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# An enhanced differential evolution algorithm for daily optimal hydro generation scheduling

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#### **Abstract**

The daily optimal hydro generation scheduling problem (DOHGSB) is a complicated nonlinear dynamic constrained optimization problem, which plays an important role in the economic operation of electric power systems. This paper proposes a new enhanced differential evolution algorithm to solve DOHGSB. In the proposed method, chaos theory was applied to obtain self-adaptive parameter settings in differential evolution (DE). In order to handle constraints effectively, three simple feasibility-based selection comparison techniques embedded into DE are devised to guide the process toward the feasible region of the search space. The feasibility of the proposed method is demonstrated for the daily generation scheduling of a hydro system with four interconnected cascade hydro plants, and the test results are compared with those obtained by the conjugate gradient and two-phase neural network method in terms of solution quality. The simulation results show that the proposed method is able to obtain higher quality solutions.

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Keywords: Differential evolution; Chaos; Hydro plant; Daily optimal scheduling

#### 1. Introduction

All major water resource systems have the capability of providing a number of water related benefits to the public at large. In hydro plants, potential energy, embodied in water head, is transformed into electrical energy by turbine/generator units. The efficient utilization of hydro resources plays an important role in the economic operation of a power system where the hydroelectric plants constitute a significant portion of the installed capacity. Determination of daily optimal hydroelectric generation scheduling is a crucial task in water resource management. By utilizing the limited water resource, the purpose of hydroelectric generation scheduling is to find out the magnitude of water releases from each reservoir and hydro plant so that the total benefit of hydro generated energy can be maximized, while the various physical and operational constraints are satisfied.

Mathematically, the daily optimal hydro generation scheduling problem (DOHGSB) is categorized as a class of large-scale, dynamic, nonlinear and non-convex constrained optimization problem. Non-linearity is due to the

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generating characteristic of hydro plant, whose outputs are generally a nonlinear function of water discharge and net hydraulic head. Non-convexity is caused by the hydro generating characteristic being below its best efficiency point. Hydropower production is essentially dynamic in nature, for two reasons: first, the more electricity a hydro plant decides to generate in the current period, the less water it will have in its reservoir for future production; second, when there are multiple hydro plants located along the same river, any amount of water released by an upstream hydro plant in a given period ends up in the reservoir of the downstream hydro plant, expanding the latter's capacity to generate power in the next period. The latter reservoir also influences the upstream hydro plant by its effect on the tail water elevation and effective head in return. The existence of multiple interconnected reservoirs and the need for multi-period optimization characterize the problem as large-scale problem.

Many methods have been developed to solve DOHGSB in the past decades. The major methods include the variational calculus [1], maximum principle [2], functional analysis [3], dynamic programming [4–6], network flow and mixed-integer linear programming [7–9], nonlinear programming [10], progressive optimality algorithm [11], mathematical decomposition method [12,13] and modern heuristics algorithms such as artificial neural networks [14, 15], evolutionary algorithm [16–18], ant colony [19], Tabu search [20], simulated annealing [21]. But these methods have one or another drawback, such as dimensionality difficulty, large memory requirement or inability to handle nonlinear characteristics, premature phenomena and trap into local optimum, taking much computational time. Therefore, improving current optimization techniques and exploring new methods to solve DOHGSB has great significance so as to efficiently utilize water resources, which can be regarded as a renewable source of energy.

In recent years, a new optimization method known as Differential evolution (DE) has gradually become more popular and has been used in many practical cases, mainly because it has demonstrated good robust, convergence properties and is principally easy to understand. DE's Parameters usually are constant throughout the entire search process. However, it is difficult to properly set control parameters in DE. In this paper, chaos theory is applied to determine the parameter setting of DE. The application of chaotic sequences in DE is a powerful strategy to diversify the DE population and improve DE's performance in preventing premature convergence to local minima. The canonical version of DE lacks a mechanism to deal with constrained search spaces. Therefore, a new constraint-handling method with DE is proposed in this paper, which relies on three simple selection criteria based on feasibility to bias the search towards the feasible region. Combining the advantages of chaos and DE respectively, we present a new improved DE algorithm to solve the daily optimal hydro generation scheduling problem. Finally, the proposed method is applied to solve the daily generation scheduling of a test hydro system with four interconnected cascade hydro plants. Simulation results demonstrate the feasibility and validity of the proposed method in terms of solution precision when compared with both of conjugate gradient and two-phase neural network methods.

This paper is organized as follows. Section 2 provides the mathematical formulation of DOHGSB. Section 3 briefly describes the standard differential evolution. Section 4 proposes an improved differential evolution algorithm for solving DOHGSB. Section 5 presents the numerical simulation example. Section 6 outlines the conclusions. Acknowledgements are made in the last section.

## 2. Problem formulations

# 2.1. Notation

To formulate the problem mathematically, the following notation used in this paper is introduced:

 $\begin{array}{ll} D_t & \text{the load demand for the time interval } t \\ P_i^t & \text{power generation of hydro plant } i \text{ at time interval } t \\ P_i^{\min} & \text{minimum power generation of hydro plant } i \\ P_i^{\max} & \text{maximum power generation of hydro plant } i \\ V_i^t & \text{water volume of reservoir } i \text{ at the end of time interval } t \\ V_i^{\min} & \text{minimum water volume of reservoir } i \\ V_i^{\max} & \text{maximum water volume of reservoir } i \\ Q_i^t & \text{water discharge of hydro plant } i \text{ at time interval } t \\ Q_i^{\min} & \text{minimum water discharge of hydro plant } i \\ Q_i^{\max} & \text{maximum water discharge of hydro plant } i \end{array}$ 

initial storage volume of reservoir i at the begin of dispatching horizon

final storage volume of reservoir i at the end of dispatching horizon

 $V_i^{\text{end}}$   $S_i^t$   $I_i^t$  Nwater spillage of hydro plant i at time interval t natural inflow into reservoir i at time interval t

number of hydro plants

M conversion factor of water discharge units into stored water units

total time horizon

time index,  $t = 1, 2 \dots T$ 

number of upstream hydropower plants directly above hydro plant i Nu

water transport delay time from reservoir m to i.  $\tau_{m,i}$ 

# 2.2. Objective function and constraints

The objective of the daily optimal hydro system generation scheduling problem is to minimize the summation of the deviation between the hourly load demand and hydro system total power generation throughout the whole day dispatching time horizon, while satisfying all kinds of physical and operational constraints. So the DOHGSB can be expressed as a constrained nonlinear optimization problem as follows:

$$\min \left\{ \sum_{t=1}^{T} \left[ D_t - \sum_{i=1}^{N} P_i^t \right]^2 \right\}. \tag{1}$$

Subject to the following constraints

# 2.2.1. Hydro plant power limits

$$P_i^{\min} \le P_i^t \le P_i^{\max} \quad i = 1, 2 \dots N; \ t = 1, 2 \dots T.$$
 (2)

## 2.2.2. Hydro plant discharge limits

$$Q_i^{\min} \le Q_i^t \le Q_i^{\max} \quad i = 1, 2 \dots N; \ t = 1, 2 \dots T.$$
 (3)

## 2.2.3. Reservoir storage volumes limits

$$V_i^{\min} \le V_i^t \le V_i^{\max} \quad i = 1, 2 \dots N; \ t = 1, 2 \dots T.$$
 (4)

# 2.2.4. Initial and terminal reservoir storage volumes

$$V_i^0 = V_i^{\text{begin}}, \qquad V_i^T = V_i^{\text{end}} \quad i = 1, 2 \dots N.$$
 (5)

#### 2.2.5. Water dynamic balance equation with travel time

$$V_i^t = V_i^{t-1} + M \cdot \left\{ I_i^t - Q_i^t - S_i^t + \sum_{m=1}^{N_u} [Q_m^{t-\tau_{m,i}} + S_m^{t-\tau_{m,i}}] \right\} \quad i = 1, 2 \dots N; \ t = 1, 2 \dots T.$$
 (6)

# 2.3. Hydropower generation characteristics

A hydropower plant can be characterized by its input–output curves. The input is in terms of water discharge (Q). The output is in terms of power generation (P). The net hydraulic head (H) is defined as the difference between the level of the reservoir and the tail water. The power generated by a hydropower plant depends on the characteristics of the net hydraulic head (or the volume of reservoir V) and water discharge. The hydro plant generation characteristics can be transformed into a characteristic surface where power generation is a function of the water release through the turbine and the net head. The general form is expressed by P = f(Q, H).

Note that the relation between the net head and the reservoir volume, H = g(V), is determined by the shape of the reservoir. Therefore the characteristic surface can also be stated in terms of water discharge and reservoir volume, i.e. P = f(Q, V), which could be non-convex.

#### 3. Differential evolution

Differential evolution (DE), invented by Price and Storn in 1995, is a simple yet powerful heuristic method for solving nonlinear, non-differentiable and multi-modal optimization problems. The DE algorithm has gradually become more popular and has been used in many practical cases, mainly because it has demonstrated good convergence properties and is principally easy to understand. This technique combines simple arithmetic operators with the classical events of crossover, mutation and selection to evolve from a randomly generated initial population to the final individual solution. The key idea behind DE is a scheme for generating trial parameter vectors. Mutation and crossover are used to generate new vectors (trial vectors), and selection then determines which of the vectors will survive into the next generation.

A set of D optimization parameters is called an individual, which is represented by a D-dimensional parameter vector. A population consists of NP parameter vectors  $X_{i,G}$ , (i = 1, 2, ..., NP for each generation G). According to Storn and Price, DE's basic strategy can be described as follows [22].

## 3.1. Mutation

For each target vector  $X_{i,G}$  (i = 1, 2, ..., NP), a mutant vector  $V_{i,G+1}$  is generated according to

$$V_{i,G+1} = X_{r_1,G} + F \cdot (X_{r_2,G} - X_{r_3,G}), \quad r_1 \neq r_2 \neq r_3 \neq i$$
(7)

with randomly chosen integer indexes  $r_1, r_2, r_3 \in \{1, 2, ...NP\}$ . Note that indexes have to be different from each other and from the running index. F is called a mutation factor between [0, 1] which controls the amplification of the differential variation  $(X_{r2,G} - X_{r3,G})$ .

## 3.2. Crossover

In order to increase the diversity of the perturbed parameter vectors, crossover is introduced. The target vector is mixed with the mutated vector, using the following scheme, to yield the trial vector  $U_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \dots u_{Di,G+1})$ , that is

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if } \operatorname{rand}(j) \le CR \text{ or } j = rnb(i) \\ x_{ji,G} & \text{otherwise} \end{cases}$$

$$j = 1, 2, \dots, D$$
(8)

where  $\operatorname{rand}(j)$  is the jth evaluation of a uniform random number generator between [0, 1]. CR is the crossover constant between [0, 1] which has to be determined by the user.  $\operatorname{rnb}(i)$  is a randomly chosen index from  $1, 2, \ldots, D$  which ensures that  $U_{i,G+1}$  gets at least one parameter from  $V_{i,G+1}$ . Otherwise, no new parent vector would be produced and the population would not alter.

#### 3.3. Selection

To decide whether or not it should become a member of the next generation G + 1, the trial vector  $U_{i,G+1}$  is compared to the target vector  $X_{i,G}$  using the greedy criterion. Assume that the objective function is to be minimized, according to the following rule:

$$X_{i,G+1} = \begin{cases} U_{i,G+1} & \text{if } f(U_{i,G+1}) \le f(X_{i,G}) \\ X_{i,G} & \text{otherwise.} \end{cases}$$

$$(9)$$

That is, if vector  $U_{i,G+1}$  yields a better evaluation function value than  $X_{i,G}$ , then  $X_{i,G+1}$  is set to  $U_{i,G+1}$ ; otherwise, the old value  $X_{i,G}$  is retained. As a result, all the individuals of the next generation are as good as or better than their counterparts in the current generation. The interesting point concerning the DE selection scheme is that a trial vector is only compared to one individual vector, not to all the individual vectors in the current population.

# 4. Improved differential evolution algorithm

Chaos, which often exists in nonlinear systems, is the highly unstable motion of a deterministic system in finite phase space. Chaotic sequences display an unpredictable long-term behaviour due to their sensitiveness to initial conditions. This feature is useful to track the chaotic variable as it travels ergodically over the space of interest, so it is can be applied in DE. In order to obtain a high-quality solution for DOHGSB, a new improved DE algorithm combination of chaotic sequences for parameter setting with a selection comparison technique based on individual feasibility for constraint handling is proposed in this paper.

## 4.1. Self-adaptive parameters setting for DE with chaos theory

Recently, some applications of chaotic sequences in EA and in optimization problems have been investigated in the literature [23]. Numerous examples and statistical results show that some chaotic sequences applied to EA are always able to increase the algorithm-exploitation capability in the search space and enhance its convergence. Chaotic optimization methods are based on ergodicity, stochastic properties, and irregularity. The concepts of chaotic optimization can be useful for DE. The DE's parameters CR and F that need to be adjusted by the user are generally the key factors affecting the DE's convergence. Choosing suitable parameter values are difficult for DE, which is usually a problem-dependent task. The trial-and-error method adopted frequently for tuning the parameters in DE requires multiple optimization runs. However, the parameter CR and F cannot ensure the optimization's ergodicity completely in the search phase because they are often constant factors in traditional DE. Therefore, this paper adopts chaotic sequences to self-adaptive adjust parameters CR and F during the evolutionary process. The utilization of chaotic sequences in DE can be useful to escape more easily from local minima than with the standard DE, and improve its global convergence. Application of chaotic sequences to obtain the DE parameters CR and F has two advantages: First, the user does not need to guess at good values for F and CR. Second, the rules for self-adapting adjusted parameters F and F are quite simple.

One of the simplest dynamic systems evidencing chaotic behaviour is the iterator named the logistic map, whose equation is the following:

$$y(t) = \mu \cdot y(t-1) \cdot [1 - y(t-1)] \tag{10}$$

where  $\mu$  is a control parameter,  $0 \le \mu \le 4$ . Eq. (10) is deterministic, displaying chaotic dynamics when  $\mu = 4$  and  $y(0) \notin \{0, 0.25, 0.5, 0.5, 0.75, 1\}$ . y(t) is distributed in the range (0, 1) provided the initial  $y(0) \in (0, 1)$ .

For improving the convergence of DE, this paper proposes to use chaotic sequences based on the above logistic map to obtain the parameters CR and F. The parameter F of Eq. (7) in DE is modified by the formula (10) through the following expresses:

$$F(G) \in (0,1)$$

$$F(G) = \mu \cdot F(G-1) \cdot [1 - F(G-1)]$$

$$v_{i,G+1} = x_{r_1,G} + F(G) \cdot (x_{r_2,G} - x_{r_3,G}).$$
(11)

Similar to setting a rule for F, parameter CR of Eq. (8) in DE is updated according to:

$$CR(G) \in (0, 1)$$

$$CR(G) = \mu \cdot CR(G - 1) \cdot [1 - CR(G - 1)]$$

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if } (\operatorname{rand}(j) \le CR(G)) \text{ or } j = rnb(i) \\ x_{ji,G} & \text{otherwise} \end{cases}$$
(12)

where G is the current generation, F(G) is the new mutation factor based on the logistic map, CR is the new crossover factor based on the logistic map and  $\mu = 4$ .

## 4.2. Constraint handling method

The daily optimal hydro system generation scheduling in Section 2 can be converted into the following constrained optimization problem:

$$\min_{\mathbf{S}, \mathbf{I}} f(Q)$$
s.t.
$$\begin{cases} g_j(Q) \le 0 \\ h_k(Q) = 0 \\ Q_{\min} \le Q \le Q_{\max} \end{cases}$$
(13)

where  $Q = [Q_1, Q_2, ..., Q_n]^T$  is a vector of n discharge decision variables of the optimization problem;  $n = (T \cdot N)$ ;  $j = 1, 2, ..., (4 \cdot T \cdot N)$ ; k = 1, 2, ..., N.

Equality constraints in formula (13) are usually handled by converting them into inequality constraints:

$$g_k(Q) = |h_k(Q)| - \delta_k \le 0 \tag{14}$$

where  $\delta_k$  is a small positive value. This makes the total number of inequality constraints to m ( $m = [4 \cdot T + 1] \cdot N$ ) including all inequality and equality constraints in (13).

When we use DE to solve the above DOHGSB, a key problem is how to handle constraints effectively. At present, the most popular strategy for handling constraints is the use of various penalty function methods. When using a penalty function, a constrained optimization problem can be easily converted into an unconstrained one. Despite the popularity of penalty functions, they have several drawbacks, among which the main one is that they require a careful fine tuning of the penalty factors that accurately estimates the degree of penalization to be applied as to approach efficiently the feasible region. In order to keep the advantages of the penalty function approach and overcome drawback of choice penalty factors, this paper has adopted an effective constraint handling method for DE, which does not require us to set any additional parameters in comparison with the original DE [24]. This constraint handling method is substituted by the selection rule in DE (formula (9)) with the following expression:

$$X_{i,G+1} = \begin{cases} \begin{cases} \forall j \in \{1, 2, \dots, m\} : g_{j}(U_{i,G+1}) \leq 0 \land g_{j}(X_{i,G}) \leq 0 \\ \land \\ f(U_{i,G+1}) \leq f(X_{i,G}) \end{cases} \\ \forall j \in \{1, 2, \dots, m\} : g_{j}(U_{i,G+1}) \leq 0 \\ \land \\ \exists j \in \{1, 2, \dots, m\} : g_{j}(X_{i,G}) > 0 \end{cases} \\ \begin{cases} \exists j \in \{1, 2, \dots, m\} : g_{j}(U_{i,G+1}) > 0 \\ \land \\ \forall j \in \{1, 2, \dots, m\} : \max(g_{j}(U_{i,G+1}), 0) \leq \max(g_{j}(X_{i,G}), 0) \end{cases} \end{cases}$$

$$(15)$$

$$X_{i,G} \quad \text{otherwise.}$$

The selection rule selects a trial vector  $U_{i,G+1}$  to replace old vector  $X_{i,G}$  in the next generation in the following three cases; Otherwise, the old vector  $X_{i,G}$  is preserved.

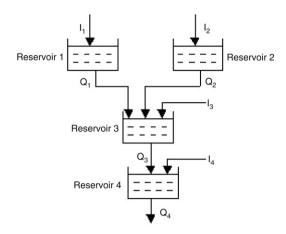


Fig. 1. Hydraulic system test network.

- (1) Both vectors are feasible and the trial vector  $U_{i,G+1}$  has at least as good value for objective than old vector  $X_{i,G}$  has.
  - (2) Old vector  $X_{i,G}$  violates at least one constraint whereas trial vector  $U_{i,G+1}$  is feasible.
- (3) Both trial vector  $U_{i,G+1}$  and old vector  $X_{i,G}$  violate at least one constraint, but the trial vector does not violate any constraints more than the old vector.

The basic idea in this selection rule is that trial vector  $U_{i,G+1}$  is required to dominate the compared old population member  $X_{i,G}$  in a constraint violation space (this comparison is made in the Pareto sense in the constraint violation space), or at least provide an equally good solution as  $X_{i,G}$ .

# 5. Numerical examples

In order to verify the feasibility and effectiveness of the proposed method, a test system taken from Ref. [15] is used. The system consists of a multi-chain cascade of four hydro plants. The scheduling period is one day/ 24 h with one-hour time intervals. The test hydro system configuration is shown in Fig. 1. This hydraulic test network models most of the complexities encountered in practical hydro networks. Details of the hydro plants' power generation characteristics, river inflows and the reservoir limits data used for the present test network are listed in Ref. [15].

With the data given, the proposed method, coded by the Microsoft Visual C++ 6.0 language on a Pentium-4 2.0 GHz-based processor with 512 MB of RAM PC computer, is applied to solve the daily optimal generation scheduling of this test hydro system. The parameters used by our experiment are the following: population size takes 80, initial mutation factor F(0) takes 0.4, crossover factor CR(0) takes 0.9 and the maximal evolutionary iteration number is 2000. Under those chosen parameters, we run our method 20 times from different initial populations in succession and select the best result as the final optimization solution. The final optimal objective evaluation value is 1401. The final hourly release from each reservoir, storage trajectories and hydro plant power generation are showed in Figs. 2–4 respectively.

To validate the results obtained with the proposed method, the same problem was solved using the conjugate gradient and two-phase neural network methods [15]. Table 1 gives the optimal objective evaluation value obtained with the three solution techniques. From Table 1, it is clear that the final optimal result obtained with the proposed method is better than those of the two-phase neural network and conjugate gradient method.

In the meantime, we examine the variation for the best objective evaluation value and the population's standard deviation during the evolutionary process, which show the convergence property of the proposed method. Fig. 5 shows the variation of the best individual solution's objective evaluation value with the number of generation during evolutionary process. Fig. 6 shows the variation of population's standard deviation with the number of generations during evolutionary process. In two figures, there are a sharp declines for the best objective evaluation value and corresponding standard deviation at the beginning evolutionary stages, while it declines slowly during later stages, and finally the best objective evaluation value and population standard deviation stabilize at constant values. From Fig. 6, it can be seen that population standard deviation is much smaller.

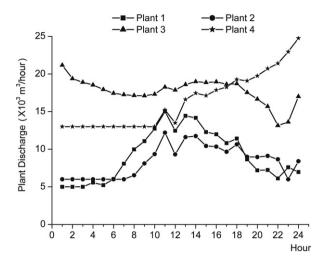


Fig. 2. Hourly hydro plant discharge.

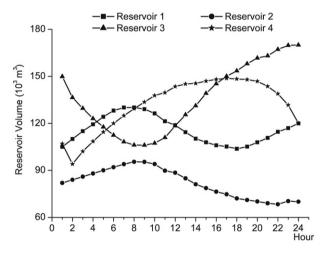


Fig. 3. Hourly reservoir volume.

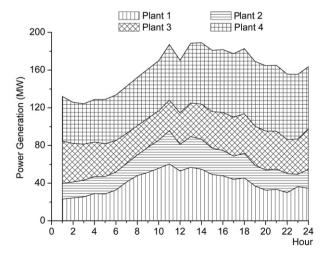


Fig. 4. Hourly hydro plant power generation.

Table 1 Comparison of result with other methods

Method	Optimal results
Conjugate gradient method	1430.0
Two-phase neural network	1429.0
Proposed method	1401.6

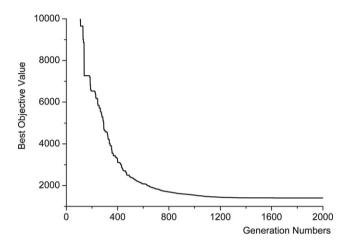


Fig. 5. Variation of the best evaluation value with generation numbers.

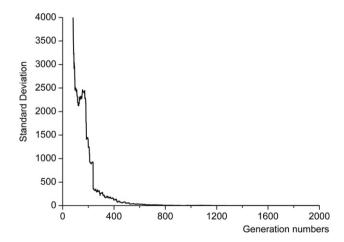


Fig. 6. Variation of standard deviation with generation numbers.

In order to check the solution's quality, we inspected the variation in objective function evaluation values and its standard deviation from 20 trials using the proposed method. The best, average and worst objective evaluation values are 1401.6, 1403.1 and 1406.4 respectively, and the corresponding standard deviation is 1.4. Fig. 7 shows the distribution of the best objective value of each trial, which generates variation in a very small range with trial numbers, thus verifying that the proposed method has better quality of solution and convergence properties.

As seen in the simulation results of the test hydro system, their solutions are optimal and also completely satisfy the constraints for DOHGSB. To sum up, the simulation results demonstrate the feasibility and effectiveness of the proposed method for solving the daily optimal hydro generation scheduling problem.

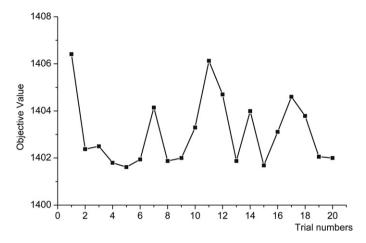


Fig. 7. Distribution of the best objective evaluation value of each trial.

## 6. Conclusions

In the daily optimal generation scheduling of hydro systems, the complexity introduced by the cascade nature of the hydraulic network, the scheduling time linkage, nonlinear non-convex relationships among the hydropower generation, turbine discharge, the net hydraulic head of the corresponding reservoir and the water transport delay time, has made this problem difficult to solve using optimization methods. In this paper, the application of chaotic sequences based on a logistic map to determine the values of parameters F and CR in DE and three simple comparison mechanisms based on feasibility to guide the search toward the optimum are devised to effectively handling constraints, which does not require a penalty function or any extra parameters to bias the search towards the feasible region of the problem. Thus, a new improved chaotic hybrid DE algorithm is proposed to solve the daily optimal hydro system generation scheduling problem. The advantage of the proposed method is that it takes care of the concurrent interaction among water discharge variables of the hydro system. Not only complicated hydraulic coupling can be dealt with conveniently, but also nonlinear non-convex relationships for hydro plant generation characteristics and the water transport delay time are all taken into account. Finally the proposed method is applied to solve the daily generation scheduling of a cascaded hydro system with 4-reservoirs. Simulation results show that the proposed method can obtain a better quality solution with higher precision and convergence properties, so it provides a new effective method to solve the daily optimal generation scheduling of a hydro system, yet it is simple as well as easy to implement.

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