in the up and one flavon in the down quark sector preserve a Z_2 group generated by BA^{k_u} and by BA^{k_d}, respectively. In the up quark sector the only renormalizable coupling generates the top quark mass

$$Q_3 t^c h_u. \tag{8}$$

Here and in the following we omit order one couplings in front of the operators. The other elements of the third column and the (32) element of the up quark mass matrix $\mathcal{M}_u$ arise at the one-flavon level through the terms

$$\frac{1}{\Lambda}(Q_D \psi^u) t^c h_u \quad \text{and} \quad \frac{1}{\Lambda} Q_3(c^c \eta^u) h_u, \tag{9}$$

respectively. We denote with $(\cdots)$ the contraction to a D_{14} invariant. The elements belonging to the upper 1–2 subblock of $\mathcal{M}_u$ are generated at the level of two-flavon insertions

$$\frac{\theta^2}{\Lambda^4}(Q_D u^c \chi^u \xi^u)h_u + \frac{\theta^2}{\Lambda^4}(Q_D u^c (\xi^u)^2)h_u + \frac{\theta^2}{\Lambda^4}(Q_D \psi^u \eta^u u^c)h_u, \tag{10}$$

$$\frac{1}{\Lambda^2}(Q_D c^c \chi^u \xi^u)h_u + \frac{1}{\Lambda^2}(Q_D c^c (\xi^u)^2)h_u + \frac{1}{\Lambda^2}(Q_D \psi^u)(\eta^u c^c)h_u. \tag{11}$$

Thereby, the (11) and (21) entries stem from the terms in Eq. (10), while the terms in Eq. (11) are responsible for the (12) and (22) elements of $\mathcal{M}_u$. Also the elements of the third column receive contributions from two-flavon insertions which, however, can be absorbed into the existing couplings (if we are in the symmetry preserving limit). Therefore, we do not mention these terms explicitly here. Only the (31) element of $\mathcal{M}_u$ vanishes in the limit of an unbroken Z_2 subgroup in the up quark sector, since the existence of the residual symmetry forbids a non-zero VEV for a flavon (a combination of flavons) in the D_{14} representation $\underline{1}_4$ for even k_u. The VEVs of the fields $\psi^u_{1,2}$, $\chi^u_{1,2}$ and $\xi^u_{1,2}$, which preserve a Z_2 symmetry generated by the element BA^{k_u}, are of the form

$$\begin{pmatrix} \langle\psi^u_1\rangle \\ \langle\psi^u_2\rangle \end{pmatrix} = v^u \begin{pmatrix} e^{-\frac{\pi i k_u}{7}} \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} \langle\chi^u_1\rangle \\ \langle\chi^u_2\rangle \end{pmatrix} = w^u e^{\frac{\pi i k_u}{7}} \begin{pmatrix} e^{-\frac{2\pi i k_u}{7}} \\ 1 \end{pmatrix},$$
$$\begin{pmatrix} \langle\xi^u_1\rangle \\ \langle\xi^u_2\rangle \end{pmatrix} = z^u e^{\frac{2\pi i k_u}{7}} \begin{pmatrix} e^{-\frac{4\pi i k_u}{7}} \\ 1 \end{pmatrix} \tag{12}$$

together with $\langle\eta_u\rangle \neq 0$. As can be read off from Eq. (1), only the difference between k_u and k_d is relevant for $|V_{ud}|$. Thus, we set $k_u = 0$. Obviously, the conserved Z_2 group in the up quark sector is then generated by B. The up quark mass matrix has the generic form

$$\mathcal{M}_u = \begin{pmatrix} -\alpha^u_1 t^2 \epsilon^2 & \alpha^u_2 \epsilon^2 & \alpha^u_3 \epsilon \\ \alpha^u_1 t^2 \epsilon^2 & \alpha^u_2 \epsilon^2 & \alpha^u_3 \epsilon \\ 0 & \alpha^u_4 \epsilon & y_t \end{pmatrix} \langle h_u \rangle \tag{13}$$

in the Z_2 symmetry limit. The couplings α^u_i and y_t are in general complex. The small expansion parameters ϵ and t are given by

$$\frac{v^u}{\Lambda}, \frac{w^u}{\Lambda}, \frac{z^u}{\Lambda}, \frac{\langle\eta^u\rangle}{\Lambda} \sim \epsilon \approx \lambda^2 \approx 0.04 \quad \text{and} \quad \frac{\langle\theta\rangle}{\Lambda} = t. \tag{14}$$

All flavon VEVs are of the order of M^u_ψ, which is the mass parameter in the flavon superpotential $w_{f,u}$ in Eq. (41), unless we assume accidental cancellations among the dimensionless parameters of $w_{f,u}$, compare Eq. (44). Eq. (14) thus holds for $M^u_\psi \approx \epsilon \Lambda$. Additionally, we take t and ϵ to be real and positive and assume

$$t \approx \epsilon \approx \lambda^2 \approx 0.04 \tag{15}$$

in the following.

We can discuss the down quark mass matrix $\mathcal{M}_d$ in a similar fashion. Taking into account only terms with one flavon we can generate apart from the (33) entry of the matrix the elements of the second column,

$$\frac{1}{\Lambda}Q_3(b^c \eta^d)h_d, \qquad \frac{\theta}{\Lambda^2}Q_3 s^c \sigma h_d \quad \text{and} \quad \frac{\theta}{\Lambda^2}(Q_D \psi^d)s^c h_d. \tag{16}$$

Actually, the first term is responsible for the (33) entry, while the second one leads to a non-vanishing (32) entry and the third one gives the dominant contribution to the (12) and (22) elements of $\mathcal{M}_d$. The flavon VEVs preserving the subgroups generated by BA^k are of the form

$$
\begin{pmatrix} \langle \psi_1^d \rangle \\ \langle \psi_2^d \rangle \end{pmatrix} = v^d \begin{pmatrix} \mathrm{e}^{-\frac{\pi i k}{7}} \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} \langle \chi_1^d \rangle \\ \langle \chi_2^d \rangle \end{pmatrix} = w^d \mathrm{e}^{\frac{\pi i k}{7}} \begin{pmatrix} \mathrm{e}^{-\frac{2\pi i k}{7}} \\ 1 \end{pmatrix} \quad \text{and}
$$
$$
\begin{pmatrix} \langle \xi_1^d \rangle \\ \langle \xi_2^d \rangle \end{pmatrix} = z^d \mathrm{e}^{\frac{2\pi i k}{7}} \begin{pmatrix} \mathrm{e}^{-\frac{4\pi i k}{7}} \\ 1 \end{pmatrix},
\tag{17}
$$

with $\langle \eta^d \rangle$ and $\langle \sigma \rangle$ being non-zero. Since we already set k_u to zero, we omitted the subscript d of the parameter k which has to be an odd integer ranging between 1 and 13. As discussed in Section 6, the value of k cannot be uniquely fixed through the superpotential w_f. The form of the down quark mass matrix is then

$$
\mathcal{M}_d = \begin{pmatrix} 0 & \alpha_1^d t \epsilon & 0 \\ 0 & \alpha_1^d \mathrm{e}^{-\pi i k/7} t \epsilon & 0 \\ 0 & \alpha_2^d t \epsilon & y_b \epsilon \end{pmatrix} \langle h_d \rangle.
\tag{18}
$$

Again, the couplings α_i^d and y_b are complex. The expansion parameter ϵ is given by

$$
\frac{v^d}{\Lambda}, \frac{w^d}{\Lambda}, \frac{z^d}{\Lambda}, \frac{\langle \eta^d \rangle}{\Lambda}, \frac{\langle \sigma \rangle}{\Lambda} \sim \epsilon \approx \lambda^2 \approx 0.04.
\tag{19}
$$

Again, the fact that all VEVs are of the same order of magnitude (for no accidental cancellations due to some special parameter choice) can be derived from the flavon superpotential $w_{f,d}$, see Eqs. (47) and (48). Eq. (19) holds for $\langle \sigma \rangle = x \approx \epsilon \Lambda$. We assume that ϵ in Eq. (19) is the same as in Eq. (14), i.e. the VEVs of all flavons are expected to be of the same order of magnitude. Eq. (15) thus also holds.

For the quark masses we find

$$
m_u^2 : m_c^2 : m_t^2 \sim \epsilon^8 : \epsilon^4 : 1,
\tag{20a}
$$
$$
m_d^2 : m_s^2 : m_b^2 \sim 0 : \epsilon^2 : 1,
\tag{20b}
$$
$$
m_b^2 : m_t^2 \sim \epsilon^2 : 1,
\tag{20c}
$$

where the third equation holds for small $\tan \beta$. As one can see, the hierarchy among the up quark masses and the ratio m_s/m_b are correctly reproduced. The down quark mass vanishes at this level and is generated by Z_2 symmetry non-conserving two-flavon insertions, see Eq. (37). The CKM matrix is of the form

$$
|V_{\mathrm{CKM}}| = \begin{pmatrix} |\cos(\frac{k\pi}{14})| & |\sin(\frac{k\pi}{14})| & 0 \\ |\sin(\frac{k\pi}{14})| & |\cos(\frac{k\pi}{14})| & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \mathcal{O}(\epsilon^4) & \mathcal{O}(\epsilon^2) \\ \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon) \\ \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon^2) \end{pmatrix}.
\tag{21}
$$

The elements $|V_{ud}|$, $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$ are determined by the group theoretical parameter k. Since k takes odd integer values between 1 and 13 we arrive at four possible scenarios: If k takes the value $k = 1$ (minimal) or $k = 13$ (maximal), we arrive at $|V_{ud}| = \cos(\frac{\pi}{14}) \approx 0.97493$. This value is very close to the central one, $|V_{ud}|_{\exp} = 0.97419^{+0.00022}_{-0.00022}$, [13]. For the other three elements of the CKM matrix, also only determined by k, we then find

$$
|V_{ud}| \approx |V_{cs}| \approx 0.97493 \quad \text{and} \quad |V_{us}| \approx |V_{cd}| \approx 0.2225,
\tag{22}
$$

which should be compared with the experimental values [13]

$$|V_{cs}|_{\exp} = 0.97334^{+0.00023}_{-0.00023}, \qquad |V_{us}|_{\exp} = 0.2257^{+0.0010}_{-0.0010}, \qquad |V_{cd}|_{\exp} = 0.2256^{+0.0010}_{-0.0010}.$$
(23)

As the experimental errors are very small, the values predicted for $|V_{ud}|$, $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$ are not within the error bars given in [13]. However, as we show in Section 5 the terms which break the residual Z_2 subgroups, change the values of the CKM matrix elements $|V_{ud}|$, $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$ by order ϵ so that the results of the model agree with the experimental data. The three other possible values for $|V_{ud}|$ which can arise are $\cos(\frac{3\pi}{14}) \approx 0.78183$ for $k = 3$ and $k = 11$, $\cos(\frac{5\pi}{14}) \approx 0.43388$ if $k = 5$ or $k = 9$ and finally $|V_{ud}|$ vanishes for $k = 7$. Thus, k has to be chosen either minimal or maximal to be in accordance with the experimental observations. The other values cannot be considered to be reasonable, since we cannot expect that the corrections coming from symmetry breaking terms change the element $|V_{ud}|$ by more than $\epsilon \approx 0.04$. For this reason, we set $k = 1$ in the following discussion. The CKM matrix elements in the third row and column are reproduced with the correct order of magnitude, apart from $|V_{ub}|$ which is slightly too small, $\epsilon^2 \approx \lambda^4$ instead of λ^3, and from $|V_{td}|$ which is slightly too large, $\epsilon \approx \lambda^2$ instead of λ^3. The value of $|V_{ub}|$ gets enhanced through the inclusion of Z_2 symmetry breaking terms. In any case by including Z_2 symmetry breaking terms it becomes possible to accommodate all experimental data, if some of the Yukawa couplings are slightly enhanced or suppressed. J_{CP}, the measure of CP violation in the quark sector [14], is of the order $\epsilon^3 \approx \lambda^6$ and thus of the correct order of magnitude.

Finally, we briefly compare the form of the mass matrices $\mathcal{M}_u$ and $\mathcal{M}_d$ to the general results we achieved in [2]. According to [2] the most general mass matrix arising from the preservation of a Z_2 subgroup generated by the element $BA^{k_{u,d}}$ for left-handed fields transforming as $\underline{2}_1 + \underline{1}_1$ and right-handed fields as three singlets is given by

$$\mathcal{M}_q = \begin{pmatrix} -A_q & B_q & C_q \\ A_q e^{-\pi i k_q/7} & B_q e^{-\pi i k_q/7} & C_q e^{-\pi i k_q/7} \\ 0 & D_q & E_q \end{pmatrix} \quad \text{for } q = u, d.$$
(24)

The parameters A_q, B_q, C_q, D_q and E_q contain Yukawa couplings and VEVs and are in general complex. Comparing Eq. (24) with Eq. (13) shows that $\mathcal{M}_u$ is of this form with $k_u = 0$. The down quark mass matrix $\mathcal{M}_d$, given in Eq. (18), equals the matrix in Eq. (24), if $k = k_d$ and the parameters A_d and C_d are set to zero. This happens, since our model only contains a restricted number of flavon fields and we do not take into account terms with more than one flavon at this level.

5. Quark masses and mixings including subgroup-breaking effects

In this section we include terms which break the residual Z_2 symmetries explicitly. These lead to corrections of the results shown in Section 4. They are generated by multi-flavon insertions in which flavon fields belonging to the set $\{\psi^d_{1,2}, \chi^d_{1,2}, \xi^d_{1,2}, \eta^d, \sigma\}$ give masses to up quarks and flavons belonging to $\{\psi^u_{1,2}, \chi^u_{1,2}, \xi^u_{1,2}, \eta^u\}$ to down quarks. Additionally, non-renormalizable terms in the flavon superpotential lead to complex shifts in the flavon VEVs in Eqs. (12) and (17). They can be parameterized as in Eq. (50) in Section 6.2 (with $k = 1$). At the same time, x remains a free parameter. The corrections to the flavon superpotential are discussed in detail in Sections 6.2 and Appendix B. The analysis given in these sections shows that the generic size of

the shifts is

$$\delta \text{VEV} \sim \mathcal{O}\left(\frac{\text{VEV}}{\Lambda}\right)\text{VEV} \sim \epsilon \text{VEV}, \tag{25}$$

if all VEVs are of the order $\epsilon \Lambda$, see Eqs. (14) and (19). Thus, the VEV shifts inserted in Yukawa terms with p flavons contribute at the same level as Yukawa terms containing $p + 1$ flavons.

In the up quark sector we find that the (11) and (21) elements receive Z_2 symmetry breaking corrections through the following operators

$$\begin{aligned}
&\frac{\theta^2}{\Lambda^4}\left[(Q_D u^c \delta \chi^u \xi^u) + (Q_D u^c \chi^u \delta \xi^u)\right]h_u + \frac{\theta^2}{\Lambda^4}(Q_D u^c \xi^u \delta \xi^u)h_u + \frac{\theta^2}{\Lambda^4}(Q_D \delta \psi^u \eta^u u^c)h_u \\
&+ \frac{\theta^2}{\Lambda^5}(Q_D \psi^d \chi^d)(\eta^d u^c)h_u + \frac{\theta^2}{\Lambda^5}(Q_D u^c (\chi^d)^3)h_u + \frac{\theta^2}{\Lambda^5}(Q_D u^c (\psi^d)^2 \xi^d)h_u \\
&+ \frac{\theta^2}{\Lambda^5}(Q_D u^c \chi^d (\xi^d)^2)h_u + \frac{\theta^2}{\Lambda^5}(Q_D u^c (\chi^d)^2 \xi^d)h_u + \frac{\theta^2}{\Lambda^5}(Q_D u^c \chi^d \xi^d)\sigma h_u \\
&+ \frac{\theta^2}{\Lambda^5}(Q_D u^c (\xi^d)^2)\sigma h_u + \frac{\theta^2}{\Lambda^5}(Q_D \psi^d)(\eta^d u^c)\sigma h_u. \tag{26}
\end{aligned}$$

The notation of, for example, $\delta \chi^u$ indicates that the VEV of the fields $\chi_{1,2}^u$, shifted through the non-renormalizable operators correcting the flavon superpotential, is used, when calculating the contribution to the up quark mass matrix. Thus, all contributions from the operators in the first line of Eq. (26) arise from the fact that the VEVs become shifted. Note that we omitted the operator stemming from the shift of the VEV of η^u, since this field only transforms as singlet under D_{14} and thus does not possess any special vacuum structure. (We also do this in the following equations.) The other operators arise from the insertions of three down-type flavon fields. There exist similar operators containing three up-type flavons. However, these still preserve the Z_2 symmetry present in the up quark sector at lowest order and therefore can be absorbed into the existing couplings. Analogously, we find that the following operators give rise to Z_2 symmetry breaking contributions to the (12) and (22) elements so that also these are no longer equal

$$\begin{aligned}
&\frac{1}{\Lambda^2}\left[(Q_D c^c \delta \chi^u \xi^u) + (Q_D c^c \chi^u \delta \xi^u)\right]h_u + \frac{1}{\Lambda^2}(Q_D c^c \xi^u \delta \xi^u)h_u + \frac{1}{\Lambda^2}(Q_D \delta \psi^u)(\eta^u c^c)h_u \\
&+ \frac{1}{\Lambda^3}(Q_D c^c \psi^d \chi^d \eta^d)h_u + \frac{1}{\Lambda^3}(Q_D c^c (\chi^d)^3)h_u + \frac{1}{\Lambda^3}(Q_D c^c (\psi^d)^2 \xi^d)h_u \\
&+ \frac{1}{\Lambda^3}(Q_D c^c \chi^d (\xi^d)^2)h_u \quad + \frac{1}{\Lambda^3}(Q_D c^c (\chi^d)^2 \xi^d)h_u + \frac{1}{\Lambda^3}(Q_D c^c \chi^d \xi^d)\sigma h_u \\
&+ \frac{1}{\Lambda^3}(Q_D c^c (\xi^d)^2)\sigma h_u + \frac{1}{\Lambda^3}(Q_D c^c \psi^d \eta^d)\sigma h_u. \tag{27}
\end{aligned}$$

Again, the operators in the first line are associated to the shifted VEVs. The rest of the operators originates from three-flavon insertions of down-type flavons. The contributions from the analogous operators with up-type flavons can again be absorbed into the existing couplings. The dominant contribution to the (13) and (23) elements which breaks the residual Z_2 symmetry stems from the VEV shift of the fields $\psi_{1,2}^u$

$$\frac{1}{\Lambda}(Q_D \delta \psi^u)t^c h_u. \tag{28}$$

All other contributions up to three flavons are either Z_2 symmetry preserving or breaking, but subdominant. The (31) element which has to vanish in the symmetry limit is generated through

the following three-flavon insertions of down-type flavons

$$
\frac{\theta^2}{\Lambda^5} Q_3 (\eta^d u^c) \sigma^2 h_u + \frac{\theta^2}{\Lambda^5} Q_3 (\eta^d u^c)(\psi^d)^2 h_u + \frac{\theta^2}{\Lambda^5} Q_3 (\eta^d u^c)(\chi^d)^2 h_u
$$
$$
+ \frac{\theta^2}{\Lambda^5} Q_3 (\eta^d u^c)(\xi^d)^2 h_u + \frac{\theta^2}{\Lambda^5} Q_3 (\eta^d u^c)(\eta^d)^2 h_u + \frac{\theta^2}{\Lambda^5} Q_3 (u^c \psi^d \chi^d \xi^d) h_u
$$
$$
+ \frac{\theta^2}{\Lambda^5} Q_3 (u^c \psi^d (\xi^d)^2) h_u. \tag{29}
$$

Note that there are symmetry-conserving couplings, i.e. operators with three up-type flavons, of the same order. These, however, vanish, if the vacuum alignment in Eq. (12) is applied. They can only contribute at the next order, if the VEV shifts are taken into account; however, such effects are subdominant. The (32) and (33) elements of the up quark mass matrix already exist at the lowest order and only receive subdominant contributions from higher-dimensional operators and VEV shifts. The up quark mass matrix can thus be cast into the form

$$
\mathcal{M}_u = \begin{pmatrix} t^2(-\alpha_1^u \epsilon^2 + \beta_1^u \epsilon^3) & \alpha_2^u \epsilon^2 + \beta_2^u \epsilon^3 & \alpha_3^u \epsilon + \beta_3^u \epsilon^2 \\ \alpha_1^u t^2 \epsilon^2 & \alpha_2^u \epsilon^2 & \alpha_3^u \epsilon \\ \beta_4^u t^2 \epsilon^3 & \alpha_4^u \epsilon & y_t \end{pmatrix} \langle h_u \rangle. \tag{30}
$$

We note that without loss of generality we can define the couplings $\alpha_{1,2,3}^u$ and $\beta_{1,2,3}^u$ in such a way that the corrections stemming from Z_2 subgroup breaking terms only appear in the first row of $\mathcal{M}_u$. Due to this and due to the absorption of subdominant contributions the couplings α_i^u only coincide at the leading order with those present in Eq. (13). This also holds for y_t. Again, all couplings are in general complex. The matrix in Eq. (30) is the most general one arising in our model, i.e. all contributions from terms including more than three flavons can be absorbed into the couplings α_i^u, β_i^u and y_t.

Similarly, we analyze the Z_2 symmetry breaking contributions to the down quark mass matrix $\mathcal{M}_d$. The (11) and (21) elements of $\mathcal{M}_d$ are dominantly generated by Z_2 symmetry breaking effects from two-flavon insertions involving one down- and one up-type flavon. We find five independent operators

$$
\frac{\theta}{\Lambda^3} (Q_D d^c \xi^d \chi^u) h_d + \frac{\theta}{\Lambda^3} (Q_D d^c \chi^d \xi^u) h_d + \frac{\theta}{\Lambda^3} (Q_D d^c \xi^d \xi^u) h_d
$$
$$
+ \frac{\theta}{\Lambda^3} (Q_D \psi^d)(\eta^u d^c) h_d + \frac{\theta}{\Lambda^3} (Q_D d^c \eta^d \psi^u) h_d. \tag{31}
$$

Since they are Z_2 symmetry breaking, the (11) and (21) entries are uncorrelated. We note that Z_2 symmetry preserving contributions can only arise, if operators with more than two flavons are considered. However, these are always subdominant compared to the operators in Eq. (31). Similar statements apply to the generation of the (13) and (23) element of $\mathcal{M}_d$. The dominant (Z_2 symmetry breaking) contributions stem from the operators

$$
\frac{1}{\Lambda^2} (Q_D b^c \xi^d \chi^u) h_d + \frac{1}{\Lambda^2} (Q_D b^c \chi^d \xi^u) h_d + \frac{1}{\Lambda^2} (Q_D b^c \xi^d \xi^u) h_d
$$
$$
+ \frac{1}{\Lambda^2} (Q_D b^c \psi^d \eta^u) h_d + \frac{1}{\Lambda^2} (Q_D \psi^u)(\eta^d b^c) h_d. \tag{32}
$$

The (12) and (22) elements which are already present at the lowest order are corrected by Z_2 symmetry breaking terms from the VEV shift of the fields $\psi_{1,2}^d$

$$
\frac{\theta}{\Lambda^2} (Q_D \delta \psi^d) s^c h_d \tag{33}
$$

as well as from two-flavon insertions with one up-type and one down-type flavon

$$\frac{\theta}{\Lambda^3}(Q_D\chi^d\psi^u)s^ch_d + \frac{\theta}{\Lambda^3}(Q_D\psi^d\chi^u)s^ch_d + \frac{\theta}{\Lambda^3}(Q_D\psi^u)\sigma s^ch_d. \tag{34}$$

The (31) entry, which must vanish in the symmetry limit, is generated dominantly by a single operator

$$\frac{\theta}{\Lambda^3}Q_3(\eta^ud^c)\sigma h_d. \tag{35}$$

Similarly to the up quark mass matrix, the (32) and (33) elements of $\mathcal{M}_d$ also receive contributions from Z_2 symmetry breaking effects, which can be absorbed into the leading order term. Eventually, the most general form of the down quark mass matrix $\mathcal{M}_d$ in our model reads

$$\mathcal{M}_d = \begin{pmatrix} \beta_1^d t\epsilon^2 & t(\alpha_4^d\epsilon + \beta_4^d\epsilon^2) & \beta_5^d\epsilon^2 \\ \beta_2^d t\epsilon^2 & \alpha_1^d e^{-\pi i/7}t\epsilon & \beta_6^d\epsilon^2 \\ \beta_3^d t\epsilon^2 & \alpha_2^d t\epsilon & y_b\epsilon \end{pmatrix} \langle h_d\rangle. \tag{36}$$

The parameters α_1^d and β_4^d have been defined so that Z_2 symmetry breaking contributions only appear in the (12) element. Note again that all parameters α_i^d, β_i^d and y_b are complex. Also note that α_i^d and y_b only coincide at leading order with the corresponding parameters in Eq. (18) due to the absorption of subdominant effects.

Before calculating quark masses and mixings the parameters β_4^u, α_4^u, y_t, β_3^d, α_2^d and y_b in the third row of $\mathcal{M}_u$ and $\mathcal{M}_d$ are made real by appropriate rephasing of the right-handed quark fields. The resulting quark masses are then (for $t \approx \epsilon$)

$$m_u^2 = 2|\alpha_1^u|^2\langle h_u\rangle^2\epsilon^8 + \mathcal{O}(\epsilon^9), \qquad m_d^2 = \frac{1}{2}\left|\beta_1^d - \beta_2^d e^{\frac{i\pi}{7}}\right|^2\langle h_d\rangle^2\epsilon^6 + \mathcal{O}(\epsilon^7), \tag{37a}$$

$$m_c^2 = 2\frac{|\alpha_3^u\alpha_4^u - y_t\alpha_2^u|^2}{y_t^2}\langle h_u\rangle^2\epsilon^4 + \mathcal{O}(\epsilon^5), \qquad m_s^2 = 2|\alpha_1^d|^2\langle h_d\rangle^2\epsilon^4 + \mathcal{O}(\epsilon^5), \tag{37b}$$

$$m_t^2 = y_t^2\langle h_u\rangle^2 + \mathcal{O}(\epsilon^2), \qquad m_b^2 = y_b^2\langle h_d\rangle^2\epsilon^2 + \mathcal{O}(\epsilon^4). \tag{37c}$$

At the subdominant level thus also the correct order of magnitude of the down quark mass is reproduced. The CKM matrix elements are given by

$$|V_{ud}| = \cos\left(\frac{\pi}{14}\right) + \mathcal{O}(\epsilon), \qquad |V_{cs}| = \cos\left(\frac{\pi}{14}\right) + \mathcal{O}(\epsilon), \tag{38a}$$

$$|V_{us}| = \sin\left(\frac{\pi}{14}\right) + \mathcal{O}(\epsilon), \qquad |V_{cd}| = \sin\left(\frac{\pi}{14}\right) + \mathcal{O}(\epsilon), \tag{38b}$$

$$|V_{cb}| = \frac{\epsilon}{\sqrt{2}}\left|\frac{\beta_5^d + \beta_6^d}{y_b} - \frac{2\alpha_3^u}{y_t}\right| + \mathcal{O}(\epsilon^2),$$

$$|V_{ts}| = \frac{\epsilon}{\sqrt{2}}\left|\frac{\beta_5^d + \beta_6^d e^{\frac{i\pi}{7}}}{y_b} - \frac{\alpha_3^u(1 + e^{\frac{i\pi}{7}})}{y_t}\right| + \mathcal{O}(\epsilon^2), \tag{38c}$$

$$|V_{ub}| = \frac{\epsilon}{\sqrt{2}}\left|\frac{\beta_5^d - \beta_6^d}{y_b}\right| + \mathcal{O}(\epsilon^2), \qquad |V_{td}| = \frac{\epsilon}{\sqrt{2}}\left|\frac{\beta_5^d - \beta_6^d e^{\frac{i\pi}{7}}}{y_b} - \frac{\alpha_3^u(1 - e^{\frac{i\pi}{7}})}{y_t}\right| + \mathcal{O}(\epsilon^2), \tag{38d}$$

$$|V_{tb}| = 1 + \mathcal{O}(\epsilon^2). \tag{38e}$$

Table 2
Driving fields of the model. The transformation properties of the driving fields under the flavor symmetry $D_{14} \times Z_3$. Similar to the flavons none of the driving fields is charged under $U(1)_{\text{FN}}$. The fields indexed with an u (d) drive the VEVs of the flavons giving masses dominantly to the up (down) quarks. Note that all these fields have a $U(1)_R$ charge $+2$.

Field	σ^{0u}	$\psi_{1,2}^{0u}$	$\varphi_{1,2}^{0u}$	$\rho_{1,2}^{0u}$	σ^{0d}	$\psi_{1,2}^{0d}$	$\varphi_{1,2}^{0d}$	$\rho_{1,2}^{0d}$
D_{14}	$\mathbf{1}_1$	$\mathbf{2}_1$	$\mathbf{2}_3$	$\mathbf{2}_5$	$\mathbf{1}_1$	$\mathbf{2}_1$	$\mathbf{2}_3$	$\mathbf{2}_5$
Z_3	1	1	1	1	ω	ω	ω	ω

As one can see, $|V_{ud}|$, $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$, which are determined by the group theoretical indices of this model, get all corrected by terms of order ϵ, so that they can be in full accordance with the experimental values [13]. The elements of the third row and column are still of the same order of magnitude in ϵ after the inclusion of Z_2 subgroup breaking terms, apart from $|V_{ub}|$ which gets enhanced by $1/\epsilon$. For this reason, $|V_{td}|$ and $|V_{ub}|$ are both slightly larger in our model, $|V_{td}|, |V_{ub}| \sim \epsilon \approx \lambda^2$, than the measured values, which are of order λ^3. However, only a moderate tuning is necessary in order to also accommodate these values. For the Jarlskog invariant J_{CP} we find

$$J_{\text{CP}} = \frac{\epsilon^2}{4 y_b^2 y_t} \sin\left(\frac{\pi}{7}\right) \left(2 y_b \text{Re}\left((\alpha_3^u)^* (\beta_5^d - \beta_6^d)\right) - y_t\left(|\beta_5^d|^2 - |\beta_6^d|^2\right)\right) + \mathcal{O}(\epsilon^3). \tag{39}$$

Similar to $|V_{ub}|$ J_{CP} gets enhanced by $1/\epsilon$ compared to the result in the symmetry limit. Thus, it has to be slightly tuned to match the experimental value, $J_{\text{CP,exp}} = (3.05^{+0.19}_{-0.20}) \times 10^{-5}$ [13], which is around $\epsilon^3 \approx \lambda^6$. However, already the factor $\sin(\frac{\pi}{7})/4 \approx 0.11$ leads to a certain suppression of J_{CP}.

6. Flavon superpotential

6.1. Leading order

Turning to the discussion of the flavon superpotential w_f we add—analogously to, for example, [15]—two additional ingredients. First, we introduce a further $U(1)$ symmetry which is an extension of R-parity called $U(1)_R$. Second, a set of so-called driving fields whose F-terms account for the vacuum alignment of the flavon fields is added to the model. Quarks transform with charge $+1$, flavon fields, $h_{u,d}$ and θ are neutral and driving fields have a charge $+2$ under $U(1)_R$. In this way all terms in the superpotential w_f are linear in the driving fields, whereas these fields do not appear in the superpotential w_q, responsible for the quark masses. Since we expect the flavor symmetry to be broken at high energies around the seesaw scale or the scale of grand unification, soft supersymmetry breaking effects will not affect the alignment so that considering only the F-terms is justified. The driving fields, required in order to construct w_f, can be found in Table 2. The flavon superpotential at the renormalizable level consists of two parts

$$w_f = w_{f,u} + w_{f,d}, \tag{40}$$

where $w_{f,u}$ gives rise to the alignment of the flavons with an index u, and $w_{f,d}$ to the alignment of the flavons coupling mainly to down quarks. $w_{f,u}$ reads

$$w_{f,u} = M_\psi^u \left(\psi_1^u \psi_2^{0u} + \psi_2^u \psi_1^{0u} \right) + a_u \left(\psi_1^u \chi_1^u \varphi_2^{0u} + \psi_2^u \chi_2^u \varphi_1^{0u} \right)$$
$$+ b_u \left(\psi_1^u \chi_2^u \psi_1^{0u} + \psi_2^u \chi_1^u \psi_2^{0u} \right) + c_u \left(\psi_1^u \xi_2^u \varphi_1^{0u} + \psi_2^u \xi_1^u \varphi_2^{0u} \right)$$
$$+ d_u \eta^u \left(\xi_1^u \varphi_1^{0u} + \xi_2^u \varphi_2^{0u} \right) + e_u \left(\psi_1^u \xi_1^u \rho_2^{0u} + \psi_2^u \xi_2^u \rho_1^{0u} \right)$$
$$+ f_u \eta^u \left(\chi_1^u \rho_1^{0u} + \chi_2^u \rho_2^{0u} \right) + g_u \sigma^{0u} \psi_1^u \psi_2^u + l_u \sigma^{0u} \chi_1^u \chi_2^u$$
$$+ n_u \sigma^{0u} \xi_1^u \xi_2^u + q_u \sigma^{0u} \left(\eta^u \right)^2. \tag{41}$$

The conditions for the vacuum alignment are given by the F-terms

$$\frac{\partial w_{f,u}}{\partial \psi_1^{0u}} = M_\psi^u \psi_2^u + b_u \psi_1^u \chi_2^u = 0, \tag{42a}$$

$$\frac{\partial w_{f,u}}{\partial \psi_2^{0u}} = M_\psi^u \psi_1^u + b_u \psi_2^u \chi_1^u = 0, \tag{42b}$$

$$\frac{\partial w_{f,u}}{\partial \varphi_1^{0u}} = a_u \psi_2^u \chi_2^u + c_u \psi_1^u \xi_2^u + d_u \eta^u \xi_1^u = 0, \tag{42c}$$

$$\frac{\partial w_{f,u}}{\partial \varphi_2^{0u}} = a_u \psi_1^u \chi_1^u + c_u \psi_2^u \xi_1^u + d_u \eta^u \xi_2^u = 0, \tag{42d}$$

$$\frac{\partial w_{f,u}}{\partial \rho_1^{0u}} = e_u \psi_2^u \xi_2^u + f_u \eta^u \chi_1^u = 0, \tag{42e}$$

$$\frac{\partial w_{f,u}}{\partial \rho_2^{0u}} = e_u \psi_1^u \xi_1^u + f_u \eta^u \chi_2^u = 0, \tag{42f}$$

$$\frac{\partial w_{f,u}}{\partial \sigma^{0u}} = g_u \psi_1^u \psi_2^u + l_u \chi_1^u \chi_2^u + n_u \xi_1^u \xi_2^u + q_u \left(\eta^u \right)^2 = 0. \tag{42g}$$

If we assume that none of the parameters in the superpotential vanishes and ψ_1^u acquires a non-zero VEV, we arrive at

$$\begin{pmatrix} \langle \psi_1^u \rangle \\ \langle \psi_2^u \rangle \end{pmatrix} = v^u \begin{pmatrix} \mathrm{e}^{-\frac{\pi i k_u}{7}} \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} \langle \chi_1^u \rangle \\ \langle \chi_2^u \rangle \end{pmatrix} = w^u \mathrm{e}^{\frac{\pi i k_u}{7}} \begin{pmatrix} \mathrm{e}^{-\frac{2\pi i k_u}{7}} \\ 1 \end{pmatrix},$$
$$\begin{pmatrix} \langle \xi_1^u \rangle \\ \langle \xi_2^u \rangle \end{pmatrix} = z^u \mathrm{e}^{\frac{2\pi i k_u}{7}} \begin{pmatrix} \mathrm{e}^{-\frac{4\pi i k_u}{7}} \\ 1 \end{pmatrix}, \tag{43}$$

with

$$w^u = -\frac{M_\psi^u}{b_u}, \qquad z^u = \frac{w^u}{2 d_u e_u} \left(c_u f_u \pm \sqrt{4 a_u d_u e_u f_u + (c_u f_u)^2} \right),$$
$$\left(v^u \right)^2 = -\frac{l_u (w^u)^2 + n_u (z^u)^2}{g_u + q_u \left(\frac{e_u z^u}{f_u w^u} \right)^2} e^{\frac{\pi i k_u}{7}} \quad \text{and} \quad \langle \eta^u \rangle = -\frac{e_u}{f_u} \frac{v^u z^u}{w^u} e^{-\frac{4\pi i k_u}{7}} \tag{44}$$

as unique solution. The flavon VEVs are aligned and their alignment only depends on the parameter k_u which is an even integer between 0 and 12 (see Section 2). Thus, all vacua conserve a Z_2 subgroup of D_{14} generated by the element BA^{k_u}. Since only the difference between k_u and k_d is relevant for the prediction of the CKM matrix element $|V_{ud}|$, we set $k_u = 0$, as it has been done in Section 4, when we study quark masses and mixings. The size of the flavon VEVs is determined by the dimensionless parameters $a_u, b_u, \ldots$ in $w_{f,u}$ and the mass parameter M_ψ^u. Assuming the dimensionless parameters are order one couplings leads to flavon VEVs which are all of the order

of $M_\psi^u \approx \epsilon \Lambda$, as done in Sections 4 and 5. Choosing the parameters in $w_{f,u}$ appropriately, we can make all VEVs in Eqs. (43) and (44) positive for $k_u = 0$.

Analogously, the flavon superpotential which drives the vacuum alignment of the fields $\psi_{1,2}^d$, $\chi_{1,2}^d$, $\xi_{1,2}^d$, η^d and σ is given by

$$
\begin{aligned}
w_{f,d} = {}& m_\psi^d \sigma \left(\psi_1^d \psi_2^{0d} + \psi_2^d \psi_1^{0d} \right) + a_d \left(\psi_1^d \chi_1^d \varphi_2^{0d} + \psi_2^d \chi_2^d \varphi_1^{0d} \right) \\
& + b_d \left(\psi_1^d \chi_2^d \psi_1^{0d} + \psi_2^d \chi_1^d \psi_2^{0d} \right) + c_d \left(\psi_1^d \xi_2^d \varphi_1^{0d} + \psi_2^d \xi_1^d \varphi_2^{0d} \right) \\
& + d_d \eta^d \left(\xi_1^d \varphi_1^{0d} - \xi_2^d \varphi_2^{0d} \right) + e_d \left(\psi_1^d \xi_1^d \rho_2^{0d} + \psi_2^d \xi_2^d \rho_1^{0d} \right) \\
& + f_d \eta^d \left(\chi_1^d \rho_1^{0d} - \chi_2^d \rho_2^{0d} \right) + g_d \sigma^{0d} \psi_1^d \psi_2^d + l_d \sigma^{0d} \chi_1^d \chi_2^d + n_d \sigma^{0d} \xi_1^d \xi_2^d \\
& + q_d \sigma^{0d} \left(\eta^d \right)^2 + p_d \sigma^{0d} \sigma^2 .
\end{aligned}
\tag{45}
$$

Setting the F-terms of the driving fields $\psi_{1,2}^{0d}$, $\varphi_{1,2}^{0d}$, $\rho_{1,2}^{0d}$ and σ^{0d} to zero we find

$$
\frac{\partial w_{f,d}}{\partial \psi_1^{0d}} = m_\psi^d \sigma \psi_2^d + b_d \psi_1^d \chi_2^d = 0,
\tag{46a}
$$

$$
\frac{\partial w_{f,d}}{\partial \psi_2^{0d}} = m_\psi^d \sigma \psi_1^d + b_d \psi_2^d \chi_1^d = 0,
\tag{46b}
$$

$$
\frac{\partial w_{f,d}}{\partial \varphi_1^{0d}} = a_d \psi_2^d \chi_2^d + c_d \psi_1^d \xi_2^d + d_d \eta^d \xi_1^d = 0,
\tag{46c}
$$

$$
\frac{\partial w_{f,d}}{\partial \varphi_2^{0d}} = a_d \psi_1^d \chi_1^d + c_d \psi_2^d \xi_1^d - d_d \eta^d \xi_2^d = 0,
\tag{46d}
$$

$$
\frac{\partial w_{f,d}}{\partial \rho_1^{0d}} = e_d \psi_2^d \xi_2^d + f_d \eta^d \chi_1^d = 0,
\tag{46e}
$$

$$
\frac{\partial w_{f,d}}{\partial \rho_2^{0d}} = e_d \psi_1^d \xi_1^d - f_d \eta^d \chi_2^d = 0,
\tag{46f}
$$

$$
\frac{\partial w_{f,d}}{\partial \sigma^{0d}} = g_d \psi_1^d \psi_2^d + l_d \chi_1^d \chi_2^d + n_d \xi_1^d \xi_2^d + q_d \left(\eta^d \right)^2 + p_d \sigma^2 = 0.
\tag{46g}
$$

These equations lead to the same VEV structure as shown in Eq. (43), if we assume that again none of the parameters in the flavon superpotential vanishes and the two fields ψ_1^d and σ get a non-vanishing VEV. Thus, $\langle \psi_{1,2}^d \rangle$, $\langle \chi_{1,2}^d \rangle$ and $\langle \xi_{1,2}^d \rangle$ have the same form as $\langle \psi_{1,2}^u \rangle$, $\langle \chi_{1,2}^u \rangle$ and $\langle \xi_{1,2}^u \rangle$ with obvious replacements $\{v^u, w^u, z^u\} \to \{v^d, w^d, z^d\}$, $k_u \to k_d$ and k_d being an odd integer. w^d, z^d and v^d are given by

$$
\begin{aligned}
& w^d = -\frac{m_\psi^d x}{b_d} \quad \text{and} \quad z^d = \frac{w^d}{2 d_d e_d} \left(c_d f_d \pm \sqrt{4 a_d d_d e_d f_d + (c_d f_d)^2} \right), \\
& (v^d)^2 = \frac{l_d (w^d)^2 + n_d (z^d)^2 + p_d x^2}{q_d \left(\frac{e_d z^d}{f_d w^d} \right)^2 - g_d} e^{\frac{\pi i k_d}{7}}
\end{aligned}
\tag{47}
$$

and the VEVs of the two singlets η^d and σ read

$$
\langle \eta^d \rangle = \frac{e_d}{f_d} \frac{v^d z^d}{w^d} e^{-\frac{4\pi i k_d}{7}} \quad \text{and} \quad \langle \sigma \rangle = x.
\tag{48}
$$

We find that the parameter x remains as free parameter in the model. However, taking the dimensionless parameters of $w_{f,d}$ to be of similar size all flavon VEVs are of the same order of magnitude, i.e. $x \approx \epsilon \Lambda$. Note further that the two possible signs appearing in Eqs. (44) and (47) are uncorrelated. We can choose the parameters such that v^d, w^d, z^d and x are positive. The parameter k_d is an odd integer in the range $\{1, \ldots, 13\}$. Similar to k_u being even, k_d is required to be odd by the transformation property of the flavon η^d under D_{14}. Especially, k_d is different from k_u so that we preserve different Z_2 subgroups in both sectors. As a consequence, the derived mixing angle is non-trivial. However, we cannot uniquely fix the parameter k_d and thus the mixing angle by the vacuum alignment deduced from w_f. As discussed in Section 4, we are left with a small number (four) of different possible values for $|V_{ud}|$. Due to the different subgroups preserved in up and down quark sector D_{14} is eventually completely broken in the whole theory. As we already set k_u to zero, we omit the index of the parameter k_d from now on also in the discussion of the flavon superpotential.

We end with a few remarks about the VEVs of the driving fields, the absence of a μ-term and the mass spectrum of the gauge singlets transforming under D_{14}. The VEVs of the driving fields are determined by the F-terms of the flavon fields. We find that the VEVs of all driving fields can vanish at the minimum, if we plug in the solutions for the VEVs of the flavons found in Eqs. (43), (44), (47) and (48). The term $\mu h_u h_d$ is forbidden by the $U(1)_R$ symmetry. It cannot be generated through terms including one driving field, h_u and h_d and an appropriate number of flavon fields (to make it invariant under the symmetry $D_{14} \times Z_3$), as long as the driving fields do not acquire non-vanishing VEVs. Thus, the μ-term should originate from another mechanism, see also [15]. Massless modes which might exist in the spectrum of flavons and driving fields are expected to become massive, if soft supersymmetry breaking masses are included into the potential.

6.2. Corrections to the leading order

In the flavon superpotential, terms containing three flavons and one driving field lead to corrections of the vacuum alignment achieved through the superpotential w_f, i.e. they induce (small) shifts in the VEVs of the flavons. Such terms are suppressed by the cutoff scale Λ. Due to the Z_3 symmetry two types of three-flavon combinations can couple to a driving field with an index u, namely either all three flavons also carry an index u or all three of them belong to the set $\{\psi_{1,2}^d, \chi_{1,2}^d, \xi_{1,2}^d, \eta^d, \sigma\}$. If the driving field has an index d, two of the three flavons have to be down-type flavons, while the third one necessarily has to carry an index u. These corrections to the flavon superpotential can be written as

$$\Delta w_f = \Delta w_{f,u} + \Delta w_{f,d}, \tag{49}$$

where the terms of $\Delta w_{f,u}$ ($\Delta w_{f,d}$) are responsible for the shifts in the VEVs of the flavons uncharged (charged) under the Z_3 symmetry. The exact form of the terms is given in Appendix B. We choose the following convention for the shifts of the VEVs

$$\langle \psi_i^u \rangle = v^u + \delta v_i^u, \qquad \langle \chi_i^u \rangle = w^u + \delta w_i^u, \qquad \langle \xi_i^u \rangle = z^u + \delta z_i^u,$$

$$\langle \eta^u \rangle = -\frac{e_u}{f_u} \frac{v^u z^u}{w^u} + \delta \eta^u, \qquad \langle \psi_1^d \rangle = \mathrm{e}^{-\frac{\pi i k}{7}}\left(v^d + \delta v_1^d\right), \qquad \langle \psi_2^d \rangle = v^d + \delta v_2^d,$$

$$\langle \chi_1^d \rangle = \mathrm{e}^{-\frac{\pi i k}{7}}\left(w^d + \delta w_1^d\right), \qquad \langle \chi_2^d \rangle = \mathrm{e}^{\frac{\pi i k}{7}}\left(w^d + \delta w_2^d\right), \qquad \langle \xi_1^d \rangle = \mathrm{e}^{-\frac{2\pi i k}{7}}\left(z^d + \delta z_1^d\right),$$

$$\langle \xi_2^d \rangle = \mathrm{e}^{\frac{2\pi i k}{7}}\left(z^d + \delta z_2^d\right) \quad \text{and} \quad \langle \eta^d \rangle = \mathrm{e}^{-\frac{4\pi i k}{7}}\left(\frac{e_d}{f_d}\frac{v^d z^d}{w^d} + \delta \eta^d\right), \tag{50}$$

while

$$\langle \sigma \rangle = x \tag{51}$$

remains as free parameter. As can be read off from the equations given in Appendix B, x is also not fixed by the corrections to the superpotential. We do not fix the parameter k in Eq. (50), although we showed in Section 4 that only $k = 1$ and $k = 13$ lead to a phenomenologically viable model. This is done, because the complexity of the calculation of the shifts does not depend on the actual value of k. (k still has to be an odd integer.) One finds that also the inclusion of the corrections to the flavon superpotential does not fix the value of k. The detailed calculations given in Appendix B show that the generic size of the shifts is

$$\delta \text{VEV} \sim \mathcal{O}\left(\frac{\text{VEV}}{\Lambda}\right) \text{VEV} \sim \epsilon \text{VEV} \tag{52}$$

for all VEVs being of the order $\epsilon \Lambda$. The shifts are expected to be in general complex, without having a particular phase.

7. Summary and outlook

We presented an extension of the MSSM in which the value of the CKM matrix element $|V_{ud}|$ or equivalently the Cabibbo angle θ_C is fixed by group theoretical quantities of the flavor symmetry D_{14}, up to the choice among four different possible values. The determination of $|V_{ud}|$ originates from the fact that residual Z_2 symmetries of D_{14} exist in the up and down quark sector. We have shown that these can be maintained by the vacuum alignment resulting from a properly constructed flavon superpotential. Furthermore, it is ensured through the choice of flavon representations that the Z_2 symmetries of the up and down quark sector do not coincide so that the quark mixing cannot be trivial. It turns out that the vacua of Z_2 symmetries generated by BA^k with k being either even or odd are degenerate so that we arrive at the mentioned four possible values for $|V_{ud}|$. Out of these only one is phenomenologically viable, namely $|V_{ud}| = \cos(\frac{\pi}{14}) \approx 0.97493$. The CKM matrix elements $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$ are as well predicted to be $|V_{us}| \approx |V_{cd}| \approx 0.2225$ and $|V_{cs}| \approx |V_{ud}| \approx 0.97493$. For the other elements we find the following orders of magnitude in $\epsilon \approx \lambda^2$ (including Z_2 subgroup breaking effects): $|V_{cb}|, |V_{ts}|, |V_{ub}|, |V_{td}| \sim \epsilon \approx \lambda^2$ and $|V_{tb}| = 1 + \mathcal{O}(\epsilon^2) = 1 + \mathcal{O}(\lambda^4)$. Thus, $|V_{td}|$ and $|V_{ub}|$ turn out to be slightly too large. The same is true for J_{CP} which is of the order of $\epsilon^2 \approx \lambda^4$ instead of λ^6. However, it only requires a moderate tuning of the parameters of the model to accommodate the experimentally measured values. All quark masses are appropriately reproduced. The large top quark mass results from the fact that the top quark is the only fermion acquiring a mass at the renormalizable level. Since the bottom quark mass stems from an operator involving one flavon, the correct ratio $m_b/m_t \sim \epsilon$ is produced without large $\tan \beta$. The hierarchy $m_u : m_c : m_t \sim \epsilon^4 : \epsilon^2 : 1$ in the up quark sector is accommodated in the Z_2 subgroup conserving limit. Thereby, the suppression of the up quark mass is (partly) due to the non-vanishing FN charge of the right-handed up quark. The correct order of magnitude of the strange quark mass can as well be achieved through the FN mechanism. The down quark mass which vanishes at the lowest order is generated by operators with two-flavon insertions. Also its correct size is guaranteed by the FN mechanism. The main problem which cannot be solved in this model is the fact that the parameter $k_{(d)}$—and therefore also $|V_{ud}|$—is not uniquely fixed, but can take a certain number of different values. We presume that a new type of mechanism for the vacuum alignment is necessary which also fixes the (absolute) phase of the VEVs of the flavons so that

the parameter $k_{(d)}$ is determined. One possibility might arise in models with extra dimensions. For a recent discussion of the breaking of a flavor symmetry with extra dimensions see [16].

As a next step, it is interesting to discuss the extension of our model to the leptonic sector. In the literature models with the dihedral flavor group D_3 ($\cong S_3$) [17] or D_4 [18] can found which also use the fact that different subgroups of the flavor symmetry are conserved in the charged lepton and neutrino (Dirac and right-handed Majorana neutrino) sector to predict the leptonic mixing angle θ_{23} to be maximal and θ_{13} to be zero. These models are non-supersymmetric and contain Higgs doublets transforming non-trivially under the flavor group in their original form. However, recently variants of [18] have been discussed whose framework is the MSSM and in which only gauge singlets break the flavor group spontaneously at high energies [19]. A possibility to combine such a variant with the model presented here by using a (possibly larger) dihedral group is worth studying.

As has been discussed in [2], the assignment $\underline{2} + \underline{1}$ for the left-handed and $\underline{1} + \underline{1} + \underline{1}$ for the right-handed fields is not the only possible one in order to predict one element of the mixing matrix in terms of group theoretical quantities only. Alternatively, we can consider a model in which both, left- and right-handed fields, are assigned to $\underline{2} + \underline{1}$. Such an assignment usually emerges when we consider grand unified theories (GUTs), e.g. in $SU(5)$ where the left- and right-handed up quarks both reside in the representation $\underline{10}$.[7] However, the following problem might occur: the product $\underline{2} \times \underline{2}$ contains an invariant of the dihedral group, if left- and right-handed fields transform as the same doublet. The group theoretical reason is the fact that all two-dimensional representations of dihedral groups are real. The existence of the invariant leads to a degenerate mass spectrum among the first two generations, e.g. in an $SU(5)$ GUT to the prediction that up quark and charm quark mass are degenerate. One possibility to circumvent this difficulty might be to resort to a double-valued dihedral group. Such a group additionally possesses pseudo-real (two-dimensional) representations. One of their properties is that the product of a representation with itself contains the invariant/trivial representation $\underline{1}_1$ in its anti-symmetric part. In an $SU(5)$ model one can then use the fact that the contribution of a Higgs field in the GUT representation $\underline{5}$ to the up quark mass matrix leads to a symmetric mass matrix, in order to avoid the invariant coupling. However, it is still not obvious whether the mass hierarchy among the up quarks can be generated (through the FN mechanism) without tuning the parameters. Even in non-unified models in which the two-dimensional representations under which left- and right-handed fields transform do not have to be equivalent, it might not be obvious that the fermion mass hierarchy can be appropriately accommodated (with an additional FN symmetry).

Finally, further interesting aspects to analyze are the anomaly conditions holding for the flavor symmetry D_{14} which in general lead to additional constraints [20] as well as the origin of such a flavor symmetry, see for instance [21].

Acknowledgements

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[7] The case in which all three generations transform as singlets is not very appealing, since the up quark sector would be merely determined by an Abelian flavor symmetry rather than a non-Abelian one.

Appendix A. Kronecker products and Clebsch–Gordan coefficients

Here we list the explicit form of the Kronecker products as well as the Clebsch–Gordan coefficients. More general results for dihedral groups with an arbitrary index n can be found in [2,12].

A.1. Kronecker products

The products $\mathbf{1}_i \times \mathbf{1}_j$ are

$$\mathbf{1}_i \times \mathbf{1}_i = \mathbf{1}_1, \qquad \mathbf{1}_1 \times \mathbf{1}_i = \mathbf{1}_i \quad \text{for } i = 1, \ldots, 4,$$
$$\mathbf{1}_2 \times \mathbf{1}_3 = \mathbf{1}_4, \qquad \mathbf{1}_2 \times \mathbf{1}_4 = \mathbf{1}_3 \quad \text{and} \quad \mathbf{1}_3 \times \mathbf{1}_4 = \mathbf{1}_2.$$

For $\mathbf{1}_i \times \mathbf{2}_j$ we find

$$\mathbf{1}_{1,2} \times \mathbf{2}_j = \mathbf{2}_j \quad \text{and} \quad \mathbf{1}_{3,4} \times \mathbf{2}_j = \mathbf{2}_{7-j} \quad \text{for all } j.$$

The products of $\mathbf{2}_i \times \mathbf{2}_i$ decompose into

$$[\mathbf{2}_i \times \mathbf{2}_i] = \mathbf{1}_1 + \mathbf{2}_j \quad \text{and} \quad \{\mathbf{2}_i \times \mathbf{2}_i\} = \mathbf{1}_2,$$

where the index j equals $j = 2i$ for $i \leqslant 3$ and $j = 14 - 2i$ holds for $i \geqslant 4$. $[\nu \times \nu]$ denotes thereby the symmetric part of the product $\nu \times \nu$, while $\{\nu \times \nu\}$ is the anti-symmetric one. For the mixed products $\mathbf{2}_i \times \mathbf{2}_j$ with $i \neq j$ two structures are possible either

$$\mathbf{2}_i \times \mathbf{2}_j = \mathbf{2}_k + \mathbf{2}_l,$$

with $k = |i - j|$ and l being $i + j$ for $i + j \leqslant 6$ and $14 - (i + j)$ for $i + j \geqslant 8$. For $i + j = 7$ we find instead

$$\mathbf{2}_i \times \mathbf{2}_j = \mathbf{1}_3 + \mathbf{1}_4 + \mathbf{2}_k,$$

where k is again $|i - j|$.

A.2. Clebsch–Gordan coefficients

For $s_i \sim \mathbf{1}_i$ and $(a_1, a_2)^T \sim \mathbf{2}_j$ we find

$$\begin{pmatrix} s_1 a_1 \\ s_1 a_2 \end{pmatrix} \sim \mathbf{2}_j, \qquad \begin{pmatrix} s_2 a_1 \\ -s_2 a_2 \end{pmatrix} \sim \mathbf{2}_j, \qquad \begin{pmatrix} s_3 a_2 \\ s_3 a_1 \end{pmatrix} \sim \mathbf{2}_{7-j} \quad \text{and} \quad \begin{pmatrix} s_4 a_2 \\ -s_4 a_1 \end{pmatrix} \sim \mathbf{2}_{7-j}.$$

The Clebsch–Gordan coefficients of the product of $(a_1, a_2)^T$, $(b_1, b_2)^T \sim \mathbf{2}_i$ read

$$a_1 b_2 + a_2 b_1 \sim \mathbf{1}_1, \qquad a_1 b_2 - a_2 b_1 \sim \mathbf{1}_2, \qquad \begin{pmatrix} a_1 b_1 \\ a_2 b_2 \end{pmatrix} \sim \mathbf{2}_j \quad \text{or} \quad \begin{pmatrix} a_2 b_2 \\ a_1 b_1 \end{pmatrix} \sim \mathbf{2}_j$$

depending on whether $j = 2i$ as it is for $i \leqslant 3$ or $j = 14 - 2i$ which holds if $i \geqslant 4$. For the two doublets $(a_1, a_2)^T \sim \mathbf{2}_i$ and $(b_1, b_2)^T \sim \mathbf{2}_j$ we find for $i + j \neq 7$

$$\begin{pmatrix} a_1 b_2 \\ a_2 b_1 \end{pmatrix} \sim \mathbf{2}_k \quad (k = i - j) \quad \text{or} \quad \begin{pmatrix} a_2 b_1 \\ a_1 b_2 \end{pmatrix} \sim \mathbf{2}_k \quad (k = j - i),$$
$$\begin{pmatrix} a_1 b_1 \\ a_2 b_2 \end{pmatrix} \sim \mathbf{2}_l \quad (l = i + j) \quad \text{or} \quad \begin{pmatrix} a_2 b_2 \\ a_1 b_1 \end{pmatrix} \sim \mathbf{2}_l \quad (l = 14 - (i + j)).$$

If $i + j = 7$ holds the covariants read

$$a_1 b_1 + a_2 b_2 \sim \underline{1}_3, \qquad a_1 b_1 - a_2 b_2 \sim \underline{1}_4, \qquad \begin{pmatrix} a_1 b_2 \\ a_2 b_1 \end{pmatrix} \sim \underline{2}_k \quad \text{or} \quad \begin{pmatrix} a_2 b_1 \\ a_1 b_2 \end{pmatrix} \sim \underline{2}_k.$$

Again, the first case is relevant for $k = i - j$, while the second form for $k = j - i$.

Appendix B. Corrections to the flavon superpotential

In this appendix we discuss the form of the VEV shifts induced by the corrections of the flavon superpotential. These corrections can be written as

$$\Delta w_f = \Delta w_{f,u} + \Delta w_{f,d}.$$

We can parameterize the shifted VEVs as shown in Eq. (50). x remains unchanged, since it is a free parameter. As mentioned, since the complexity of the calculation is not increased, if k is not fixed, it is kept as parameter in the VEVs. For the actual calculation of the shifts we choose a plus sign in z^u and z^d in front of the square root as well as the positive root for v^u and v^d, see Eqs. (44) and (47). The corrections to the flavon superpotential, which induce shifts in the VEVs of the fields with an index u, are given by

$$\Delta w_{f,u} = \frac{1}{\Lambda} \left(\sum_{l=1}^{16} r_l^u I_l^{R,u} + \sum_{l=1}^{11} s_l^u I_l^{S,u} + \sum_{l=1}^{12} t_l^u I_l^{T,u} + \sum_{l=1}^{9} p_l^u I_l^{P,u} \right). \tag{B.1}$$

The invariants $I_l^{R,u}$ read

$$
\begin{aligned}
I_1^{R,u} &= \sigma^2 \big(\psi_1^d \psi_2^{0u} + \psi_2^d \psi_1^{0u} \big), & I_9^{R,u} &= \big(\psi_1^d \psi_2^{0u} + \psi_2^d \psi_1^{0u} \big) \big(\eta^d \big)^2, \\
I_2^{R,u} &= \sigma \big(\psi_1^d \chi_2^d \psi_1^{0u} + \psi_2^d \chi_1^d \psi_2^{0u} \big), & I_{10}^{R,u} &= \big(\psi_1^u \psi_2^{0u} + \psi_2^u \psi_1^{0u} \big) \big(\eta^u \big)^2, \\
I_3^{R,u} &= \big(\psi_1^d \psi_2^{0u} + \psi_2^d \psi_1^{0u} \big) \big(\psi_1^d \psi_2^d \big), & I_{11}^{R,u} &= \big(\psi_1^d \chi_1^d \xi_2^d \psi_1^{0u} + \psi_2^d \chi_2^d \xi_1^d \psi_2^{0u} \big), \\
I_4^{R,u} &= \big(\psi_1^u \psi_2^{0u} + \psi_2^u \psi_1^{0u} \big) \big(\psi_1^u \psi_2^u \big), & I_{12}^{R,u} &= \big(\psi_1^u \chi_1^u \xi_2^u \psi_1^{0u} + \psi_2^u \chi_2^u \xi_1^u \psi_2^{0u} \big), \\
I_5^{R,u} &= \big(\psi_1^d \psi_2^{0u} + \psi_2^d \psi_1^{0u} \big) \big(\chi_1^d \chi_2^d \big), & I_{13}^{R,u} &= \eta^d \big(\chi_1^d \xi_1^d \psi_1^{0u} - \chi_2^d \xi_2^d \psi_2^{0u} \big), \\
I_6^{R,u} &= \big(\psi_1^u \psi_2^{0u} + \psi_2^u \psi_1^{0u} \big) \big(\chi_1^u \chi_2^u \big), & I_{14}^{R,u} &= \eta^u \big(\chi_1^u \xi_1^u \psi_1^{0u} + \chi_2^u \xi_2^u \psi_2^{0u} \big), \\
I_7^{R,u} &= \big(\psi_1^d \psi_2^{0u} + \psi_2^d \psi_1^{0u} \big) \big(\xi_1^d \xi_2^d \big), & I_{15}^{R,u} &= \eta^d \big((\xi_1^d)^2 \psi_2^{0u} - (\xi_2^d)^2 \psi_1^{0u} \big), \\
I_8^{R,u} &= \big(\psi_1^u \psi_2^{0u} + \psi_2^u \psi_1^{0u} \big) \big(\xi_1^u \xi_2^u \big), & I_{16}^{R,u} &= \eta^u \big((\xi_1^u)^2 \psi_2^{0u} + (\xi_2^u)^2 \psi_1^{0u} \big).
\end{aligned} \tag{B.2}
$$

For $I_l^{S,u}$ we find

$$
\begin{aligned}
I_1^{S,u} &= \sigma \big(\psi_1^d \chi_1^d \varphi_2^{0u} + \psi_2^d \chi_2^d \varphi_1^{0u} \big), & I_7^{S,u} &= \big(\psi_1^u \chi_2^u \xi_1^u \varphi_2^{0u} + \psi_2^u \chi_1^u \xi_2^u \varphi_1^{0u} \big), \\
I_2^{S,u} &= \sigma \big(\psi_1^d \xi_2^d \varphi_1^{0u} + \psi_2^d \xi_1^d \varphi_2^{0u} \big), & I_8^{S,u} &= \big((\chi_1^d)^2 \psi_2^d \varphi_2^{0u} + (\chi_2^d)^2 \psi_1^d \varphi_1^{0u} \big), \\
I_3^{S,u} &= \sigma \eta^d \big(\xi_1^d \varphi_1^{0u} - \xi_2^d \varphi_2^{0u} \big), & I_9^{S,u} &= \big((\chi_1^u)^2 \psi_2^u \varphi_2^{0u} + (\chi_2^u)^2 \psi_1^u \varphi_1^{0u} \big), \\
I_4^{S,u} &= \big((\psi_1^d)^3 \varphi_2^{0u} + (\psi_2^d)^3 \varphi_1^{0u} \big), & I_{10}^{S,u} &= \eta^d \big((\chi_1^d)^2 \varphi_1^{0u} - (\chi_2^d)^2 \varphi_2^{0u} \big), \\
I_5^{S,u} &= \big((\psi_1^u)^3 \varphi_2^{0u} + (\psi_2^u)^3 \varphi_1^{0u} \big), & I_{11}^{S,u} &= \eta^u \big((\chi_1^u)^2 \varphi_1^{0u} + (\chi_2^u)^2 \varphi_2^{0u} \big), \\
I_6^{S,u} &= \big(\psi_1^d \chi_2^d \xi_1^d \varphi_2^{0u} + \psi_2^d \chi_1^d \xi_2^d \varphi_1^{0u} \big),
\end{aligned} \tag{B.3}
$$

and for $I_l^{T,u}$

$$
\begin{aligned}
&I_1^{T,u} = \sigma\big(\psi_1^d \xi_1^d \rho_2^{0u} + \psi_2^d \xi_2^d \rho_1^{0u}\big), &&I_7^{T,u} = \big(\chi_1^d \xi_1^d \psi_2^d \rho_2^{0u} + \chi_2^d \xi_2^d \psi_1^d \rho_1^{0u}\big), \\
&I_2^{T,u} = \sigma\eta^d\big(\chi_1^d \rho_1^{0u} - \chi_2^d \rho_2^{0u}\big), &&I_8^{T,u} = \big(\chi_1^u \xi_1^u \psi_2^u \rho_2^{0u} + \chi_2^u \xi_2^u \psi_1^u \rho_1^{0u}\big), \\
&I_3^{T,u} = \eta^d\big(\big(\psi_1^d\big)^2 \rho_1^{0u} - \big(\psi_2^d\big)^2 \rho_2^{0u}\big), &&I_9^{T,u} = \big(\big(\xi_1^d\big)^2 \psi_1^d \rho_1^{0u} + \big(\xi_2^d\big)^2 \psi_2^d \rho_2^{0u}\big), \\
&I_4^{T,u} = \eta^u\big(\big(\psi_1^u\big)^2 \rho_1^{0u} + \big(\psi_2^u\big)^2 \rho_2^{0u}\big), &&I_{10}^{T,u} = \big(\big(\xi_1^u\big)^2 \psi_1^u \rho_1^{0u} + \big(\xi_2^u\big)^2 \psi_2^u \rho_2^{0u}\big), \\
&I_5^{T,u} = \big(\psi_1^d \big(\chi_1^d\big)^2 \rho_2^{0u} + \psi_2^d \big(\chi_2^d\big)^2 \rho_1^{0u}\big), &&I_{11}^{T,u} = \eta^d\big(\chi_2^d \xi_1^d \rho_1^{0u} - \chi_1^d \xi_2^d \rho_2^{0u}\big), \\
&I_6^{T,u} = \big(\psi_1^u \big(\chi_1^u\big)^2 \rho_2^{0u} + \psi_2^u \big(\chi_2^u\big)^2 \rho_1^{0u}\big), &&I_{12}^{T,u} = \eta^u\big(\chi_2^u \xi_1^u \rho_1^{0u} + \chi_1^u \xi_2^u \rho_2^{0u}\big).
\end{aligned}
\tag{B.4}
$$

Finally, $I_l^{P,u}$ read

$$
\begin{aligned}
&I_1^{P,u} = \sigma^{0u}\big(\big(\psi_1^u\big)^2 \chi_2^u + \big(\psi_2^u\big)^2 \chi_1^u\big), &&I_6^{P,u} = \sigma^{0u}\sigma\big(\eta^d\big)^2, \\
&I_2^{P,u} = \sigma^{0u}\big(\big(\chi_1^u\big)^2 \xi_2^u + \big(\chi_2^u\big)^2 \xi_1^u\big), &&I_7^{P,u} = \sigma^{0u}\sigma^3, \\
&I_3^{P,u} = \sigma^{0u}\sigma \psi_1^d \psi_2^d, &&I_8^{P,u} = \sigma^{0u}\big(\big(\psi_1^d\big)^2 \chi_2^d + \big(\psi_2^d\big)^2 \chi_1^d\big), \\
&I_4^{P,u} = \sigma^{0u}\sigma \chi_1^d \chi_2^d, &&I_9^{P,u} = \sigma^{0u}\big(\big(\chi_1^d\big)^2 \xi_2^d + \big(\chi_2^d\big)^2 \xi_1^d\big), \\
&I_5^{P,u} = \sigma^{0u}\sigma \xi_1^d \xi_2^d.
\end{aligned}
\tag{B.5}
$$

The shifts in the VEVs of the set of fields $\{\psi_{1,2}^d, \chi_{1,2}^d, \xi_{1,2}^d, \eta^d, \sigma\}$ originate from non-renormalizable terms which are of the form

$$
\Delta w_{f,d} = \frac{1}{\Lambda}\left(\sum_{l=1}^{21} r_l^d I_l^{R,d} + \sum_{l=1}^{14} s_l^d I_l^{S,d} + \sum_{l=1}^{16} t_l^d I_l^{T,d} + \sum_{l=1}^{7} p_l^d I_l^{P,d}\right).
\tag{B.6}
$$

The invariants $I_l^{R,d}$ are the following

$$
\begin{aligned}
&I_1^{R,d} = \sigma^2\big(\psi_1^u \psi_2^{0d} + \psi_2^u \psi_1^{0d}\big), &&I_{12}^{R,d} = \big(\psi_1^d \psi_2^{0d} - \psi_2^d \psi_1^{0d}\big)\eta^u \eta^d, \\
&I_2^{R,d} = \sigma\big(\psi_1^d \chi_2^u \psi_1^{0d} + \psi_2^d \chi_1^u \psi_2^{0d}\big), &&I_{13}^{R,d} = \big(\psi_1^u \psi_2^{0d} + \psi_2^u \psi_1^{0d}\big)\big(\eta^d\big)^2, \\
&I_3^{R,d} = \sigma\big(\psi_1^u \chi_2^u \psi_1^{0d} + \psi_2^u \chi_1^u \psi_2^{0d}\big), &&I_{14}^{R,d} = \big(\psi_1^u \chi_1^d \xi_2^d \psi_1^{0d} + \psi_2^u \chi_2^d \xi_1^d \psi_2^{0d}\big), \\
&I_4^{R,d} = \big(\big(\psi_1^d\big)^2 \psi_2^u \psi_2^{0d} + \big(\psi_2^d\big)^2 \psi_1^u \psi_1^{0d}\big), &&I_{15}^{R,d} = \big(\psi_1^d \chi_1^u \xi_2^d \psi_1^{0d} + \psi_2^d \chi_2^u \xi_1^d \psi_2^{0d}\big), \\
&I_5^{R,d} = \big(\psi_1^u \psi_2^{0d} + \psi_2^u \psi_1^{0d}\big)\big(\psi_1^d \psi_2^d\big), &&I_{16}^{R,d} = \big(\psi_1^d \chi_1^u \xi_2^u \psi_1^{0d} + \psi_2^d \chi_2^u \xi_1^u \psi_2^{0d}\big), \\
&I_6^{R,d} = \big(\psi_1^d \psi_2^{0d} \chi_1^u \chi_2^u + \psi_2^d \psi_1^{0d} \chi_2^u \chi_1^u\big), &&I_{17}^{R,d} = \eta^d\big(\chi_1^u \xi_1^d \psi_1^{0d} - \chi_2^u \xi_2^d \psi_2^{0d}\big), \\
&I_7^{R,d} = \big(\psi_1^d \psi_2^{0d} \chi_2^d \chi_1^u + \psi_2^d \psi_1^{0d} \chi_1^d \chi_2^u\big), &&I_{18}^{R,d} = \eta^d\big(\chi_1^d \xi_1^u \psi_1^{0d} - \chi_2^d \xi_2^u \psi_2^{0d}\big), \\
&I_8^{R,d} = \big(\psi_1^u \psi_2^{0d} + \psi_2^u \psi_1^{0d}\big)\big(\chi_1^d \chi_2^d\big), &&I_{19}^{R,d} = \eta^u\big(\chi_1^d \xi_1^d \psi_1^{0d} + \chi_2^d \xi_2^d \psi_2^{0d}\big), \\
&I_9^{R,d} = \big(\psi_1^d \psi_2^{0d} \xi_1^d \xi_2^u + \psi_2^d \psi_1^{0d} \xi_2^d \xi_1^u\big), &&I_{20}^{R,d} = \eta^d\big(\xi_1^u \xi_1^u \psi_2^{0d} - \xi_2^u \xi_2^u \psi_1^{0d}\big), \\
&I_{10}^{R,d} = \big(\psi_1^d \psi_2^{0d} \xi_2^d \xi_1^u + \psi_2^d \psi_1^{0d} \xi_1^d \xi_2^u\big), &&I_{21}^{R,d} = \eta^u\big(\big(\xi_1^d\big)^2 \psi_2^{0d} + \big(\xi_2^d\big)^2 \psi_1^{0d}\big), \\
&I_{11}^{R,d} = \big(\psi_1^u \psi_2^{0d} + \psi_2^u \psi_1^{0d}\big)\big(\xi_1^d \xi_2^d\big).
\end{aligned}
\tag{B.7}
$$

The second set reads

$$
I_1^{S,d} = \sigma\big(\psi_1^u \chi_1^d \varphi_2^{0d} + \psi_2^u \chi_2^d \varphi_1^{0d}\big), \qquad I_8^{S,d} = \big(\psi_1^u \chi_2^d \xi_1^d \varphi_2^{0d} + \psi_2^u \chi_1^d \xi_2^d \varphi_1^{0d}\big),
$$

$$I_2^{S,d} = \sigma\big(\psi_1^d \chi_1^u \varphi_2^{0d} + \psi_2^d \chi_2^u \varphi_1^{0d}\big), \qquad I_9^{S,d} = \big(\psi_1^d \chi_2^u \xi_1^d \varphi_2^{0d} + \psi_2^d \chi_1^u \xi_2^d \varphi_1^{0d}\big),$$

$$I_3^{S,d} = \sigma\big(\psi_1^u \xi_2^d \varphi_1^{0d} + \psi_2^u \xi_1^d \varphi_2^{0d}\big), \qquad I_{10}^{S,d} = \big(\psi_1^d \chi_2^d \xi_1^u \varphi_2^{0d} + \psi_2^d \chi_1^d \xi_2^u \varphi_1^{0d}\big),$$

$$I_4^{S,d} = \sigma\big(\psi_1^d \xi_2^u \varphi_1^{0d} + \psi_2^d \xi_1^u \varphi_2^{0d}\big), \qquad I_{11}^{S,d} = \big((\chi_1^d)^2 \psi_2^u \varphi_2^{0d} + (\chi_2^d)^2 \psi_1^u \varphi_1^{0d}\big),$$

$$I_5^{S,d} = \sigma\eta^d\big(\xi_1^u \varphi_1^{0d} - \xi_2^u \varphi_2^{0d}\big), \qquad I_{12}^{S,d} = \big(\chi_1^u \chi_1^u \psi_2^d \varphi_2^{0d} + \chi_2^u \chi_2^u \psi_1^d \varphi_1^{0d}\big),$$

$$I_6^{S,d} = \sigma\eta^u\big(\xi_1^d \varphi_1^{0d} + \xi_2^d \varphi_2^{0d}\big), \qquad I_{13}^{S,d} = \eta^d\big(\chi_1^d \chi_1^u \varphi_1^{0d} - \chi_2^d \chi_2^u \varphi_2^{0d}\big),$$

$$I_7^{S,d} = \big((\psi_1^d)^2 \psi_1^u \varphi_2^{0d} + (\psi_2^d)^2 \psi_2^u \varphi_1^{0d}\big), \qquad I_{14}^{S,d} = \eta^u\big((\chi_1^d)^2 \varphi_1^{0d} + (\chi_2^d)^2 \varphi_2^{0d}\big) \qquad \text{(B.8)}$$

and $I_l^{T,d}$ are given by

$$I_1^{T,d} = \sigma\big(\psi_1^u \xi_1^d \rho_2^{0d} + \psi_2^u \xi_2^d \rho_1^{0d}\big), \qquad I_9^{T,d} = \big(\chi_1^u \xi_1^d \psi_2^d \rho_2^{0d} + \chi_2^u \xi_2^d \psi_1^d \rho_1^{0d}\big),$$

$$I_2^{T,d} = \sigma\big(\psi_1^d \xi_1^u \rho_2^{0d} + \psi_2^d \xi_2^u \rho_1^{0d}\big), \qquad I_{10}^{T,d} = \big(\chi_1^d \xi_1^u \psi_2^d \rho_2^{0d} + \chi_2^d \xi_2^u \psi_1^d \rho_1^{0d}\big),$$

$$I_3^{T,d} = \sigma\eta^u\big(\chi_1^d \rho_1^{0d} + \chi_2^d \rho_2^{0d}\big), \qquad I_{11}^{T,d} = \big(\chi_1^u \xi_1^d \psi_2^u \rho_2^{0d} + \chi_2^u \xi_2^d \psi_1^u \rho_1^{0d}\big),$$

$$I_4^{T,d} = \sigma\eta^d\big(\chi_1^u \rho_1^{0d} - \chi_2^u \rho_2^{0d}\big), \qquad I_{12}^{T,d} = \big((\xi_1^d)^2 \psi_1^u \rho_1^{0d} + (\xi_2^d)^2 \psi_2^u \rho_2^{0d}\big),$$

$$I_5^{T,d} = \eta^d\big(\psi_1^d \psi_1^u \rho_1^{0d} - \psi_2^d \psi_2^u \rho_2^{0d}\big), \qquad I_{13}^{T,d} = \big(\xi_1^u \xi_1^d \psi_1^u \rho_1^{0d} + \xi_2^u \xi_2^d \psi_2^d \rho_2^{0d}\big),$$

$$I_6^{T,d} = \eta^u\big((\psi_1^d)^2 \rho_1^{0d} + (\psi_2^d)^2 \rho_2^{0d}\big), \qquad I_{14}^{T,d} = \eta^d\big(\chi_2^u \xi_1^d \rho_1^{0d} - \chi_1^u \xi_2^d \rho_2^{0d}\big),$$

$$I_7^{T,d} = \big(\psi_1^u (\chi_1^d)^2 \rho_2^{0d} + \psi_2^u (\chi_2^d)^2 \rho_1^{0d}\big), \qquad I_{15}^{T,d} = \eta^d\big(\chi_2^d \xi_1^u \rho_1^{0d} - \chi_1^d \xi_2^u \rho_2^{0d}\big),$$

$$I_8^{T,d} = \big(\psi_1^d \chi_1^u \chi_1^d \rho_2^{0d} + \psi_2^d \chi_2^u \chi_2^d \rho_1^{0d}\big), \qquad I_{16}^{T,d} = \eta^u\big(\chi_2^d \xi_1^d \rho_1^{0d} + \chi_1^d \xi_2^d \rho_2^{0d}\big). \qquad \text{(B.9)}$$

For $I_l^{P,d}$ we find

$$I_1^{P,d} = \sigma^{0d}\big((\psi_1^d)^2 \chi_2^u + (\psi_2^d)^2 \chi_1^u\big), \qquad I_5^{P,d} = \sigma^{0d}\sigma\big(\psi_1^d \psi_2^u + \psi_2^d \psi_1^u\big),$$

$$I_2^{P,d} = \sigma^{0d}\big((\chi_1^d)^2 \xi_2^u + (\chi_2^d)^2 \xi_1^u\big), \qquad I_6^{P,d} = \sigma^{0d}\sigma\big(\chi_1^d \chi_2^u + \chi_2^d \chi_1^u\big),$$

$$I_3^{P,d} = \sigma^{0d}\big(\psi_1^d \chi_2^d \psi_1^u + \psi_2^d \chi_1^d \psi_2^u\big), \qquad I_7^{P,d} = \sigma^{0d}\sigma\big(\xi_1^d \xi_2^u + \xi_2^d \xi_1^u\big),$$

$$I_4^{P,d} = \sigma^{0d}\big(\chi_1^d \xi_2^d \chi_1^u + \chi_2^d \xi_1^d \chi_2^u\big). \qquad \text{(B.10)}$$

To actually calculate the shifts of the VEVs we take the parameterization given in Eq. (50) and plug this into the F-terms arising from the corrected superpotential. We then linearize the equations in δVEV and $1/\Lambda$ and can derive the following for the shifts of the flavons with index u from the F-terms of the driving fields $\psi_{1,2}^{0u}$, $\varphi_{1,2}^{0u}$, $\rho_{1,2}^{0u}$ and σ^{0u}

$$
\begin{aligned}
b_u\big(v^u \delta w_2^u + w^u(\delta v_1^u - \delta v_2^u)\big) + \frac{1}{\Lambda}\bigg\{ & r_1^u x^2 v^d + r_2^u x v^d w^d \\
& + (v^d)^3 e^{-\frac{\pi i k}{7}}\bigg[r_3^u - r_9^u\bigg(\frac{e_d z^d}{f_d w^d}\bigg)^2\bigg] + (v^u)^3\bigg[r_4^u + r_{10}^u\bigg(\frac{e_u z^u}{f_u w^u}\bigg)^2\bigg] \\
& + r_5^u (w^d)^2 v^d + r_6^u (w^u)^2 v^u + v^d (z^d)^2\bigg[r_7^u - r_{13}^u \frac{e_d}{f_d} - r_{15}^u\bigg(\frac{e_d z^d}{f_d w^d}\bigg)\bigg] \\
& + v^u (z^u)^2\bigg[r_8^u - r_{14}^u \frac{e_u}{f_u} - r_{16}^u\bigg(\frac{e_u z^u}{f_u w^u}\bigg)\bigg] + r_{11}^u v^d w^d z^d + r_{12}^u v^u w^u z^u\bigg\} = 0, \qquad \text{(B.11)}
\end{aligned}
$$

$$b_u\big(v^u\delta w_1^u + w^u(\delta v_2^u - \delta v_1^u)\big) + \frac{1}{\Lambda}\bigg\{ r_1^u \mathrm{e}^{-\frac{\pi ik}{7}} x^2 v^d + r_2^u \mathrm{e}^{-\frac{\pi ik}{7}} x v^d w^d$$

$$+ \big(v^d\big)^3 \mathrm{e}^{-\frac{2\pi ik}{7}}\bigg[r_3^u - r_9^u\bigg(\frac{e_d z^d}{f_d w^d}\bigg)^2 \bigg] + \big(v^u\big)^3\bigg[r_4^u + r_{10}^u\bigg(\frac{e_u z^u}{f_u w^u}\bigg)^2 \bigg] + r_5^u \mathrm{e}^{-\frac{\pi ik}{7}}\big(w^d\big)^2 v^d$$

$$+ r_6^u\big(w^u\big)^2 v^u + v^d\big(z^d\big)^2 \mathrm{e}^{-\frac{\pi ik}{7}}\bigg[r_7^u - r_{13}^u \frac{e_d}{f_d} - r_{15}^u\bigg(\frac{e_d z^d}{f_d w^d}\bigg) \bigg]$$

$$+ v^u\big(z^u\big)^2\bigg[r_8^u - r_{14}^u \frac{e_u}{f_u} - r_{16}^u\bigg(\frac{e_u z^u}{f_u w^u}\bigg) \bigg] + r_{11}^u \mathrm{e}^{-\frac{\pi ik}{7}} v^d w^d z^d + r_{12}^u v^u w^u z^u \bigg\} = 0,$$

(B.12)

$$a_u\big(v^u\delta w_2^u + w^u\delta v_2^u\big) + c_u\big(v^u\delta z_2^u + z^u\delta v_1^u\big) + d_u z^u\bigg[\delta\eta^u - \bigg(\frac{e_u v^u}{f_u w^u}\bigg)\delta z_1^u\bigg]$$

$$+ \frac{1}{\Lambda}\bigg\{ \mathrm{e}^{\frac{\pi ik}{7}} s_1^u x v^d w^d + \mathrm{e}^{\frac{\pi ik}{7}} x v^d z^d\bigg[s_2^u - s_3^u\bigg(\frac{e_d z^d}{f_d w^d}\bigg) \bigg] + s_4^u\big(v^d\big)^3 + s_5^u\big(v^u\big)^3$$

$$+ v^d w^d z^d \mathrm{e}^{\frac{\pi ik}{7}}\bigg(s_6^u - s_{10}^u \frac{e_d}{f_d} \bigg) + v^u w^u z^u\bigg(s_7^u - s_{11}^u \frac{e_u}{f_u} \bigg)$$

$$+ \mathrm{e}^{\frac{\pi ik}{7}} s_8^u v^d\big(w^d\big)^2 + s_9^u v^u\big(w^u\big)^2 \bigg\} = 0,$$

(B.13)

$$a_u\big(v^u\delta w_1^u + w^u\delta v_1^u\big) + c_u\big(v^u\delta z_1^u + z^u\delta v_2^u\big) + d_u z^u\bigg[\delta\eta^u - \bigg(\frac{e_u v^u}{f_u w^u}\bigg)\delta z_2^u\bigg]$$

$$+ \frac{1}{\Lambda}\bigg\{ \mathrm{e}^{-\frac{2\pi ik}{7}} s_1^u x v^d w^d + \mathrm{e}^{-\frac{2\pi ik}{7}} x v^d z^d\bigg[s_2^u - s_3^u\bigg(\frac{e_d z^d}{f_d w^d}\bigg) \bigg] + s_4^u \mathrm{e}^{-\frac{3\pi ik}{7}}\big(v^d\big)^3$$

$$+ s_5^u\big(v^u\big)^3 + v^d w^d z^d \mathrm{e}^{-\frac{2\pi ik}{7}}\bigg(s_6^u - s_{10}^u \frac{e_d}{f_d} \bigg)$$

$$+ v^u w^u z^u\bigg(s_7^u - s_{11}^u \frac{e_u}{f_u} \bigg) + s_8^u \mathrm{e}^{-\frac{2\pi ik}{7}} v^d\big(w^d\big)^2 + s_9^u v^u\big(w^u\big)^2 \bigg\} = 0,$$

(B.14)

$$f_u w^u\delta\eta^u + e_u\bigg(v^u\delta z_2^u - \frac{v^u z^u}{w^u}\delta w_1^u + z^u\delta v_2^u \bigg) + \frac{1}{\Lambda}\bigg\{ x v^d z^d \mathrm{e}^{\frac{2\pi ik}{7}}\bigg(t_1^u - t_2^u \frac{e_d}{f_d} \bigg)$$

$$- t_3^u \mathrm{e}^{\frac{\pi ik}{7}}\big(v^d\big)^3\bigg(\frac{e_d z^d}{f_d w^d}\bigg) - t_4^u\big(v^u\big)^3\bigg(\frac{e_u z^u}{f_u w^u}\bigg) + t_5^u \mathrm{e}^{\frac{2\pi ik}{7}} v^d\big(w^d\big)^2$$

$$+ t_6^u v^u\big(w^u\big)^2 + t_7^u \mathrm{e}^{\frac{2\pi ik}{7}} v^d w^d z^d + t_8^u v^u w^u z^u$$

$$- v^d\big(z^d\big)^2 \mathrm{e}^{\frac{2\pi ik}{7}}\bigg(t_9^u + t_{11}^u \frac{e_d}{f_d} \bigg) + v^u\big(z^u\big)^2\bigg(t_{10}^u - t_{12}^u \frac{e_u}{f_u} \bigg) \bigg\} = 0,$$

(B.15)

$$f_u w^u\delta\eta^u + e_u\bigg(v^u\delta z_1^u - \frac{v^u z^u}{w^u}\delta w_2^u + z^u\delta v_1^u \bigg) + \frac{1}{\Lambda}\bigg\{ x v^d z^d \mathrm{e}^{-\frac{3\pi ik}{7}}\bigg(t_1^u - t_2^u \frac{e_d}{f_d} \bigg)$$

$$+ t_3^u \mathrm{e}^{\frac{3\pi ik}{7}}\big(v^d\big)^3\bigg(\frac{e_d z^d}{f_d w^d}\bigg) - t_4^u\big(v^u\big)^3\bigg(\frac{e_u z^u}{f_u w^u}\bigg) + t_5^u \mathrm{e}^{-\frac{3\pi ik}{7}} v^d\big(w^d\big)^2$$

$$+ t_6^u v^u\big(w^u\big)^2 + t_7^u \mathrm{e}^{-\frac{3\pi ik}{7}} v^d w^d z^d + t_8^u v^u w^u z^u$$

$$- v^d\big(z^d\big)^2 \mathrm{e}^{-\frac{3\pi ik}{7}}\bigg(t_9^u + t_{11}^u \frac{e_d}{f_d} \bigg) + v^u\big(z^u\big)^2\bigg(t_{10}^u - t_{12}^u \frac{e_u}{f_u} \bigg) \bigg\} = 0,$$

(B.16)

$$g_u v^u \left(\delta v_1^u + \delta v_2^u\right) + l_u w^u \left(\delta w_1^u + \delta w_2^u\right) + n_u z^u \left(\delta z_1^u + \delta z_2^u\right) - 2 q_u v^u \left(\frac{e_u z^u}{f_u w^u}\right)\delta \eta^u$$

$$+ \frac{1}{\Lambda}\left\{2\left(p_1^u (v^u)^2 w^u + p_2^u (w^u)^2 z^u\right) + \mathrm{e}^{-\frac{\pi i k}{7}}(v^d)^2 x \left(p_3^u - p_6^u\left[\frac{e_d z^d}{f_d w^d}\right]^2\right)\right.$$

$$\left. + p_4^u (w^d)^2 x + p_5^u (z^d)^2 x + p_7^u x^3 + 2\left(p_8^u \mathrm{e}^{-\frac{\pi i k}{7}}(v^d)^2 w^d + p_9^u (w^d)^2 z^d\right)\right\} = 0. \quad \text{(B.17)}$$

Note that we replaced the mass parameter M_ψ^u by the VEV w^u. Analogously, we replace the dimensionless coupling m_ψ^d with the VEV w^d. We also frequently use the fact that k is an odd integer in order to simplify the phase factors appearing in the formulae.

Similarly, we can deduce another set of equations from the F-terms of the driving fields $\psi_{1,2}^{0d}$, $\varphi_{1,2}^{0d}$, $\rho_{1,2}^{0d}$ and σ^{0d} which gives rise to the shifts in the VEVs of the flavons $\psi_{1,2}^d$, $\chi_{1,2}^d$, $\xi_{1,2}^d$, η^d and σ

$$b_d \left(v^d \delta w_2^d + w^d \left(\delta v_1^d - \delta v_2^d\right)\right) + \frac{1}{\Lambda}\left\{r_1^d v^u x^2 + r_2^d \mathrm{e}^{-\frac{\pi i k}{7}} v^d w^u x + r_3^d \mathrm{e}^{\frac{\pi i k}{7}} v^u w^d x\right.$$

$$+ v^u (v^d)^2\left[r_4^d + \mathrm{e}^{-\frac{\pi i k}{7}} r_5^d - r_{12}^d \mathrm{e}^{\frac{3\pi i k}{7}}\left(\frac{e_d e_u z^d z^u}{f_d f_u w^d w^u}\right) - r_{13}^d \mathrm{e}^{-\frac{\pi i k}{7}}\left(\frac{e_d z^d}{f_d w^d}\right)^2\right]$$

$$+ v^d w^d w^u \left(\mathrm{e}^{\frac{\pi i k}{7}} r_6^d + \mathrm{e}^{-\frac{\pi i k}{7}} r_7^d\right) + r_8^d v^u (w^d)^2$$

$$+ v^d z^d z^u\left[\mathrm{e}^{\frac{2\pi i k}{7}} r_9^d + \mathrm{e}^{-\frac{2\pi i k}{7}} r_{10}^d - \mathrm{e}^{\frac{2\pi i k}{7}} r_{18}^d \frac{e_d}{f_d} - \mathrm{e}^{-\frac{2\pi i k}{7}} r_{20}^d\left(\frac{e_d z^d}{f_d w^d}\right)\right]$$

$$+ v^u (z^d)^2\left[r_{11}^d + \mathrm{e}^{-\frac{3\pi i k}{7}} r_{21}^d\left(\frac{e_u z^u}{f_u w^u}\right)\right] + v^u w^d z^d\left[r_{14}^d \mathrm{e}^{\frac{\pi i k}{7}} - r_{19}^d \mathrm{e}^{-\frac{3\pi i k}{7}}\left(\frac{e_u z^u}{f_u w^u}\right)\right]$$

$$\left. + v^d w^u z^d \mathrm{e}^{\frac{\pi i k}{7}}\left[r_{15}^d - r_{17}^d\left(\frac{e_d z^d}{f_d w^d}\right)\right] + r_{16}^d \mathrm{e}^{-\frac{2\pi i k}{7}} v^d w^d z^u\right\} = 0, \quad \text{(B.18)}$$

$$\mathrm{e}^{-\frac{\pi i k}{7}} b_d \left(v^d \delta w_1^d + w^d \left(\delta v_2^d - \delta v_1^d\right)\right) + \frac{1}{\Lambda}\left\{r_1^d v^u x^2 + r_2^d v^d w^u x + r_3^d \mathrm{e}^{-\frac{\pi i k}{7}} v^u w^d x\right.$$

$$+ v^u (v^d)^2\left[r_4^d \mathrm{e}^{-\frac{2\pi i k}{7}} + \mathrm{e}^{-\frac{\pi i k}{7}} r_5^d + r_{12}^d \mathrm{e}^{\frac{2\pi i k}{7}}\left(\frac{e_d e_u z^d z^u}{f_d f_u w^d w^u}\right)\right.$$

$$\left. - r_{13}^d \mathrm{e}^{-\frac{\pi i k}{7}}\left(\frac{e_d z^d}{f_d w^d}\right)^2\right] + v^d w^d w^u \left(\mathrm{e}^{-\frac{2\pi i k}{7}} r_6^d + r_7^d\right)$$

$$+ r_8^d v^u (w^d)^2 + v^d z^d z^u\left[\mathrm{e}^{-\frac{3\pi i k}{7}} r_9^d + \mathrm{e}^{\frac{\pi i k}{7}} r_{10}^d - \mathrm{e}^{-\frac{3\pi i k}{7}} r_{18}^d \frac{e_d}{f_d} - \mathrm{e}^{\frac{\pi i k}{7}} r_{20}^d\left(\frac{e_d z^d}{f_d w^d}\right)\right]$$

$$+ v^u (z^d)^2\left[r_{11}^d + \mathrm{e}^{\frac{3\pi i k}{7}} r_{21}^d\left(\frac{e_u z^u}{f_u w^u}\right)\right] + v^u w^d z^d\left[r_{14}^d \mathrm{e}^{-\frac{\pi i k}{7}} - r_{19}^d \mathrm{e}^{\frac{3\pi i k}{7}}\left(\frac{e_u z^u}{f_u w^u}\right)\right]$$

$$\left. + v^d w^u z^d \mathrm{e}^{-\frac{2\pi i k}{7}}\left[r_{15}^d - r_{17}^d\left(\frac{e_d z^d}{f_d w^d}\right)\right] + r_{16}^d \mathrm{e}^{\frac{\pi i k}{7}} v^d w^d z^u\right\} = 0, \quad \text{(B.19)}$$

$$\mathrm{e}^{\frac{\pi i k}{7}}\left[a_d \left(v^d \delta w_2^d + w^d \delta v_2^d\right) + c_d \left(v^d \delta z_2^d + z^d \delta v_1^d\right) - d_d z^d\left(\left[\frac{e_d v^d}{f_d w^d}\right]\delta z_1^d + \delta \eta^d\right)\right]$$

$$+ \frac{1}{\Lambda}\left\{s_1^d \mathrm{e}^{\frac{\pi i k}{7}} x v^u w^d + s_2^d x v^d w^u + x v^u z^d\left[s_3^d \mathrm{e}^{\frac{2\pi i k}{7}} - \mathrm{e}^{-\frac{2\pi i k}{7}} s_6^d\left(\frac{e_u z^u}{f_u w^u}\right)\right]\right.$$

$$+ xv^d z^u \left[s_4^d e^{-\frac{\pi i k}{7}} - s_5^d e^{\frac{3\pi i k}{7}} \left(\frac{e_d z^d}{f_d w^d} \right) \right] + s_7^d (v^d)^2 v^u + s_8^d e^{\frac{\pi i k}{7}} v^u w^d z^d$$

$$+ v^d w^u z^d e^{\frac{2\pi i k}{7}} \left(s_9^d - s_{13}^d \frac{e_d}{f_d} \right) + s_{10}^d e^{-\frac{\pi i k}{7}} v^d w^d z^u + s_{11}^d e^{\frac{2\pi i k}{7}} v^u (w^d)^2$$

$$+ s_{12}^d v^d w^d w^u - s_{14}^d e^{-\frac{2\pi i k}{7}} v^u (w^d)^2 \left(\frac{e_u z^u}{f_u w^u} \right) \bigg\} = 0, \tag{B.20}$$

$$e^{-\frac{2\pi i k}{7}} \left[a_d (v^d \delta w_1^d + w^d \delta v_1^d) + c_d (v^d \delta z_1^d + z^d \delta v_2^d) - d_d z^d \left(\left[\frac{e_d v^d}{f_d w^d} \right] \delta z_2^d + \delta \eta^d \right) \right]$$

$$+ \frac{1}{\Lambda} \bigg\{ s_1^d e^{-\frac{\pi i k}{7}} x v^u w^d + s_2^d e^{-\frac{\pi i k}{7}} x v^d w^u + x v^u z^d \left[s_3^d e^{-\frac{2\pi i k}{7}} - e^{\frac{2\pi i k}{7}} s_6^d \left(\frac{e_u z^u}{f_u w^u} \right) \right]$$

$$+ x v^d z^u \left[s_4^d + s_5^d e^{\frac{3\pi i k}{7}} \left(\frac{e_d z^d}{f_d w^d} \right) \right] + s_7^d e^{-\frac{2\pi i k}{7}} (v^d)^2 v^u + s_8^d e^{-\frac{\pi i k}{7}} v^u w^d z^d$$

$$+ v^d w^u z^d e^{-\frac{3\pi i k}{7}} \left(s_9^d - s_{13}^d \frac{e_d}{f_d} \right) + s_{10}^d v^d w^d z^u + s_{11}^d e^{-\frac{2\pi i k}{7}} v^u (w^d)^2 + s_{12}^d e^{-\frac{\pi i k}{7}} v^d w^d w^u$$

$$- s_{14}^d e^{\frac{2\pi i k}{7}} v^u (w^d)^2 \left(\frac{e_u z^u}{f_u w^u} \right) \bigg\} = 0, \tag{B.21}$$

$$e^{\frac{2\pi i k}{7}} \left[e_d \left(v^d \delta z_2^d + z^d \delta v_2^d - \frac{v^d z^d}{w^d} \delta w_1^d \right) - f_d w^d \delta \eta^d \right] + \frac{1}{\Lambda} \bigg\{ t_1^d e^{\frac{2\pi i k}{7}} x v^u z^d + t_2^d x v^d z^u$$

$$- t_3^d e^{-\frac{\pi i k}{7}} x v^u z^u \left(\frac{e_u w^d}{f_u w^u} \right) - t_4^d e^{\frac{3\pi i k}{7}} x v^d z^d \left(\frac{e_d w^u}{f_d w^d} \right)$$

$$- (v^d)^2 v^u \left[t_5^d e^{\frac{2\pi i k}{7}} \left(\frac{e_d z^d}{f_d w^d} \right) + t_6^d e^{-\frac{2\pi i k}{7}} \left(\frac{e_u z^u}{f_u w^u} \right) \right] + t_7^d e^{\frac{2\pi i k}{7}} v^u (w^d)^2 + t_8^d e^{\frac{\pi i k}{7}} v^d w^d w^u$$

$$+ w^u z^d v^d e^{\frac{\pi i k}{7}} \left[t_9^d - t_{14}^d \left(\frac{e_d z^d}{f_d w^d} \right) \right] + t_{10}^d v^d w^d z^u$$

$$+ w^d z^d v^u \left[t_{11}^d e^{\frac{3\pi i k}{7}} - t_{16}^d e^{-\frac{\pi i k}{7}} \left(\frac{e_u z^u}{f_u w^u} \right) \right] - t_{12}^d e^{\frac{3\pi i k}{7}} v^u (z^d)^2$$

$$+ z^u z^d v^d e^{-\frac{3\pi i k}{7}} \left(t_{13}^d + t_{15}^d \frac{e_d}{f_d} \right) \bigg\} = 0, \tag{B.22}$$

$$e^{-\frac{3\pi i k}{7}} \left[e_d \left(v^d \delta z_1^d + z^d \delta v_1^d - \frac{v^d z^d}{w^d} \delta w_2^d \right) - f_d w^d \delta \eta^d \right] + \frac{1}{\Lambda} \bigg\{ t_1^d e^{-\frac{2\pi i k}{7}} x v^u z^d$$

$$+ t_2^d e^{-\frac{\pi i k}{7}} x v^d z^u - t_3^d e^{\frac{\pi i k}{7}} x v^u z^u \left(\frac{e_u w^d}{f_u w^u} \right) + t_4^d e^{\frac{3\pi i k}{7}} x v^d z^d \left(\frac{e_d w^u}{f_d w^d} \right)$$

$$+ (v^d)^2 v^u \left[t_5^d e^{\frac{3\pi i k}{7}} \left(\frac{e_d z^d}{f_d w^d} \right) - t_6^d \left(\frac{e_u z^u}{f_u w^u} \right) \right] + t_7^d e^{-\frac{2\pi i k}{7}} v^u (w^d)^2 + t_8^d e^{-\frac{2\pi i k}{7}} v^d w^d w^u$$

$$+ w^u z^d v^d e^{-\frac{2\pi i k}{7}} \left[t_9^d - t_{14}^d \left(\frac{e_d z^d}{f_d w^d} \right) \right] + t_{10}^d e^{-\frac{\pi i k}{7}} v^d w^d z^u$$

$$+ w^d z^d v^u \left[t_{11}^d e^{-\frac{3\pi i k}{7}} - t_{16}^d e^{\frac{\pi i k}{7}} \left(\frac{e_u z^u}{f_u w^u} \right) \right]$$

$$- t_{12}^d e^{-\frac{3\pi i k}{7}} v^u (z^d)^2 + z^u z^d v^d e^{\frac{2\pi i k}{7}} \left(t_{13}^d + t_{15}^d \frac{e_d}{f_d} \right) \bigg\} = 0, \tag{B.23}$$

$$
e^{-\frac{\pi i k}{7}} v^d \left(g_d \big(\delta v_1^d + \delta v_2^d \big) - 2 q_d \left[\frac{e_d z^d}{f_d w^d} \right] \delta \eta^d \right) + l_d w^d \big(\delta w_1^d + \delta w_2^d \big)
$$

$$
+ n_d z^d \big(\delta z_1^d + \delta z_2^d \big) + \frac{2}{\Lambda} \left\{ \cos\left(\frac{\pi k}{7} \right) \big(p_1^d e^{-\frac{\pi i k}{7}} \big(v^d \big)^2 w^u + p_4^d w^d w^u z^d + p_6^d w^d w^u x \big) \right.
$$

$$
\left. + \cos\left(\frac{2\pi k}{7} \right) \big(p_2^d \big(w^d \big)^2 z^u + p_7^d x z^d z^u \big) + \cos\left(\frac{\pi k}{14} \right) e^{-\frac{\pi i k}{14}} v^d v^u \big(p_3^d w^d + p_5^d x \big) \right\} = 0.
$$

(B.24)

One can infer the generic size of the shifts of the VEVs from these equations. In the case of no accidental cancellation among the various terms present here we expect all of them to be of the order VEV^2 / Λ which is $\epsilon \text{VEV} \approx \epsilon^2 \Lambda$ for all VEVs being of the order $\epsilon \Lambda$ with $\epsilon \approx \lambda^2 \approx 0.04$.

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