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Optimal design of FIR high pass filter based on L_1 error approximation using real coded genetic algorithm

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ABSTRACT

In this paper, an optimal design of linear phase digital finite impulse response (FIR) highpass (HP) filter using the L_1-norm based real-coded genetic algorithm (RCGA) is investigated. A novel fitness function based on L_1 norm is adopted to enhance the design accuracy. Optimized filter coefficients are obtained by defining the filter objective function in L_1 sense using RCGA. Simulation analysis unveils that the performance of the RCGA adopting this fitness function is better in terms of signal attenuation ability of the filter, flatter passband and the convergence rate. Observations are made on the percentage improvement of this algorithm over the gradient-based L_1 optimization approach on various factors by a large amount. It is concluded that RCGA leads to the best solution under specified parameters for the FIR filter design on account of slight unnoticeable higher transition width.

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1. Introduction

Digital filters are frequency selective device, which convolves the discrete signal amplitude with the specified impulse response in frequency domain. Thus, the filter extracts useful part of the input signal lying within its operating frequency range. Two broad categories in which digital filters are classified based on different criteria are finite impulse response (FIR) filter and infinite impulse response (IIR) filter [1]. The output of FIR filter depends on present and past values of input, hence there is no feedback network and are realized non-recursively. On the other hand, the output of IIR filter depends not only on previous inputs, but also on previous outputs with theoretically infinite impulse response in time and requires more storage element for the recursive IIR filter. FIR filter approaches the ideal response with the increase in filter order, thus the complexity and processing time increases. Whereas, IIR filters tends to be ideal at lower filter order on the account of obtaining non-linearity in phase and instability issues. In digital filtering applications, the FIR filters are often preferred over the IIR because of their inherent stability and the ability to provide a linear phase response over a wide frequency range.

The problem of filter design can be viewed as a constraint minimization problem, to meet all the requirement with an acceptable degree of accuracy for an optimal design. To find more efficient techniques and application based optimal solution is still an active field of research for the research community. There are different established techniques that exist for the design of FIR filter and its implementation [1,2]. The least-squares (LS) method minimizes the mean squared error (with the L_1-norm based fitness function) and is solved using the normal equations by Gaussian elimination. LS filters are popular and are extensively used in many applications [1,3–6]. Minimizing the LS error has the physical interpretation of energy minimization, which is also related to the signal to noise ratio colligated with the signals to be filtered. The resulting optimal filter demands the solution of a single linear system of equations, which can be solved efficiently. The eigenfilter method is one of the fastest ways to obtain an approximate filter [5]. This algorithm computes an eigenvector of an appropriate matrix to obtain the optimal filter coefficients in the LS sense but requires a large amount of calculations for solving the eigenvalue problem.

LS filters, however, results in overshoot at the discontinuity. Thus, the minimax measure of error is computed which minimizes the maximum absolute error value (obtained by varying the filter coefficients) in the filter response [7,8]. FIR filter design is achieved

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on minimization of Chebyshev error (with the L_∞-norm based fitness function) using linear programming techniques [9,10]. It is more efficiently accomplished with the Parks–McClellan algorithm [1,7] which renders a minimum Chebyshev error by employing the Remez exchange algorithm obtaining equal ripples in frequency domain.

Another common method, related to the least-squares approach, is the windowing technique with easy implementation [11,12]. In this, LS error approximation to an ideal low-pass filter is truncated by the multiplication of the infinite ideal impulse response and a relatively simple time domain window. They find importance in short-time Fourier analysis and their use in filter design evolved from the demand of having a simple method which reduces the Chebyshev error resulting from the Gibbs phenomenon in the LS error approximation of an ideal frequency response that has a discontinuity. However, it destroys the minimum squared error optimality of the original approximation and have inexplicit effects on the frequency response. Choice has to be made from a variety of window functions on the grounds of the amount of reduction of the ripples, acceptable range of the transition region and on the ease of calculations.

From last few decades, numerous significant results and effective algorithms have been developed in the L_1-approximation theory [13]. Conventionally, L_1-norm was adopted in several engineering applications, particularly in robust estimation problems, basis pursuit and sparse representations [14]. In the design of filters, the study of L_1-approximation is mainly concerned with the problems of uniqueness and characterization [15] and with the purposes of smoothing and deconvolution [16,17]. An L_1-approximation based method for the synthesis of digital FIR filters with the objective to optimize the filter parameters such that their frequency responses approaches to that of ideal ones was introduced in [18]. This was achieved by minimizing the L_1-norm of the error between the frequency response of the filter and the desired ideal response and forming a mathematical optimization problem such that it becomes solvable by the linear programming technique. This made the solution of the original problem practicable and efficient. Results in [19] portray that the optimal L_1 filters outcomes a flatter response in the passband and stopband than those of the L_1 and L_∞ filters, while retain a transition band which is comparable to that of the least-squares. Applying the mathematical theory of L_1 filters [15], it was demonstrated that the error function is differentiable, the Hessian matrix was deduced, condition for uniqueness was expressed and a modified globally convergent Newton algorithm was proposed to calculate the optimal filter. Further, it states that the uniqueness usually holds, and even when it does not, fast convergence will be observed.

These classical methods are related with some drawbacks due to which their computational cost increases with slow convergence rate and requires a handful experience for the tuning of filter parameters. FIR filter design being a multi-modal optimization problem, it requires a continuous and differentiable objective function. These techniques cannot optimize a non-uniform, non-differentiable, non-linear, multi-dimensional error fitness function, hence cannot converge to the global minimum results and usually diverge same local sub-optimal solution [20]. They have high sensitivity towards initial points as the number of solution parameters get increased as a result, their capability of searching decreases with an increased problem space. They also demand multiple runs to acquire optimized solutions. This necessitates algorithms with better control of parameters, fast and global convergence. This evolved to the design methods based on heuristic optimization algorithms.

In past research it is found that most of the evolutionary methods for the optimization of digital filters are computed with a differentiable fitness function such as least-square [21–25]. One of the such technique is GA which is developed by Holland [26]. It is a highly flexible population based bio-inspired global optimization technique, inspired by the Darwin's “Survival of the Fittest” and is employed for filter designing in the work reported in [27–30]. Other such algorithms used for finding the optimal filter parameters includes simulated annealing (SA), inspired from annealing in metallurgy [31]; differential evolution (DE) which is a randomized stochastic search technique based on reverse genes [32–34]; bat algorithm is based on the echolocation behavior of bats [35]; particle swarm optimization (PSO) simulates the behavior of bird flocking or fish schooling [24,36]. Filters designed with the above algorithms comprises of more ripples in the passband. To obtain a flatter passband and higher attenuation in the stopband, a novel fitness function based on the L_1 norm is defined. Finally the probabilistic optimization technique RCGA is incorporated with L_1 method to get the global solution with faster convergence.

In this paper the capability to approximate filter in L_1 sense and optimizing using RCGA fitted with L_1-norm is investigated for solving the Nth order digital FIR filter design problem. The multimodal objective function is chosen in L_1 sense under the constraints of differentiability and uniqueness in solution. The RCGA is employed to obtain nearly best solution in the designing of FIR HP filter. A good and comprehensive simulations results and their statistical analysis are showcased to justify the effectiveness of the algorithm.

The paper is organized as follows: Section 2 formulates the FIR filter design problem using the L_1 fitness function. In Section 3, the RCGA techniques using the L_1 fitness function, employed for designing the FIR filters is presented. Section 4 describes the linear phase FIR HP filter design examples along with the result analysis and comparative outcomes. Finally, the conclusions of the proposed work are highlighted in Section 5.

2. Problem formulation

The digital optimal FIR filter design procedure is based on the L_1-error approximation. The technique involves the evaluation of a weighted error function. The coefficients of the filter are then determined so as to minimize the absolute error that occurs. For the optimal design of Nth order FIR HP filter, the filter impulse response $h(n)$, $0 \le n \le N$, is approximated to the ideal frequency response, $H_{id}(e^{j\omega})$ specified as

$$H_{id}\left(e^{j\omega}\right) = \begin{cases} 0, & \omega \in [0, \omega_c) \text{ stopband} \\ 1, & \omega \in [\omega_c, \pi] \text{ passband} \end{cases} \tag{1}$$

The frequency response of the approximating filter, $H(e^{j\omega})$ obtained by computing the discrete time Fourier transform (DTFT) of filter impulse response, $h(n)$ is defined as

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{N} h(n) e^{-j\omega n} \tag{2}$$

Considering Type-I linear phase FIR filter with odd length and symmetric coefficient, $\{h(n) = h(N-n), 0 \le n \le N\}$, the amplitude response is given by [2,37].

$$H_r\left(e^{j\omega}\right) = h[M] + 2\sum_{n=1}^{M} h[M-n] \cos(\omega n) \tag{3}$$

where M = N/2 and $H_r(e^{j\omega})$ is the real valued function. Since $H_{id}(e^{j\omega})$ is zero-phase, approximating it by $H(e^{j\omega})$ is equivalent to approximating it by $H_r(e^{j\omega})$, and adding a delay of M-taps to $H_r(e^{j\omega})$ to make $H(e^{j\omega})$ causal.

Defining $a[0] = h[M]$ and $a[n] = 2h[M-n], 1 \le n \le M$, eq. (3) is rewritten as

$$H_r\left(e^{j\omega}\right) = H_r(\omega, \mathbf{a}) = \sum_{n=0}^{M} a[n]\cos(\omega n) \quad (4)$$

where $\mathbf{a} = (a(0), a(1),\ldots, a(M))$.

Now, for the approximation of the response $H_r(\omega,\mathbf{a})$ to the zero-phase ideal response $H_{id}(e^{j\omega})$, we obtain the error function $E(\omega,\mathbf{a})$ as

$$\begin{aligned} E(\omega, a) &= H_r(\omega, a) - H_{id}(\omega) \\ &= \sum_{n=0}^{M} a[n]\cos(\omega n) - H_{id}\left(e^{j\omega}\right) \end{aligned} \quad (5)$$

Various approximation methods are developed based on different definitions of the norm of the error function. The L_p norm approximation for the magnitude [38] response is defined as [38].

$$\|E(\omega, a)\|_p = \left\{ \int_0^{\pi} W(\omega)\left|\sum_{n=0}^{M} a[n]\cos(\omega n) - H_{id}\left(e^{j\omega}\right)\right|^p d\omega \right\}^{1/p} \quad (6)$$

Commonly used definitions includes L_1, L_2 or LS and L_∞ or chebyshev which are as follows.

Weighted error function in L_1-norm ($p = 1$) used for the design of FIR HP filter is given by

$$\|E(\omega, \mathbf{a})\|_1 = \int_0^{\pi} W(\omega)\left|\sum_{n=0}^{M} a[n]\cos(\omega n) - H_{\mathrm{id}}\left(e^{j\omega}\right)\right| d\omega \quad (7)$$

where $W(\omega)$ is a non-negative weighting function.

The existence of the optimal L_1 approximation is established in [13]. The mathematical analysis of L_1 approximation is more complex and challenging than least-square and Chebyshev norm mainly due to following reasons: (i) Differentiability of the L_1 norm of the error cannot be ensured, which leads to no closed-form solution of the optimal filter [12,13]. (ii) Approximation of $H_{\mathrm{id}}(e^{j\omega})$ over the entire digital frequency leads to unique solutions for both the L_2 and the L_∞ problems whereas it is not always assured in the L_1 solution [12]. (iii) There are many efficient optimization algorithms available for differentiable (such as L_2) and non-differentiable functions (such as L_∞). However, such efficient techniques were not developed for solving the L_1 non-linear optimization problem and minimizing the absolute error function due to above reasons. Grossmann et al. proposed a modified Newton's algorithm to calculate the optimal L_1 filter [19]. Under mild assumptions it examined that the L_1-norm can be differentiated and sometimes can be differentiated twice based on the first and second-order derivative theorems established in [15]. This resulted in optimal L_1 filters to have a flat response in the passband and stopband and a unique solution with a second order rate of convergence. In the next section, the design procedure of FIR L_1-norm based filter using RCGA is presented.

3. FIR design algorithms

In this section, the evolutionary search optimization scheme GA using real codes (RCGA) implemented on the L_1 fitness function is discussed in detail. The motive behind exploring and implementing L_1-norm based optimization is due to the smaller overshoot it yields around the discontinuity as compared with the norms, L_∞ and L_1 [39]. In passband, the L_1 based filter results in a flatter response than L_2 which happens to be its most desirable property. The design and optimization of linear phase FIR filters using L_1-norm based technique and its characteristic comparison with the minimax method is being demonstrated in [40].

3.1. L_1 based filter design using real coded genetic algorithm

Standard genetic algorithm is a bio-inspired optimization technique. It is based on the evolutionary ideas of natural selection and genetics wherein a set of coefficient chromosomes (analogous to the base-4 chromosomes in our own DNA) is randomly selected and are encoded as binary strings called genotypes [26]. However, binary coding reduces the precision level as it cannot represent the exact values. For higher precision optimization problems, the final local tuning potential of a binary coded GA is improved with the use of RCGA. With the use of real values, the solutions are represented very close to the natural formulation of the problem, avoiding coding and decoding processes. Thus the algorithm performs better in terms of speed of operation, efficiency and precision in results.

Based on the natural selection, the algorithm evolves through three operations after the initial population is randomly generated: selection, crossover and mutation. The selection operator gives preference to better individual genotype chromosome depending upon their fitness to produce a new generation of offspring chromosomes. Crossover refers to replacing some of the genes of one parent with that of the other. A heuristic crossover operator used, directs towards a better solution by determining the fitness values of the two parent chromosomes. Mutation chooses a subset of genes randomly and then change its allele value. The adaptive feasible mutation employed here, generates random modifications and are adaptive with respect to the last productions or abortive generations. Corresponding to each genotype, there is a decimal equivalent called phenotype which is used to evaluate cost function. According to the problem under consideration, each individual in the population is assigned by means of a cost function, a measure of its goodness. The fitness function used in the design of FIR HP filter based on L_1 approximation error criterion is expressed in (7). Best fitted chromosomes, called elite chromosomes are transmitted as it is to the next generation. With each generation, better solutions are obtained. To illustrate this algorithm, the algorithm flow is projected in Fig. 1 in the form of a flowchart for the design of FIR HP filter and the algorithm steps adopted for this work are ascertained in Table 1. RCGA with genetic operators including heuristic crossover and adaptive feasible mutation is applied for optimizing the coefficients in order to minimize the absolute magnitude error in L_1 norm.

4. Design examples and analysis

Extensive simulations have been carried out with the MATLAB 7.13 version on intel core i5, 3.20 GHz with 4 GB RAM. Filter Specifications, $\omega_s = 0.474\pi$, $\omega_p = 0.493\pi$, $\omega_c = 0.485\pi$ and $W(\omega) = 1$ are selected for the design of FIR HP filter of order 64, 52 and 40. Implementing similar steps and modifications in filter specifications, other FIR filters can also be designed. In order to demonstrate the effectiveness of the filter design method, several examples of FIR HP filter are constructed using the conventional Parks McClellan (PM) technique for the equiripple design of filters, the L_1-error optimization method [40] and the L_1-norm fitted RCGA approach.

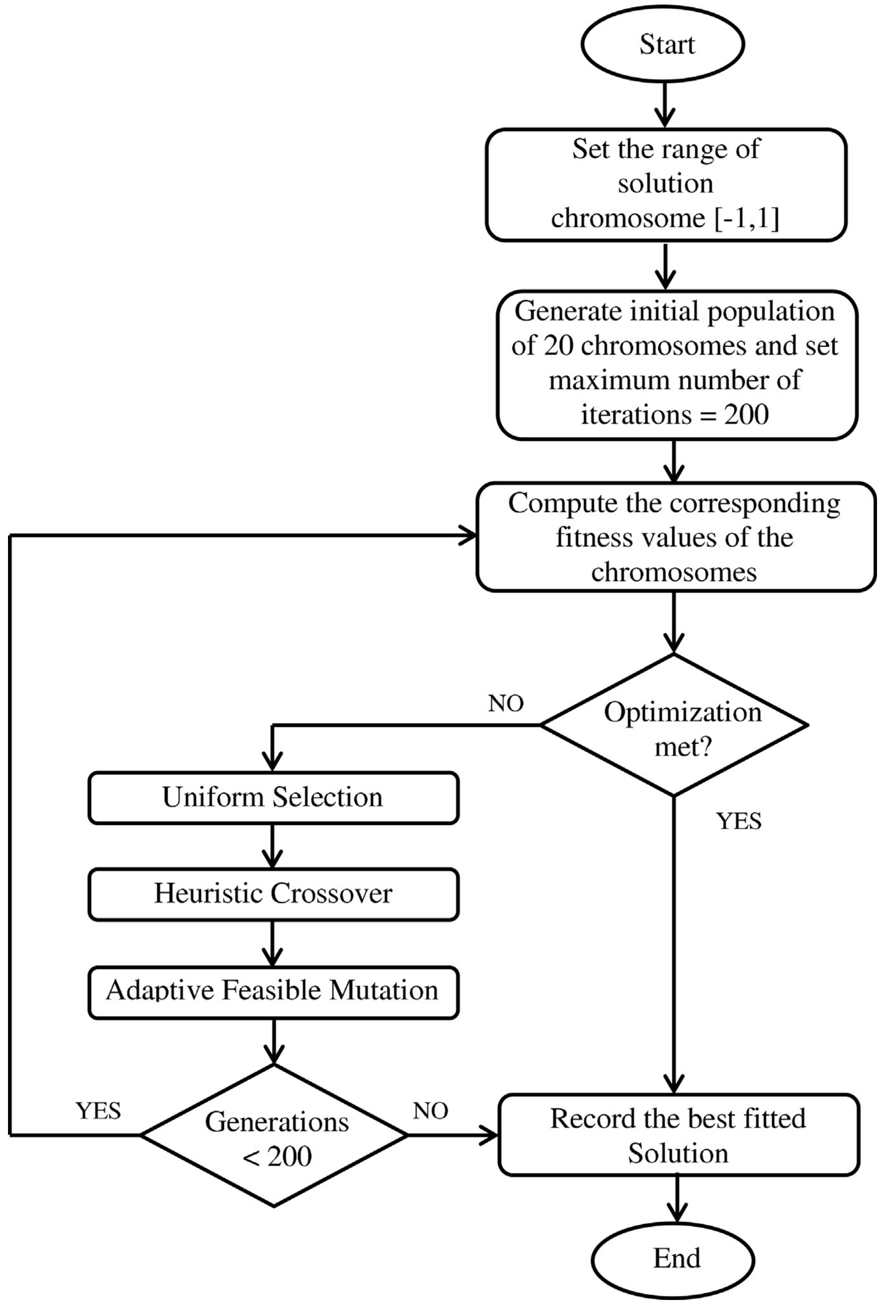

Fig. 1. Flow chart for real-coded genetic algorithm for filter design.

Table 1
Steps for real-coded genetic algorithm for filter design.

Step 1	Assign the fitness function, specifying the number of symmetric coefficients (41, 53 and 65) to be optimized for linear phase even Nth order filter. Adjust the upper and lower bound values of the unknown coefficient values as −1 and +1, respectively.
Step 2	Initialize the population size as 20. Select an initial random solution set of the chromosome strings, with each string consisting of a set of HP filter coefficients.
Step 3	Set up uniform select operator for random parent selection.
Step 4	Specify elite count at 2, guaranteeing their survival to the next generation and increasing the error fitness values from the minimum value.
Step 5	Heuristic crossover and adaptive feasible mutation are applied between two chromosomes to generate offsprings and to prevent redundancy in them, respectively.
Step 6	Genetic cycle keeps updating. Fitness function is evaluated for each coefficient and the least fitted coefficients are unexpended at each iteration.
Step 7	Cycle terminates with the achievement of fitness or else if the maximum number of generations i.e. 200 reached earlier.

Table 2
Control parameters for filter design.

Algorithm	Parameters	Symbol	Value
L_1 criteria	Accuracy of stopping condition	ε	10^{-6}
	Step size selection	σ	10^{-3}
		β	0.5
	Hessian matrix control	δ_1	10^{-15}
		δ_2	10^{15}
		μ	10^{-10}
RCGA based on L_1-norm	Population size		20
	Maximum Generations		200
	Crossover ratio		1.2
	Mutation rate		0.001
	Tolerance		10^{-6}

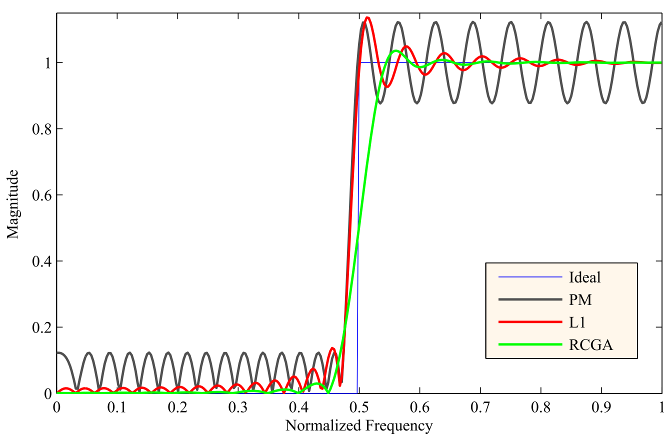

Fig. 2. Normalized magnitude response for the 64th order FIR HP filter using PM, L_1 and RCGA.

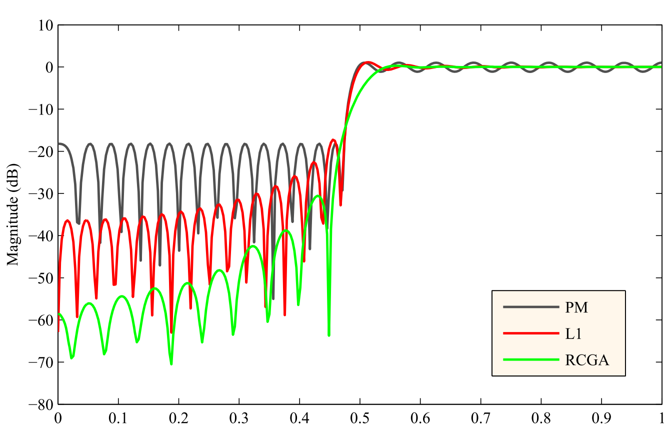

Fig. 3. Magnitude (dB) plot for the 64th order FIR HP filter using PM, L_1, RCGA.

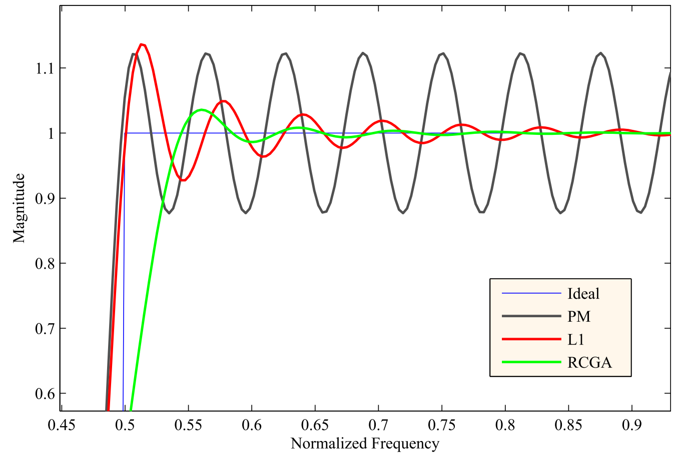

Fig. 4. Enlarged part of passband for the 64th order FIR HP filter using PM, L_1, RCGA.

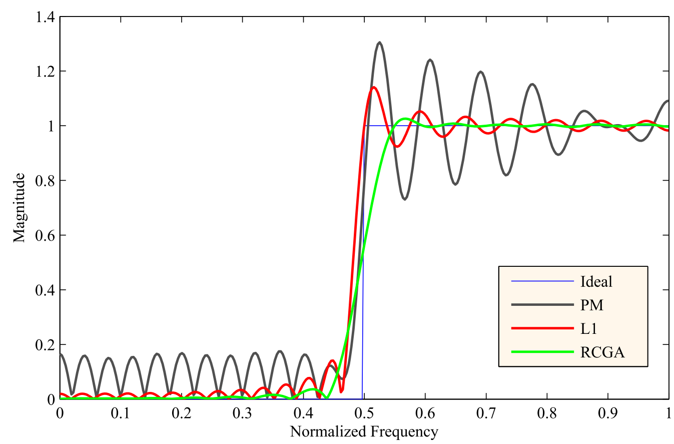

Fig. 5. Normalized Magnitude response for the 52nd order FIR HP filter using PM, L_1, RCGA.

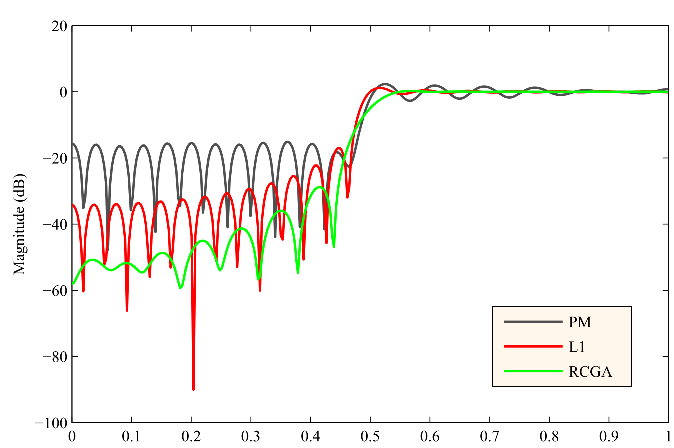

Fig. 6. Magnitude (dB) plot for the 52nd order FIR HP filter using PM, L_1, RCGA.

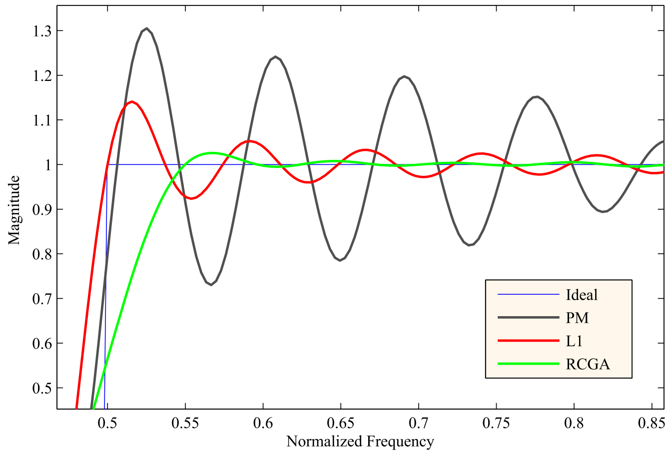

Fig. 7. Enlarged part of passband for the 52nd order FIR HP filter using PM, L_1, RCGA.

Table 3
Optimized coefficients of 64th order FIR HP filter.

Optimized Coefficients	L_1 Criteria	RCGA
$h(0) = h(64)$	0.015146255876377	−0.00744338647599136
$h(1) = h(63)$	0.001446518278107	0.01045904051077394
$h(2) = h(62)$	−0.015147386639465	0.00552532660344180
$h(3) = h(61)$	−0.002919610139636	−0.01543071660118081
$h(4) = h(60)$	0.015148333754533	−0.00664180269767027
$h(5) = h(59)$	0.004448228729674	0.03056096780597038
$h(6) = h(58)$	−0.015148907018739	−0.02234410640851608
$h(7) = h(57)$	−0.006064908594074	0.01523181095264736
$h(8) = h(56)$	0.015149552307051	−0.04081501900515115
$h(9) = h(55)$	0.007810332846147	0.04482374169896734
$h(10) = h(54)$	−0.015149965182132	0.01861081384044971
$h(11) = h(53)$	−0.009736391619451	−0.06681958255481965
$h(12) = h(52)$	0.015150443056203	−0.00034901677467067
$h(13) = h(51)$	0.011914440741549	0.12800374821245225
$h(14) = h(50)$	−0.015150787494709	−0.20197457580601488
$h(15) = h(49)$	−0.014446113906331	0.24359706715952387
$h(16) = h(48)$	0.015151125129604	−0.31257315441653477
$h(17) = h(47)$	0.017485002204957	0.30112156448937794
$h(18) = h(46)$	−0.015151434894113	−0.07991634251599974
$h(19) = h(45)$	−0.021276651832185	−0.19289802925661612
$h(20) = h(44)$	0.015151639126413	0.20650775992266740
$h(21) = h(43)$	0.026242545773922	0.03457877017111027
$h(22) = h(42)$	−0.015151923174587	−0.17734252920474555
$h(23) = h(41)$	−0.033176722925417	0.03490836368470307
$h(24) = h(40)$	0.015152000734801	0.15623888618840248
$h(25) = h(39)$	0.043776959331339	−0.12802446944049609
$h(26) = h(38)$	−0.015152252523245	−0.03515311239900284
$h(27) = h(37)$	−0.062455165180409	0.09109641605239659
$h(28) = h(36)$	0.015152216174266	−0.01172923763887420
$h(29) = h(35)$	0.105380849414094	−0.04680068968624766
$h(30) = h(34)$	−0.015152418796539	0.02078893503972301
$h(31) = h(33)$	−0.318069461439560	0.01492073993080113
$h(32)$	0.515152287788923	−0.01190284311016329

The parameters selected for the L_1 optimization method for the algorithm in [40] and for the RCGA design are listed in Table 2. In the RCGA design, the population size is fixed to a moderate value of 20 chromosomes, which will be selected in each generation from the solution space. With increase in this value, the execution time of the algorithm increases. To stop the algorithm cycle, maximum generations are set to 200 and after many runs, best solutions are reported in this work. Crossover ratio is set to 1.2, which determines the location of the next better solution depending upon the fitness calculated for the two parent chromosomes. Low mutation rate set to 0.001, avoids the state of premature convergence and yields good solutions. To illustrate the applicability of the these design methods and demonstrate their performance, simulation results have been shown for different filter orders. Figs. 2, 5 and 8 show the normalized magnitude responses of the HP FIR filters of orders 64, 52 and 40, respectively designed using PM, L_1 and RCGA approach. The enlarged normalized passband ripple plots for FIR HP filter of order 64, 52 and 40 are presented in Figs. 4, 7 and 10, respectively. It can be clearly observed from these figures that the RCGA produces lesser amount of ripples as compared to the L_1

Table 4
Optimized coefficients of 52nd order FIR HP filter.

Optimized coefficients	L_1 criteria	RCGA
$h(0) = h(52)$	−0.018512855148131	0.02365964202275418
$h(1) = h(51)$	−0.002164228249954	−0.05241824972055864
$h(2) = h(50)$	0.018514197843736	0.06140309973607933
$h(3) = h(49)$	0.004388307902221	−0.08482429372217103
$h(4) = h(48)$	−0.018515369360543	0.14917209350335284
$h(5) = h(47)$	−0.006739537445658	−0.19656942536776736
$h(6) = h(46)$	0.018516033540716	0.18012202812737632
$h(7) = h(45)$	0.009299510160187	−0.15846095056105874
$h(8) = h(44)$	−0.018516802234127	0.17710520912907765
$h(9) = h(43)$	−0.012179063250980	−0.15164006626777740
$h(10) = h(42)$	0.018517282558632	0.02018810339185055
$h(11) = h(41)$	0.015538093263302	0.10031199876345445
$h(12) = h(40)$	−0.018517791536520	−0.09606367410164711
$h(13) = h(39)$	−0.019627761076648	0.06399592779236196
$h(14) = h(38)$	0.018518205785450	−0.12007527743316666
$h(15) = h(37)$	0.024874079707452	0.13453153881376595
$h(16) = h(36)$	−0.018518480829049	0.05638166993792691
$h(17) = h(35)$	−0.032074417348470	−0.28050974146899660
$h(18) = h(34)$	0.018518862728207	0.21551282356499255
$h(19) = h(33)$	0.042930393440369	0.09516689213162123
$h(20) = h(32)$	−0.018518922082850	−0.26453358925295023
$h(21) = h(31)$	−0.061855748192403	0.11631964989411217
$h(22) = h(29)$	0.018519260820452	0.10004230098804986
$h(23) = h(28)$	0.105023652240561	−0.12461597887509612
$h(24) = h(27)$	−0.018519138053696	0.01603778197793661
$h(25) = h(26)$	−0.317950505254568	0.04084756857099279
$h(27)$	0.518519394603916	−0.01985170697742463

Table 5
Optimized coefficients of 40th order FIR HP filter.

Optimized coefficients	L_1 criteria	RCGA
$h(0) = h(40)$	0.023803297857488	0.018108946179981
$h(1) = h(39)$	0.003588480298202	0.009747329529065
$h(2) = h(38)$	−0.023804938565766	−0.093737216841601
$h(3) = h(37)$	−0.007343562526707	0.108344311600013
$h(4) = h(36)$	0.023806460746769	−0.016126414841366
$h(5) = h(35)$	0.011465484386660	0.022813847687291
$h(6) = h(34)$	−0.023807251730943	−0.259236554402040
$h(7) = h(33)$	−0.016232311806167	0.470946009593071
$h(8) = h(32)$	0.023808156869954	−0.335542133192841
$h(9) = h(31)$	0.022091382078951	−0.028921175858973
$h(10) = h(30)$	−0.023808766295215	0.214332772694192
$h(11) = h(29)$	−0.029855678370731	−0.108291526328989
$h(12) = h(28)$	0.023809193230837	−0.023939413067756
$h(13) = h(27)$	0.041238758578560	0.011910133359787
$h(14) = h(26)$	−0.023809763623805	0.025002327850544
$h(15) = h(25)$	−0.060665482593505	0.016425102702389
$h(16) = h(24)$	0.023809763947657	−0.049209133560270
$h(17) = h(23)$	0.104315963320762	0.007850484496350
$h(18) = h(22)$	−0.023810270271901	0.029660751082517
$h(19) = h(21)$	−0.317716138499056	−0.011119738508971
$h(20)$	0.523809946464530	−0.009093547648034

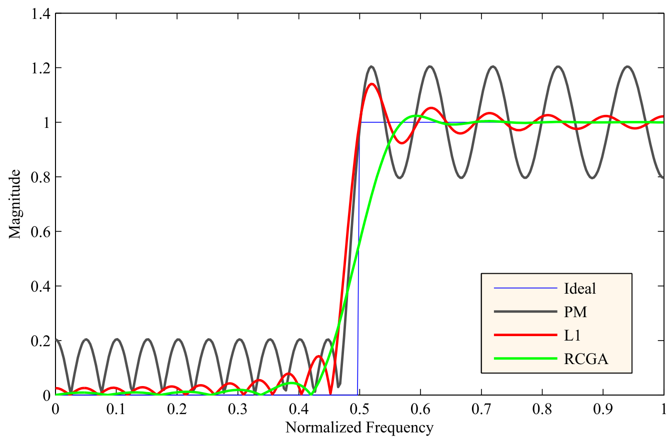

Fig. 8. Normalized Magnitude response for the 40th order FIR HP filter using PM, L_1, RCGA.

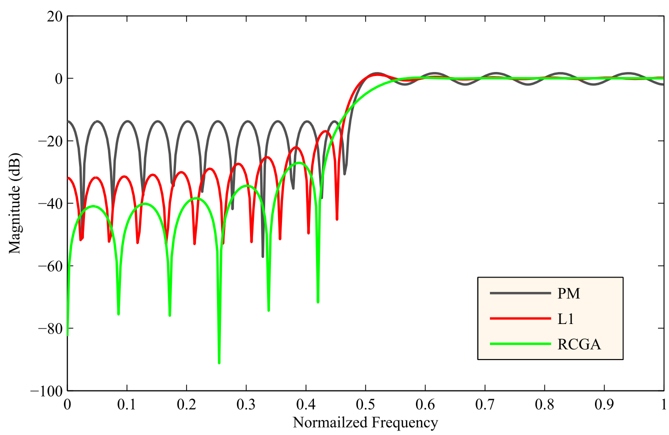

Fig. 9. Magnitude (dB) plot for the 40th order FIR HP filter using PM, L_1, RCGA.

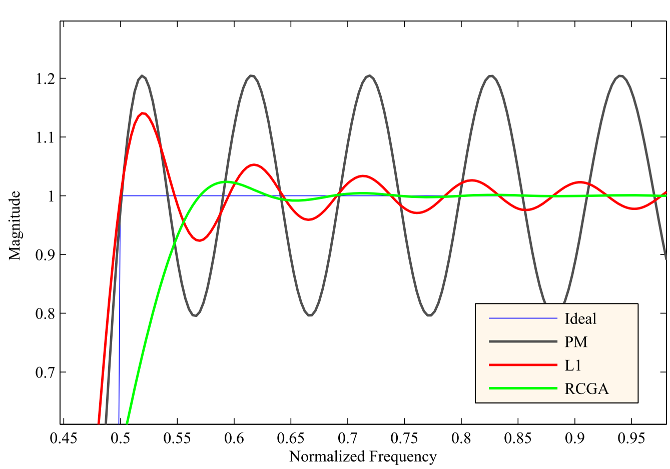

Fig. 10. Enlarged part of passband for the 40th order FIR HP filter using PM, L_1, RCGA.

method. The optimized coefficients incurred by the two design algorithms are recorded in Tables 3–5 for the filter orders 64, 52 and 40, respectively.

The magnitude (dB) plots for the HP FIR filters of orders 64, 52 and 40 are depicted in Figs. 3, 6 and 9, respectively for all the three above mentioned design techniques. An ideal FIR filter grants zero attenuation to the signal in passband and highest possible blockage in the stopband. As examined from the figures, RCGA FIR filter design tends to set about the desired ideal filter response with maximum passband attenuation, A_{pass} and least stopband attenuation, A_{stop}, as compared to that of L_1 method and the PM minimax filter design method.

The comparative result analysis for all above mentioned filter orders are testified and presented in Tables 6–8. The optimized results are obtained after 25 runs of design algorithm. Minimum A_{stop} obtained for the 64th order RCGA based filter is −30.58 dB (with an average value of −51.28 dB) whereas it is −17.27 dB (with an average of −37.13 dB) for the L_1 optimization filter and −18.29 dB (with an average of −26.68 dB) for the PM filter. This is much more than the RCGA based on L_1-norm design. Similarly, it is noticed for the other order filters and are remarked in Table 8. Also, a huge difference is noticed in maximum A_{pass} where its values are in the range 0.2–0.3 dB (much less and close to zero) in case of RCGA, 1.11–1.14 dB for the L_1 design filter and 0.9–2.3 dB for the PM filter. Considering ripples in the frequency response of the designed filters, magnitude of the normalized maximum ripple in passband is 1.02 for 40th order RCGA filter, which is majorly less than 1.14 that of the L_1 filter and 1.20 for the PM filter. Maximum stopband ripple magnitude equals to 0.04 for the 40th order RCGA filter, 0.14 for the L_1 algorithm design and 0.20 for the PM design, recorded in Table 6. The absolute magnitude error of the filter, indicated in Table 7, is 8.82 for the L_1 error employed in RCGA, 12.97 of the L_1 design and 33.99 for the PM filter. Thus, it is concluded from the above discussions, diagrams and tables that the real-coded genetic algorithm based L_1-norm design approach brings out the highest A_{stop} and nearing zero A_{pass} on account of the transition width with a minor difference of 2 decimal digits as compared with that of the L_1 optimization design. From all these analysis, significant percentage improvement of the RCGA over the L_1 method is graphically pictured in Fig. 11 for all the three orders. Here, it is seen that the RCGA fitted with L_1-norm shows improvement to a maximum of 32.04% in the absolute magnitude error (AME) with the optimized coefficients of 52nd order filter. In terms of A_{stop}, improvement is of 77.07% for the 64th order and for the algorithm runtime, 34.3% improvement is calculated for 40th order filter.

5. Conclusion

This paper showcase the optimal linear phase FIR high pass filter design using the L_1-norm based RCGA. On its comparison with the L_1 algorithm and PM technique, it is concluded that L_1 based RCGA emerges as a better approach with substantial percentage improvement in different filter specifications and algorithm execution time. Thus, L_1 based RCGA filters can be effectively employed for those applications which highly demand such filters with high attenuation in stopband and lesser passband peaks with slightly eminent transition width acceptance over L_1 filters. Other optimal FIR filters can be designed using the proposed fitness

Table 6
Statistical results for passband ripple and stopband ripple for different order FIR HP filter.

Order	Method	Passband ripple				Stopband ripple	
		Maximun	Mean	Variance	Standard deviation	Maximum	Average
64	PM	1.1214	0.0783	1.0070	1.0035	0.1217	0.0463
	L_1	1.1360	−0.0050	1.0131	1.0065	0.1369	0.0139
	RCGA	1.0360	−0.0431	0.9828	0.9914	0.0296	0.0027
52	PM	1.3049	−0.0696	1.0180	1.0089	0.1223	0.0756
	L_1	1.1410	0.0084	1.0111	1.0056	0.1410	0.0173
	RCGA	1.0260	0.0826	0.9515	0.9754	0.0361	0.0055
40	PM	1.2043	0.0235	1.0214	1.0106	0.2040	0.1001
	L_1	1.1401	−0.0019	1.0181	1.0090	0.1424	0.9960
	RCGA	1.0240	0.0734	0.9446	0.9719	0.0443	0.9985

Table 7
Qualitative results for absolute magnitude error, transition width and execution time for different order FIR HP filter.

Order	Method	Transition width	Absolute magnitude error	Execution time (s)
64	PM	0.0207	28.8036	–
	L_1	0.0414	10.1833	592.4890
	RCGA	0.0443	7.3926	465.7420
52	PM	0.0105	33.9951	–
	L_1	0.0479	12.9766	409.7261
	RCGA	0.0507	8.8189	347.6487
40	PM	0.0127	44.3881	–
	L_1	0.0637	16.0684	418.7828
	RCGA	0.0698	11.8059	275.1077

Table 8
Statistical result for stopband attenuation for different order FIR HP filter.

Order	Method	Stopband attenuation (dB)			Minimum stopband attenuation (A_{stop}) (dB)	Maximum passband attenuation (A_{pass}) (dB)
		Mean	Variance	Standard deviation		
64	PM	−26.6827	−46.3391	−23.1703	−18.29	0.995
	L_1	−27.1175	−39.1721	−19.6011	−17.27	1.112
	RCGA	−30.6283	−35.9721	−17.9857	−30.58	0.305
52	PM	−22.4295	−42.9748	−21.4526	−18.25	2.312
	L_1	−26.3083	−37.8853	−18.9427	−17.02	1.145
	RCGA	−28.8558	−35.1912	−17.5956	−28.85	0.222
40	PM	−19.7264	−36.0269	−16.6596	−13.81	1.615
	L_1	−29.8699	−44.1522	−22.0915	−16.93	1.141
	RCGA	−26.0380	−33.3109	−18.0271	−27.07	0.202

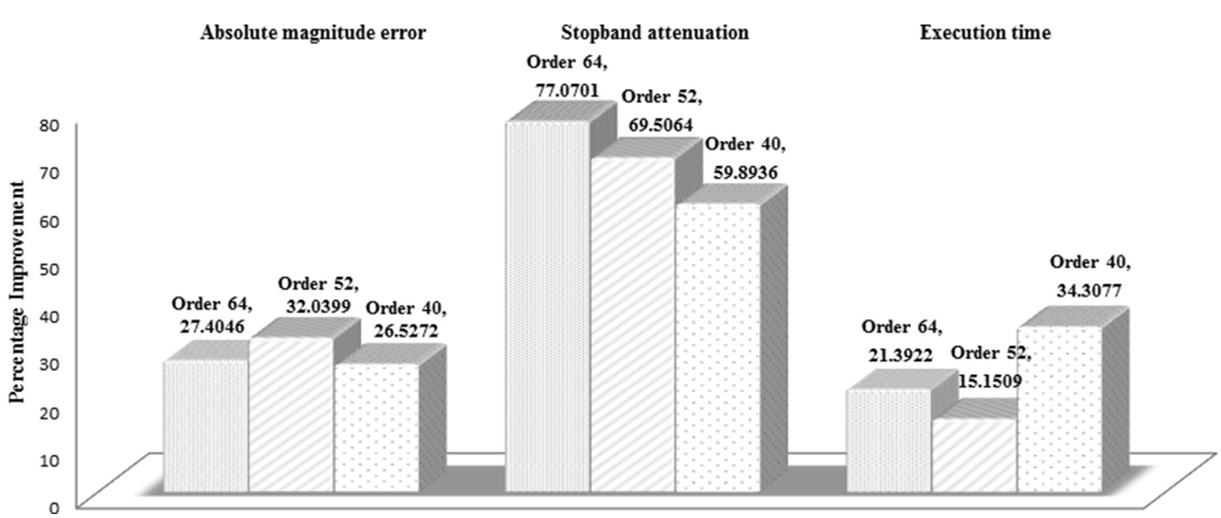

Fig. 11. Bar chart for percentage improvement in RCGA based on L_1-norm compared to the L_1 optimization method on the basis of absolute magnitude error, stopband attenuation and execution time.

function based on L_1-norm. Further this work can be extended for the design of two dimensional filters.

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