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# Simulation of spinning soccer ball trajectories influenced by altitude

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**Abstract**

In soccer, the ball is affected by aerodynamic forces and consequently by altitude. Therefore, the objective of this paper was to simulate the influence of the altitude of the different venues of the FIFA Soccer World Cup 2010 based on trajectories of a spinning soccer ball. To simulate the ball's trajectories a free kick scenario with constant launch conditions was used, implemented with an iterative solving algorithm. The results showed that altitude influenced different flight parameters such as velocity, spin parameter or final position at the soccer goal which can have a major effect on goal scoring.



*Keywords:* Free Kick Simulation; Altitude; Aerodynamics; Soccer Ball Trajectory; FIFA Soccer World Cup 2010;

## 1. Introduction

Sport is an important factor in our society. Some people play sports for fame or for money while others play just for fun. The basic physical principles, however, are always the same. The athlete, the venue and the piece of sports equipment are dynamically affected by physical principles such as aerodynamic forces. In soccer, especially the ball is highly influenced by aerodynamic forces. In the literature, numerical methods are a common tool to simulate how the soccer ball is influenced by aerodynamic forces or how the trajectory of the soccer ball is influenced by changing different parameters such as velocity or spin rate [1], [2] or [3]. Further important factors influencing the trajectory of soccer balls can be named by the air drag coefficient and Magnus force coefficient as well as by changing air densities based on different altitudes. Different soccer ball trajectories due to different altitudes can adversely affect the anticipation skills of soccer players. However, in the literature there is a lack of scientific studies which consider the influence of the altitude on soccer balls, especially on spinning soccer balls. An event which particularly highlights the need of scientific studies in this direction is the FIFA Soccer World Cup 2010 in South Africa. The FIFA Soccer World Cup 2010 in South Africa provides venues with widely separated altitudes from ~ 10 to ~ 1750 m. Due to this fact, investigations were carried out which considered the influence of altitude on straightforward kicks. It could be seen that at a non-spinning 18 m free kick (initial velocity ~ 26 m $s^{-1}$) the ball in

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high-altitude Johannesburg would be around 41 cm ahead of the ball in sea-level near Durban [4]. Based on this investigation, it could be assumed that a spinning soccer ball would exhibit different trajectories at different altitudes. Therefore, the objective of this paper was to develop a simulation which was able to capture the influence of the altitudes of the venues of the FIFA Soccer World Cup 2010 on the trajectory of a spinning soccer ball based on a direct free kick with constant launch conditions.

## 2. Methods

### *2.1. Geophysical calculation*

The air density mainly depends on the altitude above sea-level. An altitude difference of more than 1700 m causes different air densities which are able to have a major effect on the trajectories of soccer balls. In addition to this, the air density is also a function of temperature and humidity. Due to the fact that a good deal of the matches of the FIFA Soccer World Cup 2010 start in the afternoon (4.00pm: 39 %) and in the evening (8.30pm: 45 %) an average temperature (5° C) based on the average night temperatures of the different venues in June was calculated and used for the geophysical calculation [8]. Furthermore, for the simulation of the direct free kick as well as for the geophysical calculation dry air was preconditioned based on the marginal influence of humidity on air density [4]. Based on the general equation of state of an ideal gas, the density ($\rho$) of dry air is given by the following equation

$$\rho = \frac{p}{R_L T} \tag{1}$$

where $p$ is the air pressure divided by the specific gas constant for dry air ($R_L$) and the temperature in Kelvin ($T$) [5]. The specific gas constant for dry air is the quotient of the universal gas constant ($R$ = 8.314 J mol$^{-1}$ K$^{-1}$) and the molar mass for dry air ($M_L$ = 28.965 x 10$^{-3}$ kg mol$^{-1}$). For calculating the pressure as a function of the altitude the barometric height equation was needed

$$p_{(a)} = p_0 e^{\left[\frac{-ga}{R_L T}\right]} \tag{2}$$

Equation (2) consists of two parts: The first part is the pressure at sea-level ($p_0$) which can be seen as the starting point of this calculation. The second part is the exponential relationship between altitude and air pressure which is calculated by the gravitational acceleration ($g$) and the altitude ($a$), divided by the specific gas constant for dry air and the temperature [6]. The air pressure at sea level was calculated by using the following equation [6]

$$p_0 = \frac{RT}{V_M} \tag{3}$$

where $V_M$ is the molar volume given by the quotient of the Avogadro constant ($N_A$ = 6.022 x 10$^{23}$ mol$^{-1}$) and the Loschmidt constant ($N_L$ = 2.687 x 10$^{25}$ m$^{-3}$) [7]. After considering these constants and equations, the general equation of state of an ideal gas can be rewritten as

$$\rho = \frac{M_L}{V_M} e^{\left[\frac{-ga}{R_L T}\right]} \tag{4}$$

### *2.2. Aerodynamic forces*

The simulation considered three aerodynamic forces for a soccer ball in flight: the air drag ($\overrightarrow{F_d}$), the Magnus force ($\overrightarrow{F_m}$) and the weight ($m\vec{g}$). The air drag ($\overrightarrow{F_d}$) can be described as force which acts against the flight direction of the soccer ball.

Therefore, the air drag acts opposite to the velocity vector of the ball ($\vec{v}$) and can be described by Figure 1 and the following equation

$$\overrightarrow{F_d} = -\frac{1}{2}\rho A v^2 C_d \overrightarrow{e_v} \quad (5)$$

where $\rho$ is the calculated air density based on the altitude, $A$ is the cross section of the soccer ball, $v$ is the flying velocity of the soccer ball, $C_d$ is the air drag coefficient and $\overrightarrow{e_v}$ is the unit vector of the ball's velocity. The Magnus force ($\overrightarrow{F_m}$) is caused by the well known Magnus effect and occurs when the ball is spinning while flying through the air. The vector of the Magnus force is perpendicular to the plane containing the ball's velocity vector ($\vec{v}$) and the rotation axis of the spinning ball (Fig.1), calculated using the following equation

$$\overrightarrow{F_m} = \frac{1}{2}\rho A v^2 C_m \overrightarrow{e_r} \times \overrightarrow{e_v} \quad (6)$$

where $\rho$, $A$ and $v$ are the same values used for air drag. $C_m$ is the Magnus force coefficient and $\overrightarrow{e_r} \times \overrightarrow{e_v}$ is the cross product given by the unit vector of the rotation axis $\overrightarrow{e_r}$ and the unit vector of the ball's velocity $\overrightarrow{e_v}$. The weight ($m\vec{g}$) is calculated by the mass of the ball and the gravitational acceleration which acts perpendicular to the ground (Fig.1).

### 2.3. Simulation

For the simulation of the direct free kick a 3D model was developed. The settings of the direct free kick were chosen that the soccer ball should cross the soccer goal at a point at the upper right-hand corner at the venue with the highest air density (Durban), based on a launch velocity of 25 m $s^{-1}$ [1]. Due to the strong relationship between soccer ball surface and critical Reynolds number ($Re_{crit}$), there is not a clear $Re_{crit}$ in the literature, where the boundary layer of the soccer ball undergoes the transition from turbulent to laminar flow. Asai *et al.* [9] showed $Re_{crit}$ in the range of ~ 2.2 x $10^5$ to ~ 3.0 x $10^5$ (~ 14 to ~ 19 m $s^{-1}$) for different soccer balls. Furthermore, Bray and Kerwin [1] indicated that the transition to post-critical conditions usually occurs when $Re$ > 2.1 x $10^5$ (~ 13.5 m $s^{-1}$). Therefore, it was assumed that with a launch velocity of 25 m $s^{-1}$ the soccer ball stayed throughout the whole flight phase in post-critical flow regimes. To mimic further realistic free kick conditions the free kick was simulated 20 m in front of the goal with an offset of 2 m to the right side (+$y$ direction) from the middle of the goal. Furthermore, the soccer ball had to fly above a defensive wall. The distance from the opponents to the soccer ball was 9.15 m (10 yds), the minimum distance stated in the official FIFA "Laws of the Game". As a result of the defensive wall, the given launch conditions and the aim that the soccer ball should cross the soccer goal at a point at the upper right-hand corner, an elevation angle of 20°, an azimuthal angle of 14° and a spin rate of 8.5 revolutions per second (rev $s^{-1}$) were chosen.

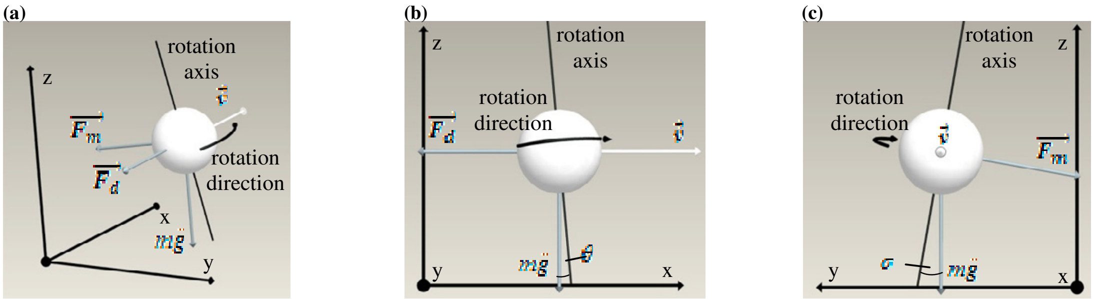


Fig. 1: Different views (a-c) of a flying and spinning soccer ball (white ball) including the forces which are acting while flying through the air. The positive x-axis represents the direction to the soccer goal. The z-axis illustrates the height and the y-axis stands for the lateral displacement. The air drag ($\overrightarrow{F_d}$) is opposite to the velocity ($\vec{v}$) and the Magnus force ($\overrightarrow{F_m}$) is perpendicular to the plane containing the ball's velocity and the rotation axis. The model shows a tilted rotation axis, $\theta$ = 5° in the x-z plane and $\sigma$ = 10° in the y-z plane. The weight ($m\vec{g}$) acts perpendicular to the ground (x-y plane).

An important parameter to describe the behavior of a spinning soccer ball during flying through the air is the dimensionless spin parameter (*Sp*) calculated by the radius *r* of the soccer ball (0.111 m) and the angular speed $\omega$ (given by the spin rate), divided by the velocity of the ball. Asai *et al.* [9] showed that for a spinning soccer ball the coefficients $C_d$ and $C_m$ are more influenced by *Sp* than by the Reynolds number in a range between ~ 22 and ~ 30 m $s^{-1}$. Additionally, a reasonably close linear relationship between *Sp* and $C_d$ as well as $Sp > 0.1$ and $C_m$ could be seen. Given this fact, a fundamental linear functional equation for both coefficients was calculated based on the investigations of Asai *et al.* [9]

$$C_d = Sp0.80 + 0.12 \tag{7}$$

$$C_m = Sp0.77 + 0.12 \tag{8}$$

This was important for variable $C_d$ and $C_m$ values during the flight of the soccer ball. Additionally, the spin rate decay (*SRD*) during the ball's flight was also taken into account. In the literature, there is a lack of scientific studies regarding the *SRD* during flights of spinning soccer balls. James and Haake [10] carried out investigations of the *SRD* of soccer balls for sub-critical conditions where the flow around the soccer ball was laminar. This can be seen as different to the assumed post-critical conditions of this simulation. However, James and Haake [10] mentioned that, as it had previously been discussed in the literature, in post-critical conditions it can be assumed that soccer balls and golf balls show a similar *SRD* based on their relatively similar surfaces. As a result, a *SRD* value of -0.00002Sp was used to calculate the reduction of the spin rate during the flight by the following equation

$$\omega = \omega_0 e^{\left[\frac{-tv0.00002}{r}\right]} \tag{9}$$

where $\omega_0$ is the initial angular speed of the soccer ball [11]. Based on Equation 9, an exponential decay of the spin rate could be calculated. A further approach to achieve realistic free kick conditions was to consider a tilted rotation axis. Therefore, a tilted angle of $\theta = 5°$ in the x-z plane was assumed based on video analyses of direct free kicks (Fig.1). This tilted angle of the rotation axis caused the fact that for the calculation of the spin parameter and the Magnus force only that part of the velocity was considered which flowed perpendicular to the rotation axis. The tilted angle ($\sigma = 10°$) in the y-z plane was the average of experiential results carried out by Bray and Kerwin [1]. For the calculation of the aerodynamic forces the cross section of the ball was $A = 0.0039$ m² with a mass of the ball of $m = 0.43$ kg. Related to Goff and Carré [3], it was assumed that the trajectories of the soccer ball were close enough to the earth's surface that the gravitational force could be seen as constant ($g = 9.81$ m $s^{-2}$).

### 2.4. Simulation process

For carrying out the simulation of the direct free kick, numerical mathematics methods were employed using an iterative algorithm ($\Delta t = 0.001$ sec) based on the described aerodynamic forces and initial conditions. The iterative algorithm consisted of three second-order ordinary differential equations for each dimension. The simulation was carried out with MATLAB R2007a (The MathWorks Inc., USA).

## 3. Results and Discussion

### 3.1. Air densities

The calculation of the different air densities of the venues of the FIFA Soccer World Cup 2010 in South Africa showed the highest air densities for Durban, Cape Town and Port Elizabeth (≈ 1.29 kg $m^{-3}$). Johannesburg was the venue at which the lowest air density could be calculated (≈ 1.04 kg $m^{-3}$). Figure 2 provides an overview of the air densities of all nine venues based on the calculated average night temperature in June (5° C). It could be clearly seen that the air densities were mainly influenced by the altitude of the different venues.

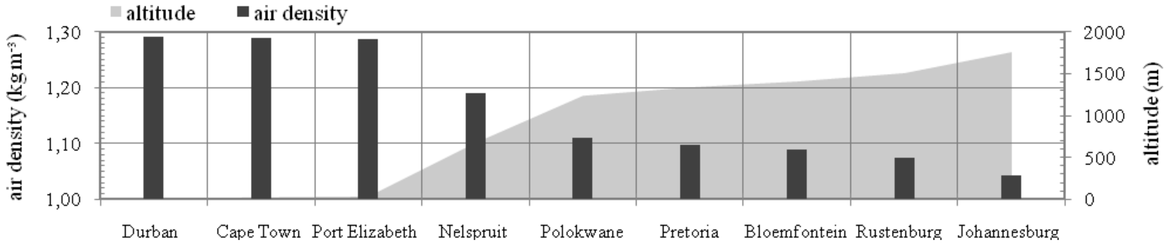


Fig. 2: Comparison of the air densities (kg $m^{-3}$) - left vertical axis - and the altitudes (m) - right vertical axis - of the different venues of the FIFA Soccer World Cup 2010 based on the average night temperature in June (5° C).

### 3.2. Comparison of soccer ball trajectories

The settings of the free kick were chosen so that at Durban (highest air density) the point of impact of the soccer ball was the upper right-hand corner of the soccer goal. Based on constant launch conditions, the point of impact varied caused by the different air densities of the venues. For providing an enhanced overview of these differences, Durban, Nelspruit, Polokwane and Johannesburg were chosen to demonstrate the influence of the altitude on a direct free kick (Fig. 3). Figure 3 (a) shows the different trajectories of the soccer ball influenced by altitude. The comparison of the points of impact at the soccer goal is displayed in Figure 3 (b). It could be calculated that only the free kick in Durban would result in a goal whereas in Nelspruit the ball would hit the post which could possibly result in a goal. In Polokwane and Johannesburg the free kick would miss the target. All in all, in only three (without the ball which hit the post) of nine venues of the FIFA Soccer World Cup 2010 the direct free kick would be successful based on the given free kick settings. The overall difference of the points of impact of the soccer ball only caused by altitude was ≈ 0.39 m (≈ 18 %) - altitudinal and ≈ 0.87 m (≈ 27 %) - lateral.

In addition to the different points of impact of the soccer ball, the altitude influenced also the flight parameters. Figure 4 (a) shows the decay of the ball velocity during the flight at different venues. The most decrease in velocity at the soccer goal of 9.99 m $s^{-1}$ (≈ 40 %) to 15.01 m $s^{-1}$ was obtained for Durban which confirmed the assumption that the soccer ball stayed throughout the whole flight phase in post-critical flow regimes. The difference of the velocity at the soccer goal between Durban and Johannesburg was 0.90 m $s^{-1}$ (≈ 6 %). Based on the high velocity decay (a mean of ≈ 38 %) and the low decrease of the spin rate (a mean of ≈ 0.5 %) an increasing spin parameter during the flight could be obtained (Fig. 4 (b)). Goff and Carré [3] proposed a decrease of the spin rate for a soccer ball flying a similar length to this simulation of about 10 %. Consequently, the considered *SRD* value for a soccer ball based on a golf ball is questionable. Moreover, it can be assumed that different air densities would also cause different *SRD* values, which highlights the need of further studies in this direction. However, the highest increased spin parameter of ≈ 0.18 (≈ 71 %) to ≈ 0.42 was obtained for Durban. The difference of the increased spin parameters after 20 m between Durban and Johannesburg was ≈ 0.03 (≈ 8 %). As discussed above, the spin parameter and the air drag coefficient ($C_d$) as well as Magnus force coefficient ($C_m$) show a reasonably close linear relationship. Therefore, Figure 4 (c) and (d) display an increasing of the coefficients (a mean of ~ 40 %) during the ball's flight. The difference between Durban and Johannesburg for both coefficients after 20 m was ≈ 0.03 (≈ 6 %).

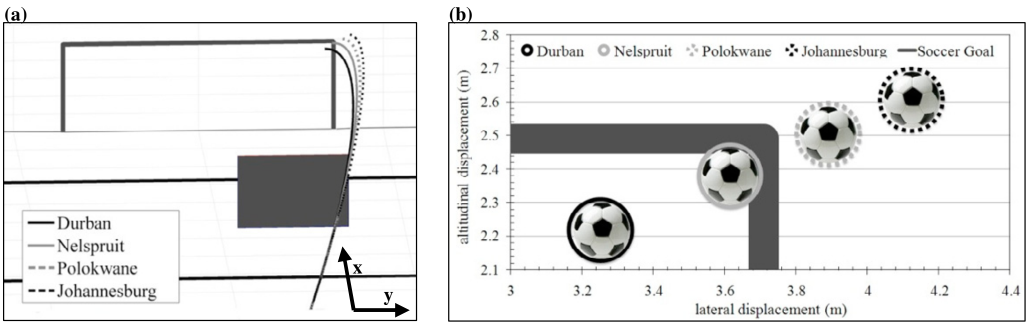


Fig. 3: (a) The different trajectories of the soccer ball influenced by altitude. (b) Comparison of the points of impact at the upper right-hand of the soccer goal.

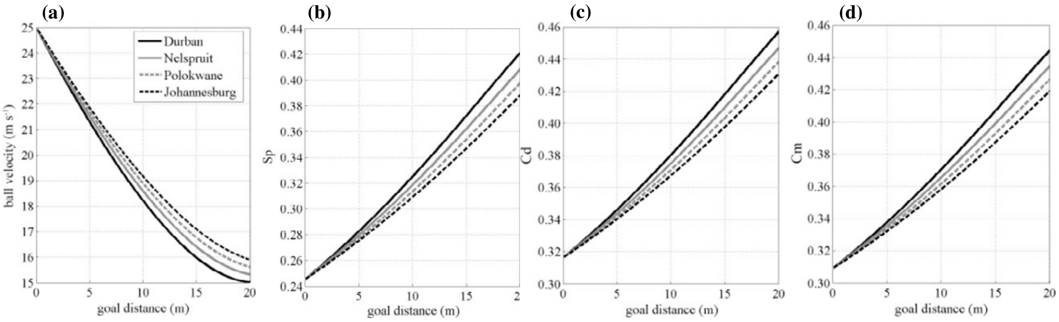


Fig. 4: Comparison of venues of the FIFA Soccer World Cup 2010 regarding ball velocity (a), spin parameter (b), air drag coefficient (c) and Magnus force coefficient (d) after 20 m (at soccer goal).

## 4. Conclusion

The objective of this paper was to develop a simulation which was able to capture the influence of the altitudes of the venues of the FIFA Soccer World Cup 2010 on the trajectory of a spinning soccer ball. It could be seen that for a direct free kick with constant launch conditions based a successful at Durban (at the upper right-hand corner of the soccer goal) only three of nine venues would see a successful kick. Anyway, this result mainly depends on the chosen and simulated free kick settings and can therefore not be directly related to other free kick scenarios. Overall, an altitudinal difference of ≈ 0.39 m as well as a lateral difference of ≈ 0.87 m of the soccer ball at the soccer goal was predicted across the venues. An influence of the altitudes on flight parameters such as velocity, spin parameter as well as air drag coefficient and Magnus force coefficient could also be seen. In particular, an overall velocity difference of ≈ 0.9 m $s^{-1}$ or ~ 3.2 km $h^{-1}$ can have a major effect on free kicks or shots. In addition to the effect on the anticipation skills of outfield players and goalkeepers, the different flight characteristics could also influence the free kick strategy. Which means that in low-altitude Durban it is probably more effective to use swerve kicks whereas in high-altitude Johannesburg little or non-spinning high-speed kicks could be more successful.

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