Prospects for sparticle discovery in variants of the MSSM

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Abstract

We discuss the prospects for detecting supersymmetric particles in variants of the minimal supersymmetric extension of the Standard Model (MSSM), in light of laboratory and cosmological constraints. We first assume that the lightest supersymmetric particle (LSP) is the lightest neutralino χ, and present scatter plots of the masses of the two lightest visible supersymmetric particles when the input scalar and gaugino masses are constrained to be universal (CMSSM), when the input Higgs scalar masses are non-universal (NUHM), and when the squark and slepton masses are also non-universal and the MSSM is regarded as a low-energy effective field theory valid up to the GUT scale (LEEST) or just up to 10 TeV (LEEST10). We then present similar plots in various scenarios when the LSP is the gravitino. We compare the prospects for detecting supersymmetry at linear colliders (LCs) of various energies, at the LHC, and as astrophysical dark matter. We find that, whilst a LC with a centre-of-mass energy $E_{CM} \leqslant 1000$ GeV has some chance of discovering the lightest and next-to-lightest visible supersymmetric particles, $E_{CM} \geqslant 3000$ GeV would be required to 'guarantee' finding supersymmetry in the neutralino LSP scenarios studied, and an even higher E_{CM} might be required in certain gravitino dark matter scenarios. Direct dark matter experiments could explore part of the low-mass neutralino LSP region, but would not reveal all the models accessible to a low-energy LC.
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1. Introduction

When considering projects for new high-energy accelerators, the prospects for discovering supersymmetry are among the issues frequently considered. Since even the minimal supersymmetric extension of the Standard Model (MSSM) has over 100 free parameters

including those characterizing supersymmetry breaking, these prospects are difficult to assess globally in a convincing way, and simplifying assumptions are often made. A common assumption is that R parity is conserved, in which case the lightest supersymmetric particle (LSP) is stable, and a possible candidate for the cold dark matter postulated by astrophysicists and cosmologists [1]. The LSP presumably has no strong or electromagnetic interactions, but otherwise its nature is ambiguous. It is often assumed that the LSP is

the lightest neutralino χ, but another generic possibility is that the LSP is the gravitino $\tilde{G}$ [2–5].

We consider both possibilities in this Letter, constraining them using laboratory, astrophysical and cosmological data. Specifically, we require that the constraints from colliders (particularly LEP) and $b \to s\gamma$ be obeyed,[1] as well as the constraints from WMAP and other cosmological data on the cold dark matter density, and (in the case of a gravitino LSP) we require consistency between the baryon-to-entropy ratio inferred from big-bang nucleosynthesis (BBN) and the cosmic microwave background (CMB) [6].

The impacts of these constraints are often explored in the framework of the CMSSM, in which the input scalar and gaugino masses are constrained to be universal, and the LSP is assumed to be the lightest neutralino [7–9]. We also include this scenario in our analysis, but our scope is broader, since we also analyze neutralino LSP models in which the input Higgs scalar masses are allowed to be non-universal (NUHM) [10,11], and in which the squark and slepton masses are also non-universal and the MSSM is regarded as a low-energy effective theory (LEEST) [12]. We also consider gravitino dark matter models (GDMs) in which different assumptions are made about the gravitino mass relative to the input scalar and gaugino masses [3–5].

In each case, we make a scatter plot of the masses of the lightest visible supersymmetric particle (LVSP) and the next-to-lightest visible supersymmetric particle (NLVSP). We do not consider the LSP itself to be visible, nor any heavier neutral sparticle that decays invisibly inside the detector, such as $\tilde{\nu} \to \nu\chi$ when $\tilde{\nu}$ is the next-to-lightest sparticle in a neutralino LSP scenario,[2] or is metastable and decays outside the detector, such as $\chi \to \gamma\tilde{G}$ in a GDM scenario. The LVSP and the NLVSP are the lightest sparticles likely to be observable in collider experiments. Since the masses of the selectron and smuon are identical in all the (simplified) models we study, one would actually get 'two for the price of one' in cases where a charged slepton is the LVSP or NLVSP.

At a generic linear e^+e^- collider (LC), the physics reach for any visible supersymmetric particle is likely to be a mass close to the beam energy. As is apparent from the scatter plots shown later in this Letter, a LC with $E_{CM} = 500$ GeV has some chance of producing and detecting one or two sparticle types, particularly in models obeying the cosmological and astrophysical constraints, but this cannot be guaranteed. A LC with $E_{CM} = 1000$ GeV clearly has a greater chance of producing sparticles, but this still cannot be guaranteed. Only a LC with $E_{CM} = 3000$ GeV seems 'guaranteed' to produce and detect sparticles, within the variants of the MSSM with a neutralino LSP studied here, namely, the CMSSM, NUHM, LEEST and LEEST10, but an even higher E_{CM} might be required in some GDM scenarios. For related studies, see [13].

For comparison, we also indicate the range of neutralino LSP models in which supersymmetric dark matter may be observable directly in elastic scattering experiments, assuming a sensitivity to the spin-independent χ–N scattering cross section $\gtrsim 10^{-8}$ pb. We find that some fraction of the models with a light neutralino LSP that are accessible to a low-energy LC might give an observable dark matter signal, but not all. Thus, a low-energy LC would add value by exploring the low-mass part of the parameter space more completely.

2. Methodology

Our procedure for analyzing the parameter spaces in each of the supersymmetric models we study is to generate a sample with 50 000 random choices of mass parameters, up to an upper limit of 2 TeV for the soft supersymmetry-breaking squark and slepton mass parameters m_Q, m_D, m_U and m_L, m_E. We also allow the gaugino mass parameter $m_{1/2}$ (which is assumed to be universal for the SU(3), SU(2) and U(1) factors) to vary over this range. The soft Higgs masses $m_{1,2}^2$ are varied from -4 to 4 TeV². The physical Higgs masses squared, which include both the soft supersymmetry-breaking contribution and the μ-dependent contribution, are constrained to be positive up to some high energy scale (either the GUT scale or 10 TeV as described below). We allow the trilinear soft supersymmetry breaking parameter A_0 to vary over the

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[1] Note that we do not apply any constraint from $g_\mu - 2$, though we comment below on the possible effect of this constraint.

[2] However, when the sneutrino has visible decays it is regarded as a possible NLVSP.

range -1 TeV $< A_0 < 1$ TeV. We treat $|\mu|$ and the pseudoscalar Higgs mass m_A as dependent parameters that are fixed by the electroweak vacuum conditions. The arbitrary upper limits on the mass parameters are crude reflections of the upper limits that are supposed to be motivated by naturalness arguments [14]. However, in many of the models under study, they are ample to include all the models that obey the cosmological constraints described below. We sample $1.8 < \tan\beta < 58$ for $\mu > 0$ and $1.8 < \tan\beta < 43$ for $\mu < 0$: above these upper limits, we no longer find solutions of the electroweak vacuum conditions in generic regions of parameter space.

Our procedure for implementing the laboratory constraints on supersymmetric models follows that described elsewhere [7]. The most relevant constraints are those due to the LEP lower limits on the chargino mass $m_{\chi^\pm}$ and the Higgs mass m_h, and the agreement of $b \to s\gamma$ decay with the prediction of the Standard Model, within experimental and theoretical errors. Note that we use here the recent update on the top-quark mass [15], $m_t = 178$ GeV, which has a significant impact on the interpretations of the Higgs limit in the various model parameter spaces. For example, in the CMSSM, the increase from $m_t = 175$ GeV decreases the lower limit on the universal gaugino mass from $m_{1/2} \sim 300$ to ~ 250 GeV for $\tan\beta = 10$ as calculated using FeynHiggs [16]. Changing m_t has other important impacts on model parameter spaces, such as moving rapid-annihilation poles [17] and focus-point regions [18]. While the former are certainly included in our samples, the sensitivity of the focus point is well known [19], and it is pushed to values of m_0 far beyond our sampling range. For example, at $m_{1/2} = 300, \tan\beta = 10$, and $A_0 = 0$, we find that the focus point moves from ~ 2.5 TeV to greater than 4.8 TeV when m_t is increased from 175 to 178 GeV. Bearing in mind this sensitivity of the focus-point region and the fact that it lies beyond our sampling range for our default choice of m_t, we do not discuss it further in this Letter. We do note however, that unless our range for $m_{1/2}$ is increased, the focus point would yield a LVSP and NLVSP which is either a neutralino or chargino and would not go beyond the bounds already considered.

We do not take explicitly into account the possible constraint from $g_\mu - 2$ [20], in view of the persistent uncertainties in the estimate of the contribution from hadronic vacuum polarization. However, we do note that, generically, the regions of the parameter spaces with $\mu > 0$ are normally compatible with experiment at the 2σ level. Including this constraint would have very little effect the models we display for $\mu > 0$, and the constraint would have no effect at all on models with large LVSP and NLVSP masses. In contrast, regions with $\mu < 0$ are normally incompatible with $g_\mu - 2$ at the 2σ level, and essentially all models shown for $\mu < 0$ are excluded by the $g_\mu - 2$ constraint. Thus, although we show results for both signs of μ, only positive values of μ are formally consistent with this constraint.

Our procedures for implementing cosmological and astrophysical constraints also follow those discussed elsewhere [7]. For the cold dark matter density, we use the range $0.094 < \Omega_{CDM}h^2 < 0.129$ preferred by a joint analysis of first-year WMAP and other data [21]. In the case of neutralino LSP models, we identify $\Omega_{CDM} = \Omega_\chi$: allowing other contributions to Ω_{CDM} [3] would, in general, allow also somewhat smaller sparticle masses, but the effect is not large. In the case of GDM, we require the density of gravitinos produced in the decays of heavier sparticles not to exceed the upper limit $\Omega_{CDM}h^2 = 0.129$, but we do allow values below 0.094, since gravitinos are likely to have also been produced by generic thermal or other mechanisms in the very early Universe. A further important constraint on GDM scenarios is that on the Standard Model decay products X accompanying the decays of sparticles $\tilde{Y}$ into gravitinos: $\tilde{Y} \to X + \tilde{G}$. These cannot perturb greatly the abundances of light elements, since astrophysical observations agree with their abundances calculated from big-bang nucleosynthesis using the baryon-to-entropy ratio inferred from WMAP and other measurements of the cosmic microwave background (CMB). We implement this constraint following the analysis in [4,6].

We close this section with some comments on the possible natures of the LVSP and NLVSP. In different regions of the parameter spaces for neutralino LSP models these might include the lighter stau $\tilde{\tau}_1$, the $(\tilde{e}_R, \tilde{\mu}_R)$, the lightest chargino $\chi^\pm$ or the second neutralino χ_2, and in GDM models the lightest neutralino

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[3] These might arise from non-thermal mechanisms such as moduli decays in specific scenarios for supersymmetric cosmology [22].
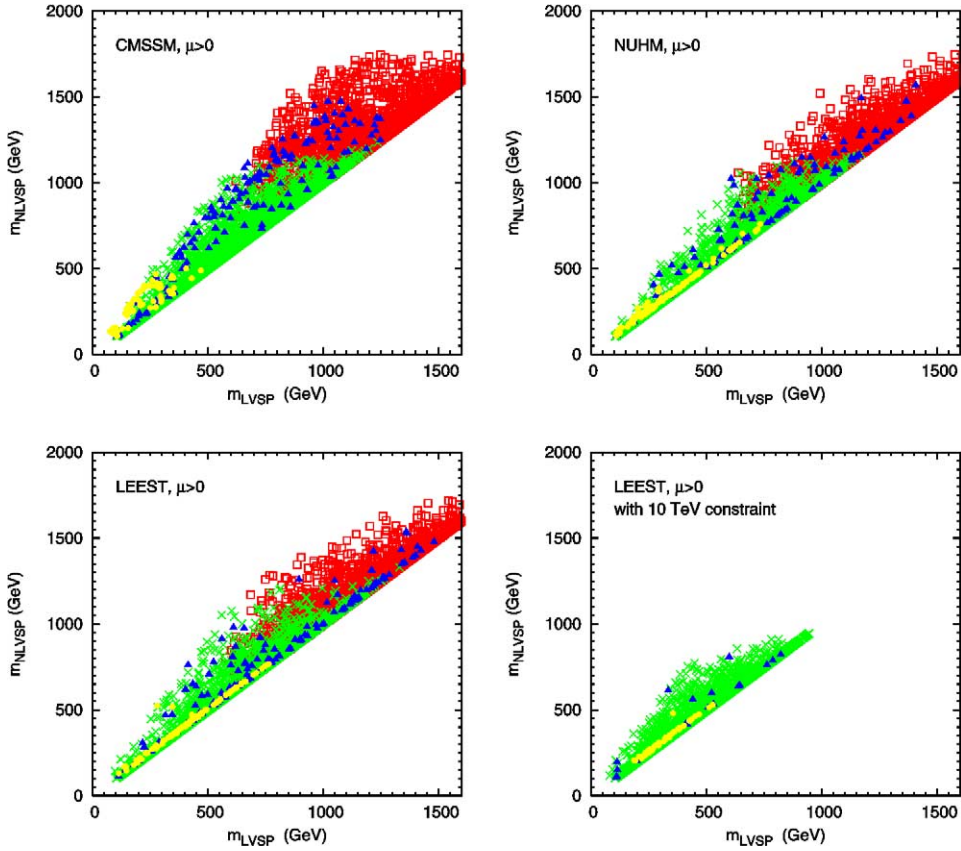
Fig. 1. Scatter plots of the masses of the lightest visible supersymmetric particle (LVSP) and the next-to-lightest visible supersymmetric particle (NLVSP) in (a) the CMSSM, (b) the NUHM, (c) the LEEST and (d) the LEEST10, all for $\mu > 0$. The darker (blue) triangles satisfy all the laboratory, astrophysical and cosmological constraints. For comparison, the dark (red) squares and medium-shaded (green) crosses respect the laboratory constraints, but not those imposed by astrophysics and cosmology. In addition, the (green) crosses represent models which are expected to be visible at the LHC. The very light (yellow) points are those for which direct detection of supersymmetric dark matter might be possible according to the criterion discussed in the text.

χ also becomes a candidate. Depending on the model, the $\tilde{\tau}_1$ may have quite a different mass from the $\tilde{e}_R$ and $\tilde{\mu}_R$, but the latter are degenerate in our analysis, because we assume degenerate sfermion masses before renormalization and neglect the e and μ Yukawa couplings. Thus, in parameter regions where these are the LVSP or NLVSP, one actually observes two sparticles for the price of one.

3. Results for collider searches

Our first set of results is shown in Fig. 1 for the choice $\mu > 0$, with panel (a) displaying our findings for the CMSSM. All points shown satisfy the phe-

nomenological constraints discussed above. The dark (red) squares represent those points for which the relic density is outside the WMAP range, and for which all coloured sparticles (squarks and gluinos) are heavier than 2 TeV. The CMSSM parameter reach at the LHC has been analyzed in [23], which used ISAJET v7.64 and CMSJET v4.801 to simulate the prospective CMS signals in many channels. To within a few percent accuracy, the CMSSM reach contours presented in [23] for different choices of $\tan\beta$ and the sign of μ coincide with the 2 TeV contour for the lightest squark (generally the stop) or gluino, so we regard the dark (red) points as unobservable at the LHC. Most of these points have $m_{\mathrm{NLVSP}} \gtrsim 1.2$ TeV. Conversely, the medium-shaded (green) crosses represent points

where at least one squark or gluino has a mass less than 2 TeV and should be observable at the LHC, according to [23]. The spread of the dark (red) squares and medium-shaded (green) crosses, by as much as 500 GeV or more in some cases, reflects the maximum mass splitting between the LVSP and the NLVSP that is induced in the CMSSM via renormalization effects on the input mass parameters. The amount of this spread also reflects our cutoff $|A_0| < 1$ TeV, which controls the mass splitting of the third generation sfermions.

The darker (blue) triangles are those points respecting the cosmological cold dark matter constraint.[4] Comparing with the regions populated by dark (red) squares and medium-shaded (green) crosses, one can see which of these models would be detectable at the LHC, according to the criterion in the previous paragraph. We see immediately that the dark matter constraint restricts the LVSP masses to be less than about 1250 GeV and NLVSP masses to be less than about 1500 GeV. In most cases, the identity of the LVSP is the lighter $\tilde{\tau}$. While pair-production of the LVSP would sometimes require a CM energy of about 2.5 TeV, in some cases there is a lower supersymmetric threshold due to the associated production of the LSP χ with the next lightest neutralino χ_2 [9]. Examining the masses and identities of the sparticle spectrum at these points, we find that $E_{CM} \gtrsim 2.2$ TeV would be sufficient to see at least one sparticle, as shown in Table 1. Similarly, only a LC with $E_{CM} \geqslant 2.5$ TeV would be 'guaranteed' to see two visible sparticles (in addition to the χ LSP), somewhat lower than the 3.0 TeV one might obtain by requiring the pair production of the NLVSP. We note that, in this and other cases, it is possible that some points with higher m_{LVSP} and/or m_{NVSP} might be found in a larger sample of models. Larger masses may occur in the focus-point region, as noted above, as well as when the neutralino and some other sparticle are nearly degenerate (such as the stop when A is large) and coannihilation controls the relic LSP density [24].[5] Our points with $m_{LVSP} \gtrsim 700$ GeV

Table 1
Centre-of-mass energy (in TeV) required to observe one or two sparticles at a future LC in each of the models discussed in the text

Model	Sgn(μ)	One sparticle	Two sparticles
CMSSM	$\mu > 0$	2.2	2.6
	$\mu < 0$	2.2	2.5
NUHM	$\mu > 0$	2.4	2.8
	$\mu < 0$	2.6	2.9
LEEST	$\mu > 0$	2.6	3.0
	$\mu < 0$	2.5	3.2
LEEST10	$\mu > 0$	1.2	1.6
	$\mu < 0$	1.1	1.5
GDM $m_{3/2} = 10$ GeV	$\mu > 0$	1.1	1.7
	$\mu < 0$	1.1	1.4
GDM $m_{3/2} = 100$ GeV	$\mu > 0$	2.6	2.9
	$\mu < 0$	2.6	3.5
GDM $m_{3/2} = 0.2m_0$	$\mu > 0$	2.5	2.7
	$\mu < 0$	2.6	3.0
GDM $m_{3/2} = m_0$	$\mu > 0$	1.7	1.8
	$\mu < 0$	1.7	1.9

are predominantly due to rapid annihilation via direct-channel H, A poles, while points with 200 GeV $\lesssim$ $m_{LVSP} \lesssim 700$ GeV are largely due to χ-slepton coannihilation. If either of these effects were overlooked, the upper limits on m_{LVSP} and m_{NLVSP} would be considerably tighter.

An $E_{CM} = 500$ GeV LC would be able to explore the 'bulk' region at low $(m_{1/2}, m_0)$, which is represented by the small cluster of points around $m_{LVSP} \sim 200$ GeV. It should also be noted that there are a few points with $m_{LVSP} \sim 100$ GeV which are due to rapid annihilation via the light Higgs pole. These points all have very large values of m_0 which relaxes the Higgs mass and chargino mass constraints, particularly when $m_t = 178$ GeV. A LC with $E_{CM} = 1000$ GeV would be able to reach some way into the coannihilation 'tail', but would not cover all the WMAP-compatible dark (blue) triangles. Indeed, about a third of these points are even beyond the reach of the LHC in this model. Finally, the light filled circles are points for which the elastic $\chi - p$ scattering cross section is larger than 10^{-8} pb. All of these points have $\Omega h^2 < 0.129$. For those points with $\Omega h^2 < 0.0945$, the cross section has been scaled downward by $\Omega h^2/0.0945$, to allow for another component of cold dark matter which populates proportionally our galactic halo. We discuss these points in more detail in the next section.

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[4] We see in the bottom-left part of this and subsequent scatter plots some lighter (yellow) points which also have $\Omega_{CDM}h^2 < 0.129$, but may have $\Omega_{CDM}h^2 < 0.094$.

[5] This is just one reason why our 'guarantees' are in quotation marks.

Panel (b) of Fig. 1 displays a corresponding scatter plot for the NUHM, in which the soft supersymmetry-breaking masses of the Higgs bosons are allowed to float relative to those of the squarks and sleptons, which are still assumed to be universal. We again use the 2 TeV mass criterion motivated by [23] to distinguish models that are unobservable at the LHC (dark, red) from those that are unobservable. No analysis as detailed as [23] has been made in the NUHM, but we do not expect large differences from the CMSSM. The 'footprint' of the darker (blue) points that respect the cosmological cold dark matter constraint is similar in shape and origin from that in the CMSSM shown in panel (a). Once again, the dark (blue) triangles with large masses are predominantly due to rapid s-channel annihilation through the H, A poles. Because we allow the two soft Higgs masses to take values different from m_0, μ and m_A take on a significantly broader range of values in the NUHM as compared to the CMSSM. Thus, the rapid annihilation funnels appear more frequently at all values of $\tan\beta$, in contrast to the CMSSM, where the funnels appear only at high $\tan\beta$. The nearly linear track of points with $m_{\rm LVSP} \simeq m_{\rm NLVSP}$ corresponds to points with large m_0 for which the LVSP and NLVSP are a nearly degenerate pair of charginos and neutralinos. Points with smaller m_0 are dispersed to higher $m_{\rm NLVSP}$ where the LVSP, NLVSP pair is typically the stau and the selectron/smuon.

The LVSP could be as heavy as ~ 1400 GeV and the NLVSP as heavy as ~ 1600 GeV in the NUHM case. In the NUHM, production of a χ_1, χ_2 pair at an LC with $E_{\rm CM} \geqslant 2.4$ TeV is sufficient to guarantee the detection of at least one visible sparticle (in addition to the χ LSP), whilst only a LC with $E_{\rm CM} \gtrsim 2.8$ TeV (corresponding to the pair production of the LVSP) would be 'guaranteed' to see at least two visible sparticles. As in panel (a), a LC with $E_{\rm CM} \sim 500$ GeV or 1000 GeV would see sparticles in only a corner of the overall footprint, though this might be the portion favoured by some naturalness arguments. Also as before, we note that a low-energy LC would be able to spot models inaccessible to direct searches for dark matter.

Panels (c), (d) of Fig. 1 display the corresponding scatter plots for the LEEST, in which no universality is assumed between the soft supersymmetry-breaking squark and slepton masses with different gauge quan-

tum numbers. On the other hand, as motivated but not mandated by upper limits on flavour-changing neutral interactions [25], we do assume universality between squarks and sleptons that have the same gauge quantum numbers but are in different generations. We require that the low-energy effective supersymmetric theory remain viable, with a stable electroweak vacuum, all the way up to some higher energy scale, taken in panel (c) to be the GUT scale (LEEST) and 10 TeV (LEEST10) in panel (d).[6] While the identity of the LVSP, NLVSP pair is predominantly a chargino and neutralino or a stau, selectron/smuon pair as in the NUHM, many other combinations are possible now. For example, one of the sneutrinos is often the NLVSP. For LEEST10, we only require the theory to remain viable up to 10 TeV, and we have made the analogous restriction that scalar masses (at 10 TeV) lie between 0 and 2 TeV. This constraint removes many of the points from the initial set of data. This is the reason for the paucity of points in panel (d). This constraint further makes it highly likely that at least one coloured sparticle exists with a mass below 2 TeV, thus making all points potentially observable at the LHC.

The conclusions to be drawn from the LEEST panel (c) do not differ qualitatively from those in the CMSSM and NUHM panels (a), (b): we use the same criterion [23] for observability at the LHC, and the upper limits on the LVSP and the NLVSP are about 1500 GeV. Including χ_1, χ_2 production, the LEEST parameter space scanned here could be covered by a LC with $E_{\rm CM} > 2.6$ TeV (one sparticle) and $E_{\rm CM} > 3.0$ TeV (two sparticles), as seen in Table 1. On the other hand, both the darker (blue) and lighter (green) points in panel (d) for the LEEST10 model extend up to somewhat smaller masses than seen previously: $m_{\rm LVSP} \sim 850$ GeV, $m_{\rm NLVSP} \sim 850$ GeV. This is due to the fact that the renormalization of the soft supersymmetry-breaking parameters between 10 TeV and the electroweak scale is considerably less than that between the GUT scale and the electroweak scale. For this reason, sparticle masses are generally larger

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[6] Compared with [12], one technical difference is that here the random sample is generated with input parameters at the high scale, which are then run down to low scales using the renormalization-group equations, whereas previously the random sample was generated at the electroweak scale. This does not affect the conclusions in any essential way.
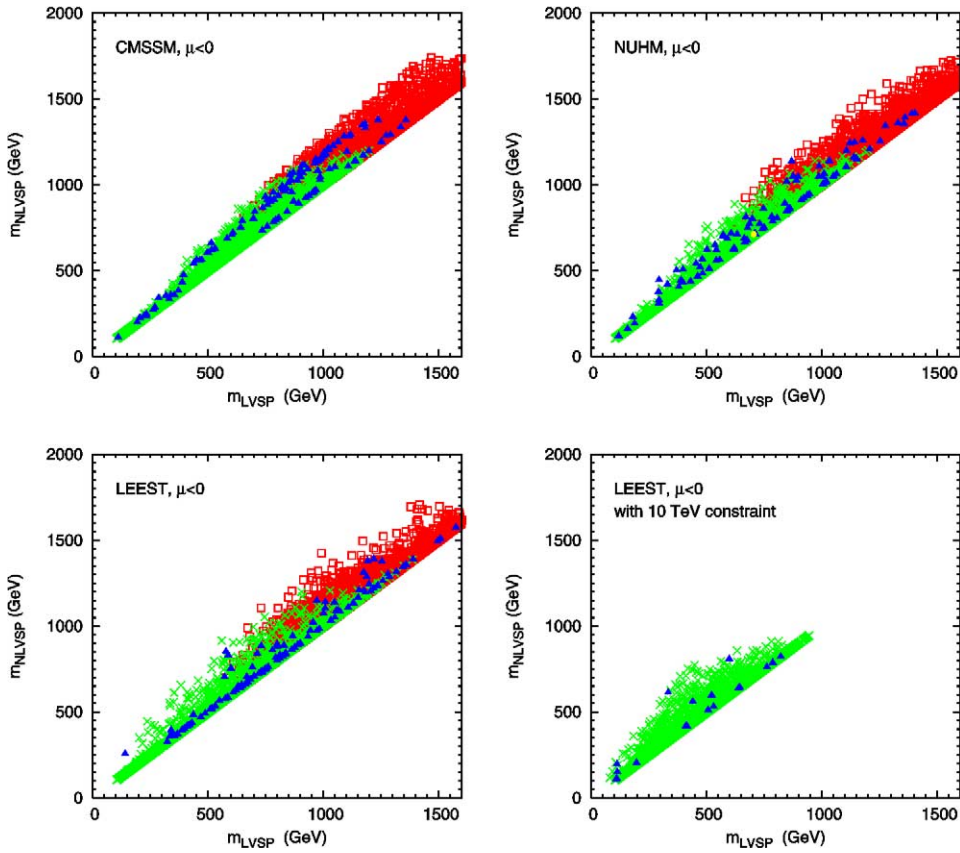
Fig. 2. As in Fig. 1, but for $\mu < 0$.

in LEEST than in LEEST10. Correspondingly, one would be more optimistic about the physics reach of lower-energy LC if one did not require the MSSM to remain valid all the way up to the GUT scale. In this case, a LC with $E_{CM} > 1.2$ TeV (one sparticle) and $E_{CM} > 1.6$ TeV (two sparticles) is sufficient.

The panels of Fig. 2 display the corresponding scatter plots for the CMSSM, NUHM, LEEST and LEEST10 in the case that $\mu < 0$. Although the scatter plots are qualitatively similar to those in Fig. 1, there are some differences of detail between the 'sister' plots for the two signs of μ. In particular, the upper bounds on the LVSP and NLVSP masses are somewhat different: $(m_{LVSP}, m_{NLVSP}) \lesssim$ (1350, 1400), (1400, 1400), (1600, 1600), (800, 800) GeV in the (a) CMSSM, (b) NUHM, (c) LEEST and (d) LEEST10 cases, respectively. In the CMSSM, the division between the dark (blue) triangles whose

relic density is controlled by coannihilations and rapid s-channel annihilations now occurs at a lower value of $m_{LVSP} \sim 500$ GeV. The two nearly linear tracks of points with large m_{LVSP} corresponds to points with large m_0 for which the LVSP and NLVSP are a nearly degenerate pair of charginos and neutralinos (lower track), and points with smaller m_0 where the LVSP, NLVSP pair is the stau and selectron/smuon. However, the overall conclusions about the physics reaches of LCs with different E_{CM} are similar: low-energy LCs with $E_{CM} \leqslant 1000$ GeV reach part of the allowed parameter space, whereas a LC with $E_{CM} = 3200$ GeV would be 'guaranteed' to find sparticles in all of these models. The required centre-of-mass energies for each case are individually summarized in Table 1.

The remaining figures display scatter plots in various scenarios with a gravitino LSP, assuming scalar-mass universality. In the absence of any better-tailored
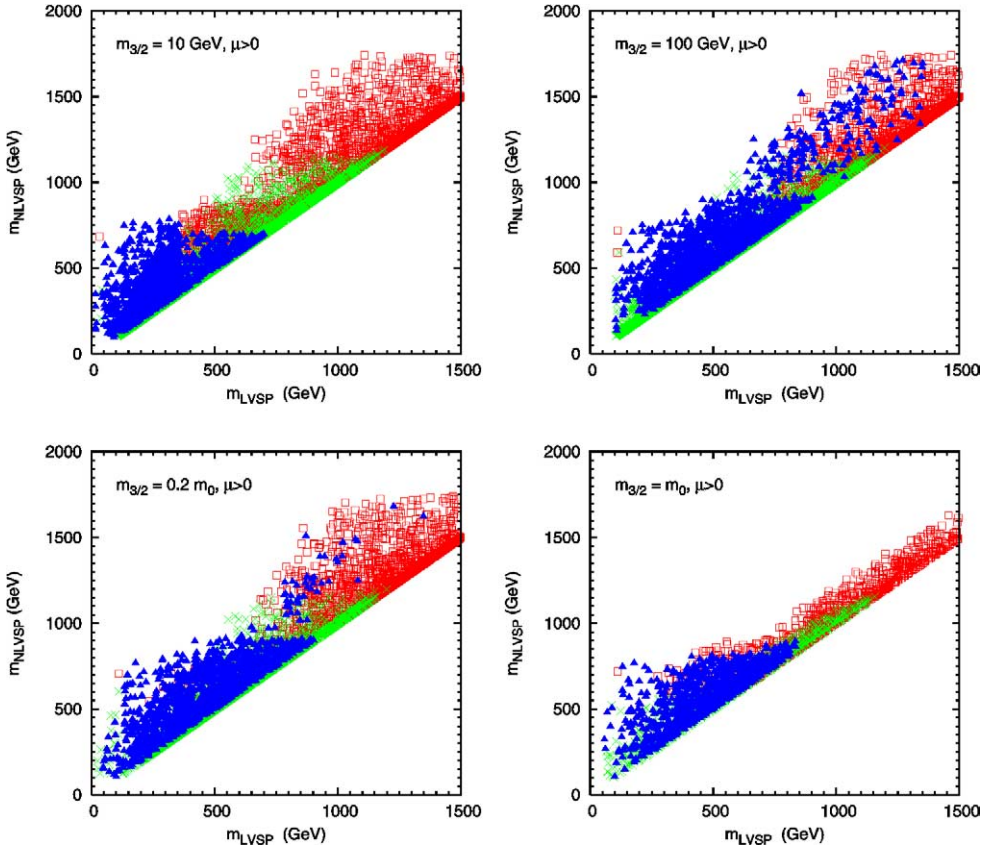
Fig. 3. Scatter plots of the masses of the lightest visible supersymmetric particle (LVSP) and the next-to-lightest visible supersymmetric particle (NLVSP) in the GDM with (a) $m_{3/2} = 10$ GeV, (b) $m_{3/2} = 100$ GeV, (c) $m_{3/2} = 0.2m_0$ and (d) $m_{3/2} = m_0$, all for $\mu > 0$. The darker (blue) triangles satisfy all the laboratory, astrophysical and cosmological constraints. For comparison, the dark (red) squares and medium-shaded (green) crosses respect the laboratory constraints, but not those imposed by astrophysics and cosmology. In addition, the (green) crosses represent models which are expected to be visible at the LHC.

analysis, we use the same criterion [23] for observability at the LHC. We recall that the allowed regions of the $(m_{1/2}, m_0)$ planes in such GDM scenarios are very different from those allowed in the CMSSM [4]. Our own studies of the GDM have been restricted to a few specific scenarios for the gravitino mass $m_{3/2}$ [4].[7] We only consider cases where the next-to-lightest supersymmetric particle (NSP) has a lifetime exceeding 10^4 s [6], since we have not yet incorporated the effects of hadron showers in the early Universe, which are expected to be important for shorter lifetimes [26].

These limitations restrict our analysis here artificially to portions of the GDM parameter space. For this reason, we do not exclude the possibility that heavier LVSP and NLVSP masses might be permitted, and the ranges of masses quoted below should be interpreted as implying that a LC with E_{CM} at least twice as large would be needed for any 'guarantee' of discovering supersymmetry in these scenarios. In the specific case $m_{3/2} = 10$ GeV shown in panel (a) of Fig. 3, we find LVSP and NLVSP masses up to 700 and 800 GeV, respectively, implying that a LC with $E_{CM} \gtrsim 1700$ GeV would be needed for even a limited 'guarantee' of discovery. However, this case in particular suffers from our restriction on the NSP lifetime. For a fixed value of m_0, the $\tilde{\tau}_1$ mass is limited by the gaugino mass, $m_{1/2}$, which is in turn limited by our restriction on the

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[7] Moreover, the computer time required to generate a useful sample in the higher-dimensional space with $m_{3/2}$ a free parameter would be prohibitive.
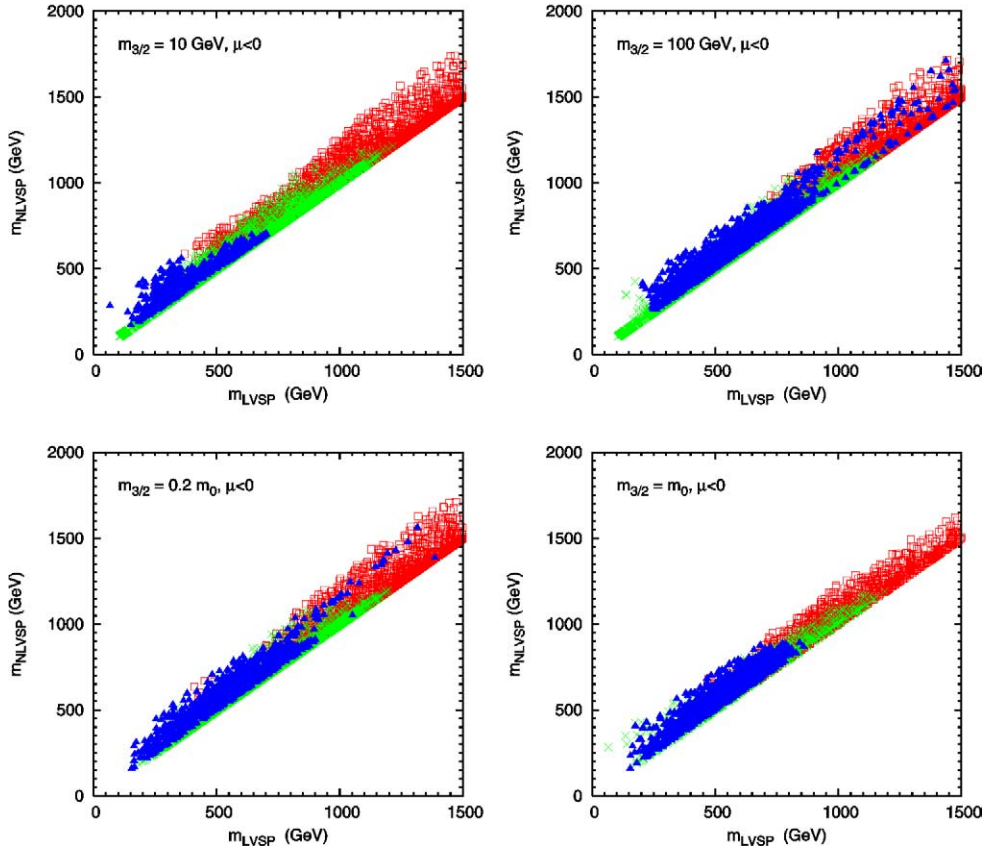
Fig. 4. As in Fig. 3, but for $\mu < 0$.

NSP lifetime. This causes most of the allowed points to appear below $m_{LVSP} \lesssim 400$ GeV, as occurs when the (LVSP, NLVSP) pair are either $(\tilde{\tau}_1, \chi)$ or $(\tilde{\tau}_1, \tilde{e}_R)$. However, some extension beyond $m_{LVSP} \sim 400$ GeV is possible for larger values of m_0. In these cases, the maximum allowed mass is determined by the gravitino relic density constraint: $\Omega_{3/2}h^2 = (m_{3/2}/m_\chi)\Omega_\chi h^2 < 0.129$, and the (LVSP, NLVSP) pair are either $(\chi_1^\pm, \chi_2)$ or $(\tilde{\tau}_1, \tilde{e})$.

When $m_{3/2} = 100$ GeV, as shown in Fig. 3(b), the restriction due to the NSP lifetime is much less severe, and the LVSP and NLVSP masses are allowed to roam to much higher values. Here, the discontinuity at $m_{NLVSP} \sim 900$ GeV is simply a result of our chosen range of $m_{1/2} < 2$ TeV. Although the dark (red) squares extend to much higher masses, they have $m_\chi < m_{\tilde{\tau}_1, \tilde{e}_R}$ and, for the most part, have $\Omega_{3/2}h^2$ above the WMAP limit. However, a smattering of points with high $\tan\beta$ are allowed in the

rapid-annihilation funnel regions. Most of these points would not be observable at the LHC. These same features are seen for $m_{3/2} = 0.2m_0$ in Fig. 3(c) and in the corresponding plots for $\mu < 0$ (Fig. 4(b), (c)). However, this feature is not found in panels (d) for either sign of μ (see, e.g., [4]), as the funnel is no longer present when $m_{3/2} = m_0$ because of the assumption that the LSP is the gravitino and the limit on the on gravitino relic density.

We find no suggestion that a low-energy LC would be a safer bet in this and other GDM scenarios than in the neutralino LSP scenarios discussed earlier. In the cases $m_{3/2} = 100$ GeV, $m_{3/2} = 0.2m_0$ and $m_{3/2} = m_0$ shown in panels (b)–(d), respectively, we find $(m_{LVSP}, m_{NLVSP}) \lesssim (1400, 1750)$, $(1400, 1700)$, $(850, 900)$ GeV, respectively. We recall that minimal supergravity (mSUGRA) models have scalar-mass universality, $m_{3/2} = m_0$ and a specific value for the universal trilinear supersymmetry-breaking parame-

ter A, and typically have neutralino and gravitino LSPs in different regions of parameter space [27]. They are not equivalent to either the CMSSM or the GDM scenario discussed here. This remark serves to emphasize that many other scenarios for the masses of the MSSM particles and the gravitino could be entertained, beyond those presented here, including also scenarios with scalar masses that are non-universal to some degree, as discussed earlier in connection with a neutralino LSP.

The ranges of visible sparticle masses in the corresponding scenarios with $\mu < 0$ are shown in Fig. 4. Here we find in the cases (a) $m_{3/2} = 10$ GeV, (b) $m_{3/2} = 100$ GeV, (c) $m_{3/2} = 0.2m_0$ and (d) $m_{3/2} = m_0$, that $(m_{\mathrm{LVSP}}, m_{\mathrm{NLVSP}}) \lesssim (700, 700)$, $(1500, 1700)$, $(1400, 1600)$, $(900, 900)$ GeV, respectively. The centre-of-mass energies in each of these cases, as well as those for $\mu > 0$, are summarized in Table 1.

4. Prospects for direct detection of supersymmetric dark matter

One of the principal competitors with colliders for the discovery of supersymmetry is the search for astrophysical dark matter, assuming this to be composed of LSPs. Gravitino dark matter is very difficult to observe, but there are interesting prospects for detecting neutralino dark matter, either directly via scattering on nuclei, or indirectly via the products of annihilations in various astrophysical environments, such as the centres of the Earth, Sun or Galaxy, or in our galactic halo: for a recent review, see [28]. Here, so as to minimize the astrophysical uncertainties, we focus on direct detection.[8]

There are two important contributions to generic χ-nucleus scattering, one that is spin-independent and related to quark contributions to the nucleon mass, and one that is spin-dependent and related to quark contributions to the nucleon spin. Since the former appears more promising in many experiments, we concentrate here on this spin-independent contribution.

Matrix elements for spin-independent χ-nucleon scattering depend on $\langle p|\bar{s}s|p \rangle$, which may be estimated on the basis of the σ term in π-nucleon scattering. Recent evaluations of this quantity appear to favour larger values than often assumed previously [29], which may also be favoured by estimates based on the possible spectroscopy of exotic baryons as treated in the chiral soliton model [30]. Accordingly, in this Letter we use a larger estimate of $\langle p|\bar{s}s|p \rangle$ than in our previous work:[9] $y \equiv 2\langle p|\bar{s}s|p \rangle/(\langle p|\bar{u}u|p \rangle + \langle p|\bar{d}d|p \rangle) = 0.44$, corresponding to $\sigma_{\pi N} = 64$ MeV.

Within the near future, searches for spin-independent χ-nucleus scattering are expected to reach a sensitivity $\sim 10^{-8}$ pb for a range of m_χ. We indicate in Figs. 1 and 2 by light (yellow) the randomly-selected models which have cross sections above 10^{-8} pb. These populate the regions of low m_{LVSP} and m_{NLVSP} that would be particularly accessible to a low-energy LC. Note that in the CMSSM, the elastic scattering cross section for $\mu < 0$ is generally smaller than the corresponding case when $\mu > 0$ (see, e.g., [32]). Furthermore, for $\mu < 0$, the $b \to s\gamma$ constraint also eliminates points with large elastic scattering cross sections. As such, no points in Fig. 2(a), rise above the 10^{-8} pb threshold.

However, many of these models make an excessive contribution to $g_\mu - 2$. In fact if we applied the upper limit to $\delta a_\mu < 31 \times 10^{-10}$, roughly half of the light (yellow) circles are removed in panel (a) of Fig. 1 for the case of the CMSSM. Of those remaining, roughly half have a relic density below 0.0945. Not all the supersymmetric models accessible to a low-energy LC would be detectable at this cross section level, so such a LC would certainly add value in this region of parameter space, and the absence of a signal in this generation of direct searches for supersymmetric dark matter should not be taken as evidence that such a low-energy LC could not see supersymmetry. In the NUHM (panel (b)) of Fig. 1, only a few ($\sim 5\%$) of the light (yellow) circles would be removed by the $g_\mu - 2$ constraint. However, in this case, most of the points ($\sim 80\%$) have $\Omega h^2 < 0.0945$. These points typically correspond to a LSP which is Higgsino-like. As a consequence the relic density is small, due to the relatively large anni-

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[8] Direct detection alone cannot unambiguously discover supersymmetry, as other non-supersymmetric dark matter candidates are possible.

[9] This point is discussed in more detail in [31].

hilation cross section and the Higgs exchange channel makes a strong contribution to the total elastic cross section.

5. Summary

We have explored the prospects for discovering one or more supersymmetric particles in a number of models with either a neutralino or a gravitino LSP. We have considered various hypotheses for relations between soft supersymmetry-breaking scalar masses with differing degrees of universality. In all the models studied, we find that a low-energy LC with $E_{CM} \leqslant 1000$ GeV has a chance to produce and detect one or more sparticles, but this cannot be guaranteed. However, a high-energy LC with $E_{CM} \geqslant 3000$ GeV would be needed to 'guarantee' the detection of supersymmetry in neutralino LSP models, and we cannot exclude the possibility that an even higher E_{CM} might be required in some models with a gravitino LSP.

It is clear that the naturalness of the electroweak symmetry-breaking scale favours lower sparticle masses to some extent [14], but there is no clear criterion how this aesthetic requirement should be imposed. One might strike lucky with some search for supersymmetric dark matter, either direct (as discussed here) or indirect, but this is not guaranteed, even if the supersymmetry breaking scale is relatively low. The next clear information on the sparticle mass scale may have to wait for data from the LHC.

Acknowledgements

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