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New smoothing procedures in contact mechanics

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Abstract

This paper presents recent methods to improve numerical simulation of contact problems by smoothing. The main idea is to combine contact surfaces regularization with an automatic adjustment of both penalty parameter and load step. The underlying goal is to provide handle situations frequently met in an industrial context.

Keywords: Contact; Finite element method; Implicit method; Diffuse approximation; Penalty method; Load step

1. Introduction

Many difficulties due to geometrical and material nonlinearities arise when dealing with numerical simulation of contact problems. In a FEM context, the contact zone is represented by a surface which is only piecewise differentiable. When large slips occur, the lack of smoothness may create undesirable abrupt changes of both normal and tangential vector components. Moreover, these effects may alter the residual vector and impede convergence. A number of authors have proposed to smooth or average the normal vector [4,11]. Interpolation techniques based on curved, patches can be used [9,13]. The approach presented here consists in creating a smooth representation of the contact surface with the use of a meshfree technique denoted as diffuse approximation. The idea of building a geometrical model using a second-order equation has been first introduced by Rassineux [10] in a remeshing context of discrete data. We have extended the technique to smoothing of the contact geometry [2]. Recently, Belytschko has proposed a similar approach [1].

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The contact formulation leads to variational inequalities. To solve this problem, a variety of numerical methodologies has been proposed. Among them, penalty method can be easily implemented in any finite element code but the optimum choice of the penalty parameter may be difficult [8]. In order to reduce this problem, an automatic adjustment of the penalty parameter appears necessary [7]. We propose in this paper, an adjustment with respect to penetration criterion.

Strong nonlinearities occur with contact phenomena. Newton methods are generally used to solve these problems. However, when too many changes of contact conditions occur, Newton Raphson algorithms may not converge. In this context, an algorithm for automatic load incrementation for path-dependent problems with contact can be used [5,12]. The main idea of our method is to limit the number of changes of contact status at each load step.

This paper is organized as follows: Section 2 presents the contact surface smoothing procedure. The automatic adjustments both of penalty parameter and load step are described in Sections 3 and 4. In Section 5, examples demonstrate the efficiency of the proposed algorithms.

2. Contact surface smoothing procedure

Let us consider two potential contact zones beforehand definite, denoted as master and slave zone. The master zone is made of a collection of facets resulting from the finite element mesh of the master solid. Let $\boldsymbol{x}^i$ denote the position of a node from this set. The slave zone is made of a set of nodes likely to come into contact with the master zone. The aim is to determine an approximation S_g^{d} of the contact surface using only the data of nodes $\boldsymbol{x}^i$ with the diffuse approximation technique. The determination of the diffuse surface S_g^{d} requires the definition of a reference frame R^{d} linked to all nodes $\boldsymbol{x}^i$. This reference plane is a least-squares fit plane calculated from nodes $\boldsymbol{x}^i$. Coordinates of a master node $\boldsymbol{x}^i$ in this reference frame are written in capital letters (X_1^i, X_2^i, X_3^i) and those of a slave node $\boldsymbol{x}^c$ with the exhibitor c, (X_1^c, X_2^c, X_3^c). This diffuse surface is built from a succession of local approximations. The approach associates with each slave node $\boldsymbol{x}^c$, a local approximation S^{d} of the contact surface.

2.1. Diffuse approximation and contact

The local approximation at node $\boldsymbol{x}^c$ is described by the following surface equation:

$$f_{\boldsymbol{x}^c}(X_1, X_2) = X_3. \tag{1}$$

The expression of $f_{\boldsymbol{x}^c}$ is given by using a polynomial basis $\boldsymbol{p}$, of k terms:

$$\begin{aligned} f_{\boldsymbol{x}^c}(X_1, X_2) &= \sum_{j=1}^{j=k} p_j(\boldsymbol{x} - \boldsymbol{x}^c)\alpha_j \\ &= \boldsymbol{p}^{\mathrm{T}}(\boldsymbol{x} - \boldsymbol{x}^c)\boldsymbol{\alpha}. \end{aligned} \tag{2}$$

In this expression, p_j, $j = 1, \ldots, k$, is a term of the polynomial basis. α_j, $j = 1, \ldots, k$ is the jth component of vector $\boldsymbol{\alpha}$ which must be given. We use a quadratic basis:

$$\boldsymbol{p} = [1, X_1 - X_1^c, X_2 - X_2^c, (X_1 - X_1^c)^2, (X_2 - X_2^c)(X_1 - X_1^c), (X_2 - X_2^c)^2]^{\mathrm{T}}. \tag{3}$$

The objective is now to determine $\boldsymbol{\alpha}$. This vector is calculated by using a weighted least-squares method, based on the minimization of the difference between the altitude X_3^i of $\boldsymbol{x}^i$ and function $f_{\boldsymbol{x}^c}$ evaluated at this node. This leads to the following $J_{\boldsymbol{x}^c}$ criterion:

$$J_{\boldsymbol{x}^c}(\boldsymbol{\alpha}) = \sum_{i=1}^{i=n} w(\boldsymbol{x}^i, \boldsymbol{x}^c)\{(\boldsymbol{p}^{\mathrm{T}}(\boldsymbol{x}^i - \boldsymbol{x}^c)\boldsymbol{\alpha} - X_3^i\}^2. \tag{4}$$

In this expression, $w(\boldsymbol{x}^i, \boldsymbol{x}^c)$ for $i = 1, \ldots, n$, denoted as w^i is the weight function associated with node $\boldsymbol{x}^i$. The determination of vector $\boldsymbol{\alpha}$ requires the calculation of the minimum of $J_{\boldsymbol{x}^c}$ given by

$$\frac{\partial J_{\boldsymbol{x}^c}}{\partial \boldsymbol{\alpha}} = 0. \tag{5}$$

Vector $\boldsymbol{\alpha}$ must be the solution of the system defined by

$$\mathscr{P}^{\mathrm{T}}\mathscr{W}\mathscr{P}\boldsymbol{\alpha} = \mathscr{P}^{\mathrm{T}}\mathscr{W}\mathscr{Z}. \tag{6}$$

$\mathscr{Z}$ is vector of the third components of all nodes $\boldsymbol{x}^i$, used in the approximation and $\mathscr{P}$ is formed by the polynomial basis $\boldsymbol{p}$ evaluated at each node $\boldsymbol{x}^i$:

$$\mathscr{Z} = [X_3^1, X_3^2, \ldots, \ldots, \ldots, X_3^n]^{\mathrm{T}} \quad \text{and} \quad \mathscr{P} = [\boldsymbol{p}^1, \boldsymbol{p}^2, \ldots, \ldots, \ldots, \boldsymbol{p}^n]^{\mathrm{T}}, \tag{7}$$

$$\boldsymbol{p}^i = [1, X_1^i - X_1^c, X_2^i - X_2^c, (X_1^i - X_1^c)^2, (X_2^i - X_2^c)(X_1^i - X_1^c), (X_2^i - X_2^c)^2]^{\mathrm{T}}.$$

$\mathscr{W}$ is the diagonal by block matrix of the weights:

$$\mathscr{W} = \begin{bmatrix} w^1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & w^2 & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots & w^n \end{bmatrix}. \tag{8}$$

The resolution of system (6) leads to the determination of $\boldsymbol{\alpha}$ and thus to the knowledge of the local approximation of the contact surface associated with node $\boldsymbol{x}^c$.

2.2. New description of the contact geometry

Initially, the definition of the gap is given by the new description of the contact surface. This distance is usually defined as the minimal distance between a slave node $\boldsymbol{x}^c$ and the master surface. It would thus be necessary to associate with the slave node, a point $\boldsymbol{x}_g^{\mathrm{d}}$ belonging to the diffuse surface S_g^{d}. This point should be the projection of $\boldsymbol{x}^c$ on S_g^{d}. However, the equation of the diffuse surface is not explicit and a succession of local approximations is necessary to define it. The idea is to use a local approximation at node $\boldsymbol{x}^c$ instead of the global approximation. Therefore, we introduce $\boldsymbol{x}^{\mathrm{d}}$ defined as the projection of $\boldsymbol{x}^c$ on S^{d}. A statement is made that the vector between $\boldsymbol{x}^c$ and $\boldsymbol{x}^{\mathrm{d}}$

and normal vector $\boldsymbol{v}^{\mathrm{d}}$ (associated with the surface S^{d} at the point $\boldsymbol{x}^{\mathrm{d}}$) are collinear which leads to

$$\boldsymbol{x}^{c} - \boldsymbol{x}^{\mathrm{d}} = g_n^{\mathrm{d}} \boldsymbol{v}^{\mathrm{d}}. \tag{9}$$

Scalar g_n^{d} (the diffuse gap) describes the contact state on the regularized surface.

2.3. Description of the three-dimensional diffuse contact element

In this part, the regularization of the contact surface is associated with the finite element method in order to build the three-dimensional diffuse contact element. The major difference between the traditional node-facet approach and our technique is that the contact is driven by potential contact nodes and not only by the nodes of the potential contact facet. Indeed, both slave and master surfaces can be seen as a set of nodes: a slave node $\boldsymbol{x}^{c}$ is supposed in contact with a master zone composed of n nodes $\boldsymbol{x}^{i}$.

The contact element described here is composed of a slave node $\boldsymbol{x}^{c}$ and n nodes $\boldsymbol{x}^{i}$. These nodes are the nodes which have been used to build the local approximation associated with $\boldsymbol{x}^{c}$. The vector of the unknowns of this element is $\boldsymbol{u}^{\mathrm{d}}$:

$$\boldsymbol{u}^{\mathrm{d}} = [\boldsymbol{x}^{c}, \boldsymbol{x}^{1}, \boldsymbol{x}^{2}, \dots \ \dots \boldsymbol{x}^{n}]^{\mathrm{T}}. \tag{10}$$

To define a three-dimensional diffuse contact element the contact elementary residual vector and the tangent matrix must be explicitly given. General expression of $\boldsymbol{R}_c^{\mathrm{d}}$ and K_c^{d} depends on the new expression of the contact surface.

It can be proved that the quantity necessary for the determination of the contact residual vector $\boldsymbol{R}_c^{\mathrm{d}}$ is the first variation of g_n^{d} [6]. Variation δg_n^{d} is evaluated by using the intrinsic properties of the regularized contact surface to obtain

$$\delta g_n^{\mathrm{d}} = \delta \boldsymbol{u}^{\mathrm{d}^{\mathrm{T}}} \mathcal{N}_c^{\mathrm{d}}, \tag{11}$$

where $\mathcal{N}_c^{\mathrm{d}}$ is a $3(n+1)$ components vector and $\delta \boldsymbol{u}^{\mathrm{d}}$ is the variation of $\boldsymbol{u}^{\mathrm{d}}$.

When a penalty method is used, the contact residual vector is thus a $3(n+1)$ components vector given by

$$\boldsymbol{R}_c^{\mathrm{d}} = -\varepsilon_n g_n^{\mathrm{d}} \mathcal{N}_c^{\mathrm{d}}, \tag{12}$$

where ε_n is the penalty parameter.

The expression of the second variation of g_n^{d} can be written as

$$\Delta\delta g_n^{\mathrm{d}} = \delta \boldsymbol{u}^{\mathrm{d}^{\mathrm{T}}} \mathcal{M}_c^{\mathrm{d}} \Delta \boldsymbol{u}^{\mathrm{d}}. \tag{13}$$

The contact tangent matrix is a matrix belonging to $\mathbb{M}_{3(n+1),3(n+1)}$ and it is given by

$$K_c^{\mathrm{d}} = \varepsilon_n g_n^{\mathrm{d}} \mathcal{M}_c^{\mathrm{d}} + \varepsilon_n \mathcal{N}_c^{\mathrm{d}} \mathcal{N}_c^{\mathrm{d}^{\mathrm{T}}}. \tag{14}$$

All expressions of these quantities can be found in Ref. [3].

3. Adjustment of the penalty parameter

The numerical treatment of the contact constraints is based on two main strategies: the penalty method and the Lagragian multiplier methods. Both approaches have their advantages and their

drawbacks. The penalty method can be easily implemented in an existing finite element code. This is the reason why it is used so frequently. The penalty method estimates the contact force as

$$T_n = \begin{cases} -\varepsilon_n g_n & \text{if } g_n \leqslant 0, \\ 0 & \text{otherwise,} \end{cases} \tag{15}$$

where ε_n is the penalty parameter and g_n, the gap. The main disadvantage of this technique is the adequate choice of the penalty parameter. A bad choice can lead to unconditioned stiffness matrix when the penalty parameter is too important or, to unacceptable penetration if it is too small. In this situation, an algorithm to adjust the penalty parameter must be developed.

The proposed algorithm adjusts the penalty parameter with respect to the required accuracy of the penetration. On the one hand, a too important penetration, i.e. unacceptable from a physical point of view, means that the penalty parameter is too small. So, it appears necessary to increase this parameter. On the other hand, if the penetration is acceptable, the penalty parameter can be decreased. For a converged node $\boldsymbol{x}^k$ at time t, ${}^{t+\Delta t}\varepsilon_n^k$ the penalty parameter at time $t+\Delta t$ will be governed by a law of the following type:

$${}^{t+\Delta t}\varepsilon_n^k = \mathscr{F}(|{}^t g_n^k|, g_{\min}, g_{\max})\,{}^t\varepsilon_n^k \text{ with } \mathscr{F} = \begin{cases} \dfrac{|{}^t g_n^k|}{g_{\max}} & \text{if } |{}^t g_n^k| > g_{\max}, \\ \dfrac{|{}^t g_n^k|}{g_{\min}} & \text{if } |{}^t g_n^k| < g_{\min}, \\ 1 & \text{otherwise,} \end{cases} \tag{16}$$

where ${}^t g_n^k$ and ε_n^k are the gap and the penalty parameter associated with $\boldsymbol{x}^k$ at time t. Function $\mathscr{F}$ depends on both the absolute value of the gap of $\boldsymbol{x}^k$ at time t and on two quantities specified by the user. $g_{\min}$ and $g_{\max}$ are, respectively, the minimum and maximum penetration allowed by the user.

The penalty is adjusted by controlling the gap. This choice is driven by the convergence of the displacements requirement of the Newton method. We notice that the control of $g_{\min}$ and $g_{\max}$ improve both the accuracy and the reliability of the technique.

4. Adjustment of the time step

Numerical problems can occur when too many nodes come into contact during the same time step. Ideally, one node at a time should come into contact. The aim of our technique is to limit the number of nodes coming into contact during a time step by using a prediction–correction algorithm. Initially, a prediction of the configuration at time $t+\Delta t$ is realized. A corrected load step Δt_{cor} is performed. Then, the calculation is repeated with this new load step and the configuration at the time $t+\Delta t_{\text{cor}}$ is evaluated.

Let us consider a slave node likely to come into contact with a master face. We suppose that this reference face is unchanged between times t and $t+\Delta t$ and $\boldsymbol{x}^k$, that the studied node remains in contact with the same face. The gap at $\boldsymbol{x}^k$ at time t (resp. $t+\Delta t$) is denoted as ${}^t g_n^k$ (resp. ${}^{t+\Delta t} g_n^k$). We seek a new load step Δt_{cor} such as the considered node comes exactly into contact at time

$t + \Delta t_{\text{cor}}$. For each node likely to come into contact, a coefficient β_k is defined by

$$\beta_k = \frac{{}^t g_n^k}{{}^t g_n^k - {}^{t+\Delta t} g_n^k}. \tag{17}$$

For all the nodes which are likely to come into contact between times t and $t + \Delta t$, coefficients β_k are evaluated and sorted in an ascending order:

$$\beta_1 \leqslant \beta_2 \leqslant \cdots \cdots \leqslant \beta_{N_c} \leqslant \cdots \leqslant \beta_{N_{pc}}, \tag{18}$$

where N_{pc} is the number of nodes likely to come into contact. The corrected load step Δ_{cor} is given by the following rule:

$$\Delta t_{\text{cor}} = \beta \Delta t. \tag{19}$$

The goal of the proposed method is to limit the number of nodes coming into contact to N_c. So, the coefficient is obtained by considering the relation:

$$\beta = \beta_{N_c}. \tag{20}$$

5. Numerical examples

The algorithms have been implemented in the finite element code SYSTUS and tested on both academic and industrial examples. In the studied situations, the contact is assumed to be frictionless.

5.1. Pinch of pipes

This example deals with contact problem between two deformable bodies in three dimensions. A pipe is pinched between two identical parallel plates. The dimensions of the pipe and the plates are as follows: $l = 200$ mm, $r = 90$ mm, $R = 100$ mm, $L = 75$ mm, $e = 20$ mm. Only one quarter of the model is used because of symmetry reason (Fig. 1(a)). The plates and the pipe have

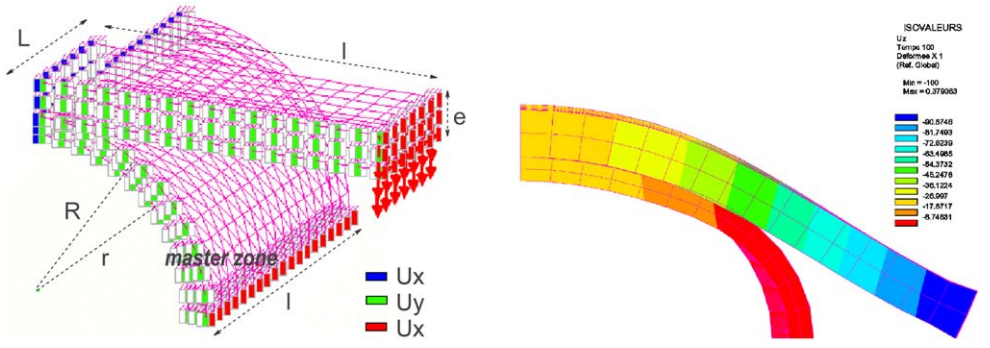


Fig. 1. Pinch of pipes: (a) finite element model and (b) deformed configuration.

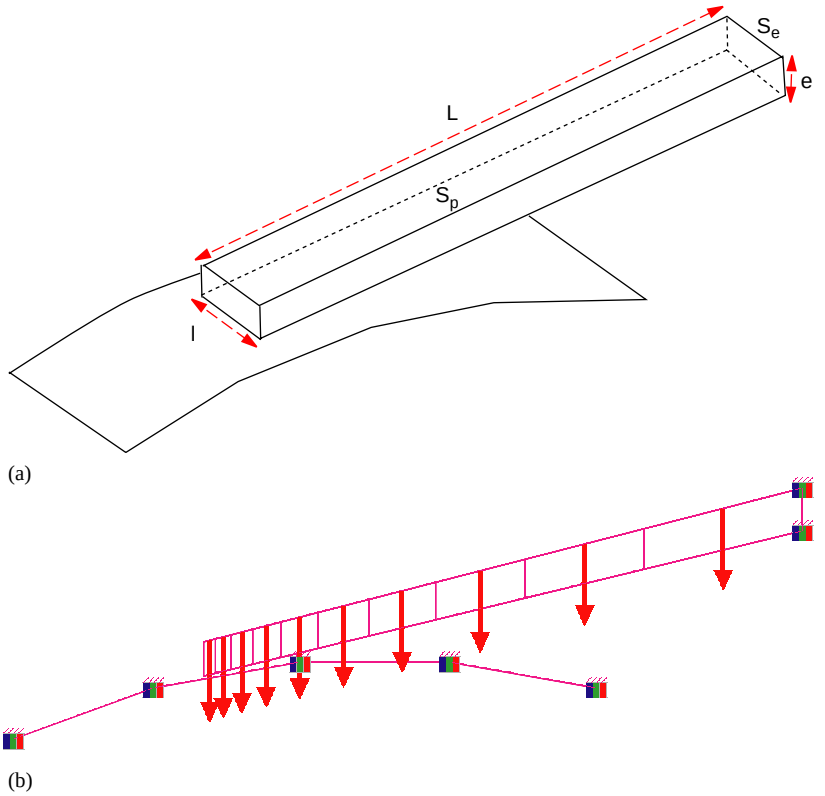


Fig. 2. Contact between a plate and a rigid cylinder: (a) geometry and (b) finite element model.

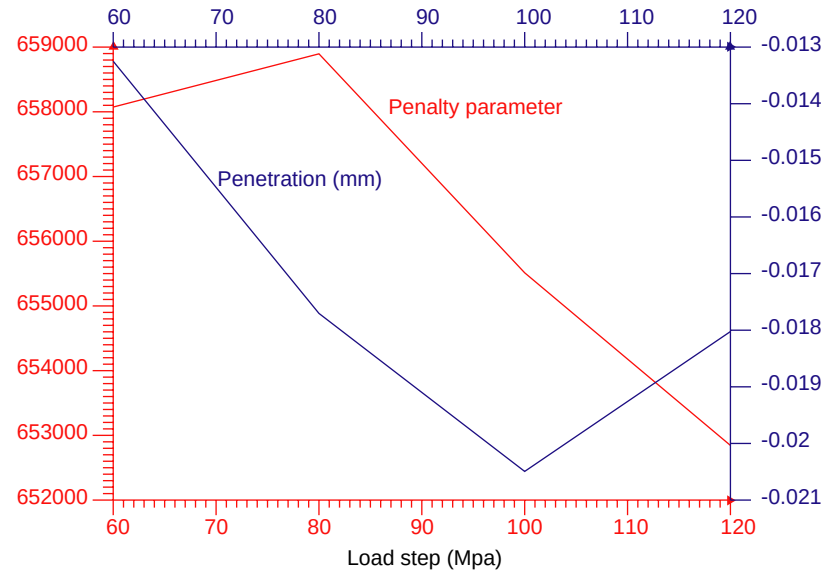
the same mechanical properties: $E = 20\,000$ MPa and $v = 0.3$. The load consists in a vertical imposed displacement of 100 mm at the end of the plate (Fig. 1(a)).

The test was performed with a traditional contact algorithm and with our surface regularization technique. The computations have been performed for both cases. We observe that the number of iterations can be reduced when the surface smoothing procedure is used. The deformed configuration can be seen in Fig. 1(b). The CPU time is not increased.

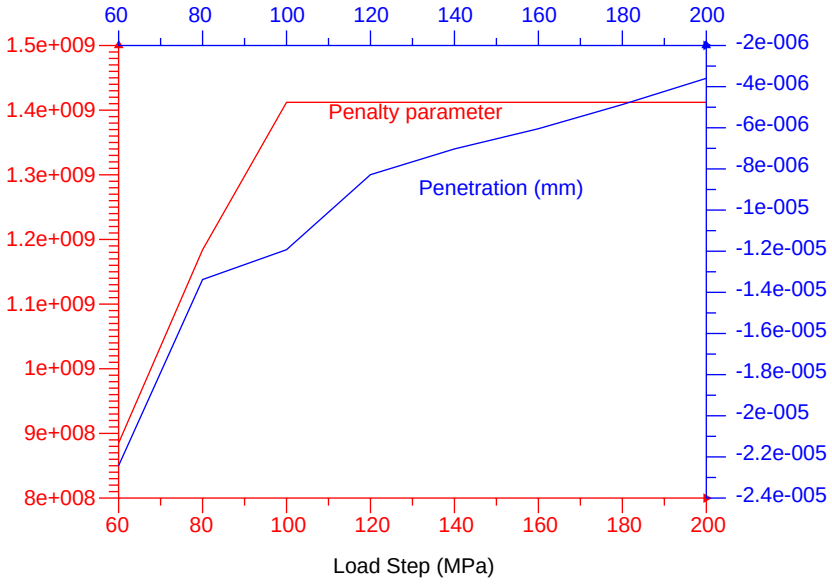
5.2. *Contact between a plate and a rigid cylinder*

We consider the contact between a plate and a rigid cylinder. The dimensions of the plate are as follows: $L = 70$ mm, $e = 4$ mm and $l = 20$ mm. The plate is submitted to an imposed pressure on S_p and S_e is embedded (Fig. 2(a)). The pressure is displayed gradually until $P = 200$ MPa by steps of 20 MPa. The mechanical properties of the plate are: $E = 200\,000$ MPa, $v = 0.3$. The finite element model can be seen in Fig. 2(b).

This test was performed with a traditional contact algorithm and with our technique of automatic adjustment of the penalty parameter. The displacements requirement is 10^{-6} mm. So, $g_{\min}$ and $g_{\max}$ have been chosen close to this reference value, $g_{\min} = 10^{-6}$ mm and $g_{\max} = 5 \times 10^{-6}$ mm.



(a)



(b)

Fig. 3. Evolution of the penalty parameter and of the penetration: (a) classical approach and (b) automatic adjustment.

With a traditional contact algorithm, the computations have not been performed. The required precision was not reached for $P = 120$ MPa. With the proposed adjustment, numerical difficulties have been reduced and the computations have been performed. Fig. 3(a) shows the evolution of the penalty parameter and of the penetration for specific contact node with a classical approach. It

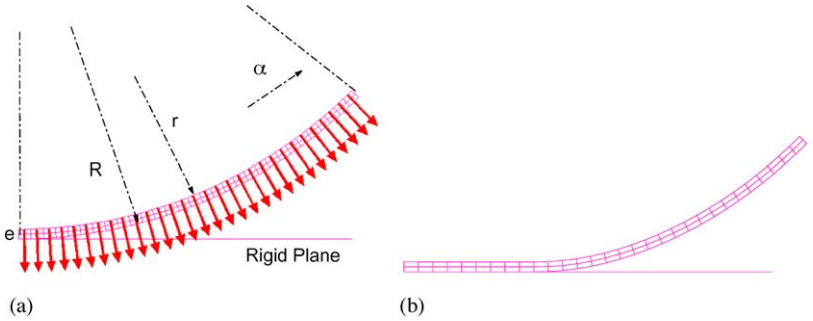


Fig. 4. Flattening of a spherical cup: (a) finite element model and (b) deformed configuration.

has to be noted that the penalty parameter strongly decreases, which can explain the strong increase of the penetration. Fig. 3(b) shows the penetration and penalty parameter with our strategy. In this case, the penalty parameter is increased to obtain the penetration between $g_{\min}$ and $g_{\max}$. We can talk about "penetration convergence".

5.3. Flattening of a spherical cup

The studied structure is a spherical shape with the following characteristics: $R=101$ mm, $r=99$ mm, $e=2$ mm and $\alpha=45^\circ$ (Fig. 4(a)). Material properties are $E=50\,000$ MPa and $v=0.3$. The structure is subjected to an internal pressure $P=200$ MPa. The contact between the spherical shape and the rigid plane is assumed to be frictionless. An axisymmetric two-dimensional model is used. The internal pressure is displayed gradually until $P=200$ MPa by steps of 5 MPa. The finite element model can be seen in Fig. 4(a) and the deformed configuration in Fig. 4(b). The test is carried out with an automatic adjustment of the load step. The data of this adjustment is $N_c=5$.

The technique is very efficient when the load step given by the user is too small. Our initial choice has led to 40 steps. It can be reduced to 9 steps by using our approach. Fig. 5 illustrates this phenomenon and emphasizes a non-linear evolution of load step.

6. Conclusion

A new approach for the regularization of frictionless contact surfaces has been proposed in this paper. Unlike a traditional approach in which the contact nodes are given by a reference master face, here contact nodes are searched with a neighbourhood criterion. A set of nodes likely to come into contact is used to create a local geometrical mode by diffuse approximation. Two strategies of adjustment have also been presented. The adjustment of penalty is based on a control of the gap. The load step is adjusted in order to limit the number of changes of contact status during each load step.

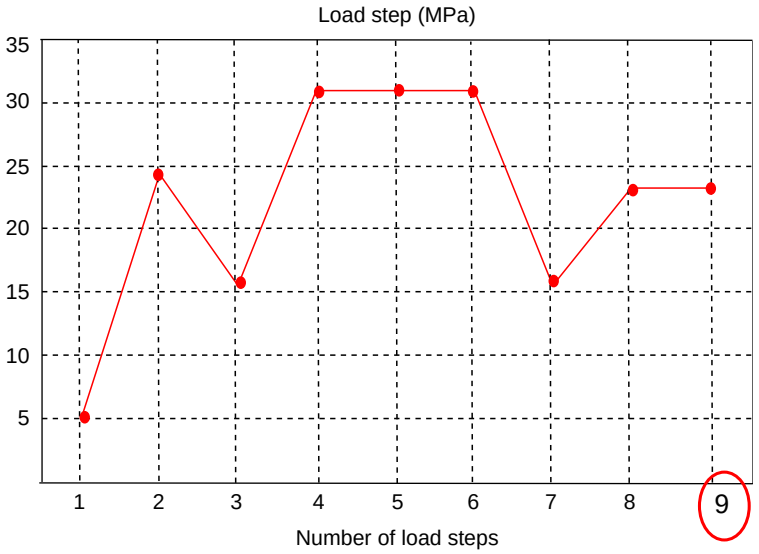


Fig. 5. Evolution of the load step.

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