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Fast Algorithm for Rotation and Translation Two-Dimensional Computed Tomography Accurate Reconstruction

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Abstract

RT (rotation and translation) scan mode for 2D (two-dimensional) CT (computed tomography), based on multiple axes, can scan large-sized work pieces and components with paraxial X-ray beam. Compared with the traditional second generation scan mode of TR (translation and rotation), it can save about 3/4 times of the scan time. The accurate reconstruction algorithm by region, can remove the artifacts around inner and outer borders in general reconstruction images. But a mass of zero values participate back-projection operations, and slow down the accurate algorithm operation. In this paper, projection data are grouped according to the regions they located before being back-projected. Data are selected or not according to the regions ID determined in advance during back-projection. So zero values are kicked out by their locations, before doing some time-consuming works such as reading, judging or accumulating the values and so on. And computer simulations show that the reconstruction algorithm saves at least half of the operation time according to the number of rotation centers.

Keywords:industrial CT; RT scan mode; large-sized work pieces; accurate reconstruction; fast algorithm

1. Introduction

X-ray CT (computed tomography) tests for large-sized work pieces and components are absolutely necessary in manufacturing industries, such as national defense, aeronautics and astronautics industry.[1] RT (rotation and translation) scan mode[2] for 2D (two-dimensional) CT tests can scan large-sized work pieces and components with paraxial X-ray beam. As shown in Figure 1(a), the scanned work piece rotates 360° at each of the *N* rotation centers in turn, and the *N* centers are equispaced in a line and parallel with the detector. X-ray is emitted at *M* angular positions circumferential equispaced during rotation, and the detector receives the X-ray and records the projection data simultaneously. Compared with the traditional second generation scan mode of TR (translation and rotation)[3], it can save about 3/4 times of the scan time.

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The accurate reconstruction algorithm by region, can remove the artifacts around inner and outer borders in general reconstruction images. N groups of data got at the N rotation centers by RT scan mode can be used by the algorithm: Data of No. i region maintains as original ($1 \le i \le N$), and the other (N-1) regions of data are interpolated, as shown in Figure 1(b), to filter the data of No. i region in the large; and the filtered data of No. i region $\tilde{P}_i(s,\beta)$ can participate back-projection.[4] $\tilde{P}_i(s,\beta)$ is required to rotate $\Delta\beta_i$, which is the (i-1) times of the fan beam angle γ_m of the X-ray source. As shown in Figure 2, after rotation, the angle:

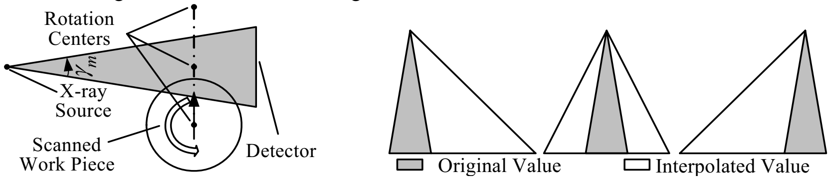

Fig. 1. (a) Scanning principle schematics (b) Accurate reconstruction schematics

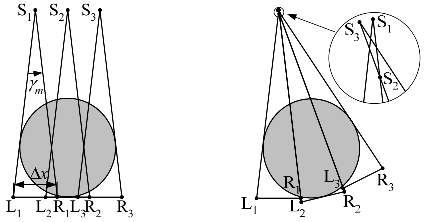

Fig. 2. (a) The filtered data of No. i region $\tilde{P}_i(s,\beta)$ (b) The rotated data of No. i region $\tilde{\tilde{P}}_i(s,\beta)$

$$\beta_i = \beta + \Delta\beta_i, \tag{1}$$

and the N regions of data $\tilde{\tilde{P}}_i(s,\beta) = \tilde{P}_i(s,\beta_i)$ ($1 \le i \le N$), in which, $\tilde{P}_i(s,\beta_i)$ is the result of $\tilde{P}_i(s,\beta)$'s unlimited extension on β axis in an 2π cycle. The reconstruction image can be got utilizing the third generation reconstruction formula[5] N times:

$$a(r,\theta) = \int_0^{2\pi} \sum_{i=1}^{N} \tilde{\tilde{P}}_i\left[s_i(\beta),\beta\right] U^2(\beta)\mathrm{d}\beta \tag{2}$$

in which $s_i(\beta)=U(\beta)[r\cos(\theta-\beta)-\tau_i]$, $U(\beta)=\mathrm{D}/[\mathrm{D}+r\sin(\beta-\theta)]$, and τ_i is the distance from No. i rotation center to the main X-ray. [4]

It's known from Equation (2) that a mass of zero values participate back-projection operations, and slow down the accurate algorithm operation. So projection data are grouped according to the regions they located before being back-projected, and data are selected or not according to the regions ID determined in advance during back-projection to speed up the algorithm.

2. Judgment of the projection region ID of any point in reconstruction image

Images in computer are expressed with rectangular coordinates, so utilizing the relation between rectangular coordinates and polar coordinates:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases},$$

Equation (2) can be transformed to:

$$a(x,y) = \int_0^{2\pi} \sum_{i=1}^{N} \tilde{\tilde{P}}_i\left[s_i(\beta),\beta\right] U^2(\beta)\mathrm{d}\beta \,, \tag{3}$$

in which $s_i(\beta)=U(\beta)(x\cos\beta+y\sin\beta-\tau_i)$, $U(\beta)=\mathrm{D}/(\mathrm{D}+x\sin\beta-y\cos\beta)$, (x, y) is rectangular coordinates of reconstruction images.

As shown in Figure 2(b), $\tilde{P}_i^{(\beta)}(s,\beta)$ ($1 \le i \le N$) is close to data of other regions in geometry, so when (x, y) and β are certain, $s_i(\beta)$ and projection need to be calculated N times. But it's known from Equation (3) that $\tilde{P}_i^{(\beta)}[s_i(\beta),\beta]$ is nonzero only once in N back-projections, and the other (N-1) back-projections are zero-valued accumulations and wastes of time. So an algorithm need to be found to judge whether $\tilde{P}_i^{(\beta)}[s_i(\beta),\beta]$ is zero or not before the calculation of $s_i(\beta)$ according to certain (x, y) and β. That is to judge the region ID of the point (x, y) in Figure 2(b), when β is certain.

The necessary and sufficient conditions for judging the region ID of the point (x_0, y_0) to be i are:
1) The point (x_0, y_0) and X-ray source S_i locate on the same side of the line $\overline{L_iR_i}$;
2) The points (x_0, y_0) and L_i locate on the same side of the line $\overline{S_iR_i}$;
3) The points (x_0, y_0) and R_i locate on the same side of the line $\overline{S_iL_i}$.

Which side of the line:

$$b + x\sin\alpha - y\cos\alpha = 0 \tag{4}$$

the point (x_0, y_0) locates on, can be judged by the flowing formula:

$$d = b + x_0\sin\alpha - y_0\cos\alpha, \tag{5}$$

in which α is the inclination of the line (α=0 means the positive direction of x axis, and counter-clockwise is positive), b is the directed distance from the origin of the coordinates to the line, d is the parameter to judge the side direction (d>0 or d<0 means the point (x_0, y_0) locates on one or the other side of the line respectively, d=0 means the point (x_0, y_0) is on the line, and $|d|$ is the distance from the point to the line), as shown in Figure 3.

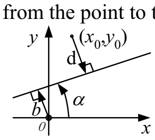

Fig. 3. Judgment of the relation between the point (x_0, y_0) and the line ($b+x\sin\alpha-y\cos\alpha=0$).

It's known from Figure 2(a) that all points in the reconstruction field of view satisfy the following conditions:
1) All points in No. i region and X-ray source S_i locate on the same side of the line $\overline{L_iR_i}$ ($1 \le i \le N$);
2) All points and the point L_N locate on the same side of the line $\overline{S_NR_N}$;
3) All points and the point R_1 locate on the same side of the line $\overline{S_1L_1}$;

It's also known from Figure 2(b) that the lines $\overline{S_iR_i}$ and $\overline{S_{i+1}L_{i+1}}$ ($1 \le i < N$) are coincident in the reconstruction field of view. Accordingly which region the point (x_0, y_0) locates can be judged by the following procedure shown in Figure 4, in which whether the point (x_0, y_0) and L_i locate on the same side of the line $\overline{S_iR_i}$ can be judged by determining if the two points in Equation (5) can get the two values of d to be the same sign.

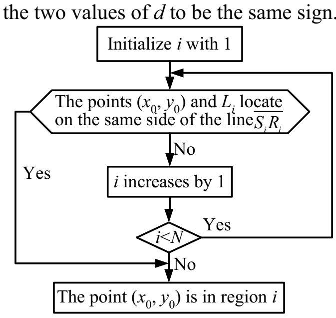

Fig. 4. Procedure for judging which region the point (x_0, y_0) locates

3. Simplification of judging the projection region ID of any point in reconstruction image

When β is certain, as shown in Figure 2(b), it's known from the relation of rotation that:

$$\begin{cases} x_i^R(\beta) = x_i^{R0}\cos\beta_i - y_i^{R0}\sin\beta_i \\ y_i^R(\beta) = x_i^{R0}\sin\beta_i + y_i^{R0}\cos\beta_i \\ \alpha_i^R(\beta) = \alpha_0 + \beta_i \end{cases} \tag{6}$$

in which, $R_i(x_i^R(\beta), y_i^R(\beta))$ is a point on the line $\overline{S_iR_i}$, $\alpha_i^R(\beta)$ is the inclination of the line $\overline{S_iR_i}$, (x_i^{R0}, y_i^{R0}) and α_0 are the location of R_i and the inclination of the line $\overline{S_1R_1}$ in Figure 2(a) respectively. When Equation (6) is put in Equation (3), the line $\overline{S_iR_i}$ can be expressed as:

$$b_i^R + x\sin(\alpha_0+\beta_i) - y\cos(\alpha_0+\beta_i) = 0,$$

in which $b_i^R = \cos(\alpha_0+\beta_i)(x_i^{R0}\sin\beta_i + y_i^{R0}\cos\beta_i) - \sin(\alpha_0+\beta_i)(x_i^{R0}\cos\beta_i - y_i^{R0}\sin\beta_i)$. Then the directed distance d from any point (x, y) in the field of view to the line $\overline{S_iR_i}$ is:

$$d_i(\beta) = \sin(\alpha_0+\beta_i)(x - x_i^{R0}\cos\beta_i + y_i^{R0}\sin\beta_i) - \cos(\alpha_0+\beta_i)(y - x_i^{R0}\sin\beta_i - y_i^{R0}\cos\beta_i).$$

The coordinate $(x_i^L(\beta), y_i^L(\beta))$ of the point L_i shown in Figure 2(b) can be expressed as:

$$\begin{cases} x_i^L(\beta) = x_i^{L0}\cos\beta_i - y_i^{L0}\sin\beta_i \\ y_i^L(\beta) = x_i^{L0}\sin\beta_i + y_i^{L0}\cos\beta_i \end{cases},$$

in which (x_i^{L0}, y_i^{L0}) is the coordinate of the point L_i in Figure 2(a). Then the directed distance d from the point L_i to the line $\overline{S_iR_i}$ is:

$$d_i^L(\beta) = (y_i^{R0} - y_i^{L0})\cos\alpha_0 - (x_i^{R0} - x_i^{L0})\sin\alpha_0.$$

The length of the detector is:

$$\Delta x = x_i^{R0} - x_i^{L0},$$

it's also known from Figure 2 that:

$$y_i^{R0} = y_i^{L0},$$

so,

$$d_i^L(\beta) = -\Delta x\sin\alpha_0.$$

It's known from Figure 2 that Δx and $\sin\alpha_0$ are permanent positive, so $d_i^L(\beta)$ is permanent negative. Then whether the points (x_0, y_0) and L_i locate on the same side of the line $\overline{S_iR_i}$ can be judged by determining if the following:

$$b_i^R + x_0\sin\alpha_i^R - y_0\cos\alpha_i^R$$

is also negative.

4. Back-projection according to the region ID

By the methods described above, which region the point (x_0, y_0) locates in Figure 2(b) can be judged. But doing this for all the points cost a lot of time. That can be simplified by doing this line by line. Points in a line are grouped by the regions they located, and the points are selected or not according to the regions ID determined in advance during back-projection. When β is certain, as shown in Figure 5, data in line j are grouped by the following steps.

1) Calculate the region ID of the two end points (P and S) of line j and name them Kp and Ks respectively. Kp and Ks can be assumed as $Kp \le Ks$ as well. Rename P and S to M_{Kp-1} and M_{Ks} respectively.
2) Calculate the intersection points M_i of the lines $\overline{M_{Kp-1}M_{Ks}}$ and $\overline{S_iR_i}$, $i=Kp, Kp+1, ..., Ks-1$.
3) The line $\overline{M_{Kp-1}M_{Ks}}$ is divided into (Ks-Kp+1) sections by M_i.

Points are grouped into regions $\overline{M_{i-1}M_i}$ in batches, in which i is the region ID and ranges from Kp to Ks. In this way, every line in the field of view can be easily divided into sections by calculating the region ID of the only two end points. Judgments of the points' region ID in the innermost loops are avoided, and reconstruction time is saved.

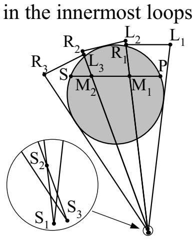

Fig. 5. Division on field of view for fast accurate reconstruction algorithm

5. Conclusion

Computer simulation results show its effects in Table 1. The same filtered projection data got from RT scan are back-projected and reconstructed by the fast accurate algorithm and the common accurate algorithm in the simulation.

Table 1. Reconstruction time (in millisecond) of comparison between the fast accurate reconstruction algorithm and the accurate reconstruction algorithm by region

reconstruction algorithm	N=3	N=4	N=5	N=6
fast accurate reconstruction algorithm	10953	11090	11152	11190
the accurate reconstruction algorithm by region	20766	23625	26265	28625

The reconstruction images' size, the total pixels to be back-projected and the numbers of the innermost loops are the same according to the two algorithms. Calculation of determining one point's region ID increases proportionally along with the number of rotation centers N. Judgments are in the innermost but one loop to reduce the increase of calculation in the fast accurate reconstruction algorithm. It's known from Table 1 that, compared with the accurate reconstruction algorithm by region, the fast accurate reconstruction algorithm saves at least half of the operation time, and the ratio of the saved time increases along with the number of rotation centers N.

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