Abstract

For meaningful learning to take place, it depends on, among other things, what dominates learners’ attention. What are they attending to and how are they attending to it? For learners to appreciate and understand mathematical topics and to have a meaningful experience of learning mathematics, it is necessary to create opportunities for them to make use of their powers to contact important mathematical structures and to become aware of dimensions and depth of the topic. Stage One pupils in three schools in Penang will be invited to construct relevant mathematical examples meeting specified constraints in various mathematical topics in order to investigate their understanding. This concept paper reports on the design, development and exploitation of a rich framework to assess mathematical thinking of Stage One pupils and for what it means to understand and appreciate a mathematical topic.

Keywords: Mathematical thinking; Mathematical examples; Attention and awareness

1. Introduction

Student difficulties in many areas of mathematics have been the concern for researchers in the field of mathematics education for decades. One of the central consensuses among researchers is that understanding of mathematical ideas appears to be cognitively situated, in the sense that learners may be well-equipped to work on the standard textbook problems in a familiar context and yet become incapacitated when faced with novel situations. This may explain why learners do not seem to be able to cope with situations beyond what they are familiar with. Rote-learning of mathematical rules and procedures could lead to confusion when learners could not cope with the unfamiliarity of the novel situation. In this regard, I see conceptual understanding as not only the ability to use situated knowledge to solve routine problems correctly but more importantly, as the ability to extend that “situatedness” appropriately and efficiently into unfamiliar situations. Dealing effectively with novel situations is likely to depend on which aspects of the concept/idea becomes the focus of learners’ attention namely, what they regard as important. Activities in which learners engage often reveals something about the scope and structure of their awareness and so can inform research.
Understanding in mathematics involves learners getting a sense of it in relation to their past experience. Informed by Gattegno’s (1987) assertion that ‘only awareness is educable’ and enriching the notion of concept image (Tall & Vinner, 1981) with the three interwoven dimensions of human psyche (cognition, affect, enaction), Mason developed a framework referred to as the ‘Structure of a Topic’ (SoaT) (see Mason and Johnston-Wilder, 2004) to describe how a mathematical topic is conceived. The framework comprises three strands: behaviour, emotion and awareness, which are closely associated with the more familiar terms enaction, affect and cognition. Behaviour is trained through practice but training alone renders the individual inflexible. Flexibility arises from awareness which informs an directs behaviour. Learning then involves educating awareness which in turn directs appropriate behaviour. Energy and motivation to learn arise from the harnessing of individuals’ emotions. Behaviour which is to be flexible and responsive to subtle changes must be guided by active awareness.

The ‘Structure of a Topic’ (SoaT) framework

Marton (Marton & Booth, 1997) regards learning as making distinctions, both discerning something from, and relating it to, a context. This fits with a view of mathematics as being essentially about the study of invariance in the midst of change. Integral to effective mathematics instruction is the use of examples to illustrate and clarify mathematical concepts. While teachers may use examples to illustrate definitions and exemplify the use of a particular rule or theorem, learners may focus on the specific details of examples and may develop restricted thinking that only those kinds of examples are appropriate. So, they may overlook the generic sense of exemplification which the teacher intends for them. What learners make of mathematical examples and their awareness of what can vary and what is kept constant to maintain the exemplary nature of the examples can reveal dimensions and depth of their awareness and promote and enrich their appreciation of the mathematical topics. One way of finding out which aspects of a topic dominates learners’ attention and whether they have grasped what the teacher intended for them to understand is by looking at what they make of mathematical examples.

Working out given examples involves different cognitive skills from constructing examples meeting specified constraints. Dahlberg & Housman (1997) showed how learners who generated examples and reflected on the process attained a more complete understanding of mathematical concepts by refining and expanding their evoked concept image. Hazzan & Zazkis (1997) showed how learners had difficulty managing degrees of freedom of generated examples. According to Watson & Mason (2005), encouraging learners to generate examples of mathematical objects can expand their example space and shift their attention away from the particularities of examples to generalizations. By prompting learners to construct examples, what they choose to change reveals dimensions, depth and scope of their awareness. Constructing examples forces them to attend to form in the example and disregard details that make up the example. Discerning generality with an awareness of particular details in mathematical examples requires learners to be sensitive to what can change and what must remain constant. Example construction activities may gradually lead learners to notice different things about the examples they produced. I conjecture that example construction itself could develop learners’ ability to discern dimensions-of-possible-variation and could reveal their awareness of the concept, which could give insight into the structure of their understanding.
Producing children who can think mathematically must be the main focus of school mathematics teaching. Thinking mathematically involves exercising mathematical powers such as imagining and expressing, specializing and generalizing, conjecturing and convincing, stressing and ignoring, ordering and characterizing, seeing sameness and seeing difference, assenting and asserting, among others. In order to investigate learners’ ability to engage in such activities, which can inform us about their mathematical thinking strategies, it is necessary to find out what sense they make of mathematical topics and which aspects of the topic they emphasize or dominate their attention. As such, in this research I am using the Structure of a Topic (SoaT) framework to find out the nature and structure of Stage One pupils understanding of mathematical topics. I am using the framework to characterise and to profile the structure of Stage One pupils’ mathematical thinking to see which aspects of a mathematical topic dominates their attention, and thus influence their understanding. I am also using the awareness element of the framework as a basis to enhance these children’s performance in mathematics by asking them to construct mathematical examples.

2. Objective(s) of the Research

The objectives of this research are:

(i) to plan, design and develop a model for profiling the structure of mathematical thinking of Stage One pupils in Malaysia using the Structure of a Topic (SoaT) framework.
(ii) to incorporate example-construction tasks with the current method of teaching mathematics.
(iii) to train teachers to incorporate example-construction in their mathematics classrooms.
(iv) to evaluate the effectiveness of the use of SoaT framework and to refine the framework to include Malaysian context and flavours.
(v) to assess the long-term effects of example construction and its impact on student understanding.

3. Methodology

The research will undergo three main phases of implementation: Planning and Designing, Profiling Structure of Mathematical Understanding and Implementation of Example Construction Tasks.

3.1 Implementation

(i) Planning & Designing

The planning and designing phase of the research will commence with discussions concerning the design of enhanced features of the Structure of a Topic (SoaT) framework. A workshop was conducted to collate all the ideas needed for this phase as well the delegation of works for the next phase. Major ideas consisted of the design, development and administration of the framework.

Next, an analysis of Stage One curriculum was carried out in order to gather data to determine the scope of Stage One mathematics and view recent reforms in the curriculum. This data is needed to carry out the needs analysis in terms of current and innovative strategies for the teaching of Mathematics so that design of the teaching and learning materials can be matched to attain maximal impact.

(ii) Profiling the structure of mathematical understanding of Stage One pupils

The Structure of a Topic (SoaT) framework will be used both as a generative tool to design questions and tasks in the study and as an analytic tool with which to analyse the data that emerges. Information from this will be used to characterize and profile the mathematical thinking and understanding of the pupils. The instrument with which to elicit responses will be based on the six strands of the framework. Questions will focus on behaviour (technique and language associated with the topic), awareness (images/sense-of and misconceptions associated with the topic) and emotion (root problems and contexts in which the topic is used). For the purpose of this research, the mathematics topic that will be focused on is Number Operations (addition, subtraction, multiplication, division).
Prior to using the asking questions based on the framework, a pre-test consisting of 20 questions will be conducted with all Year One, Year Two and Year Three pupils in three selected schools in Penang. This will give an insight into the pupils’ competence in Number Operations.

(iii) Implementation of example construction tasks in mathematics classrooms

One class of each (Year 1, Year 2 and Year 3) in each of the three schools will implement the example construction tasks at the end of a lesson for a subtopic. A book will be provided to each student to do the tasks. By prompting learners to construct mathematical objects (examples), what they choose to vary reveals the depth, dimensions and structure of their understanding, thereby structuring their awareness and thinking.

The implementation of the example construction tasks will take place in conjunction with the traditional method or current practices of the mathematics teacher so that time will not be taken from their schedules. However, initially one day of the week will be allotted to use the tasks to familiarize students (and teachers) of the innovative features of the modules (example construction). Later, it is hoped that the teachers and students will embrace the features until example construction becomes their ‘habit of mind’.

At the end of the data collection period, these books will be collected and marked (by the researcher) based on the richness of the examples constructed, namely the number of dimensions varied. In order to obtain a deeper understanding of their thinking, an interview will be conducted with a few selected pupils from each year and each school to explain about the examples they have constructed and the impact of the example construction tasks on their understanding/thinking. A post-test will also be administered with all the pupils to see the difference (if any) in the thinking/understanding of these pupils after being exposed to example construction tasks.

3.2 Research Instrument

The instrument that will be used to profile the pupils’ thinking/understanding is in the form of questions generated from the Structure of a Topic (SoaT) framework. Sample questions are as follows:

a) What do you understand by the word “addition” (subtraction, multiplication, division)?

b) What comes to mind when you see the sign ‘+’ (‘–’, ‘×’, ‘÷’)?

c) How were you introduced to the symbol ‘+’ (‘–’, ‘×’, ‘÷’)?

d) What are some of the things you need to watch out for when doing addition (subtraction, multiplication, division)?

e) What words in a problem tell you that addition (subtraction, multiplication, division) is relevant?

f) What problem(s) does addition (subtraction, multiplication, division) help to solve?

g) What are some of the techniques of addition (subtraction, multiplication, division) that you can think of?

4. Data Analysis

Analysis of data will be done in two phases:

(i) The first phase will be the profiling of the thinking/understanding of the Stage One pupils. This will be done by examining the answers to the questions based on each of the strands of the Structure of a Topic (SoaT) framework. The answers will be analysed to say something about the depth and richness of the answers. Theses answers will be analysed using a rubric, ranging from ‘superficial’, ‘emergent’ and ‘deep’ understanding. Profiles of thinking/understanding will then be mapped to say whether the pupils are behaviour-dominated, awareness-dominated or emotion-dominated or a mixture of all.
(ii) The next phase will be looking at the effect of example construction tasks as a result of ‘educating awareness’. Pupils will be asked to construct examples at the end of every subtopic learned for a certain period. Data will be analysed based on the richness and depth of the examples constructed. The more dimensions varied, the pupil is taken to have displayed deeper understanding.

5. Conclusion

This research attempts to map ways of thinking/understanding of Stage One pupils in selected schools in Penang. Profiles of their understanding will be attained through first, asking questions based on the Structure of a Topic (SoaT) framework. The framework outlines the three strands that describe how a mathematical topic is conceived: behaviour, awareness, emotion. Depths/richness of answers to the questions generated from the framework will be used to inform about the structure of the pupils’ understanding/thinking.

Mapping or profiling of mathematical thinking/understanding can give insight to the ways in which children’s understanding/thinking is structured – whether technique, awareness or emotion dominates their attention. This information will be useful to teachers in order to know whether their students have understood the topics taught superficially (procedure-dominated) or have deeper understanding of the concepts (awareness-dominated). Dominance of procedure can incapacitate students, especially when questions are slightly modified than what they are used to or familiar with. Teachers can then modify their teaching strategies or questioning techniques so that learners’ awareness is educated, along with the training of behaviour through technique acquisition.

One way to educate awareness is through asking learners to construct examples. Constructing examples can not only train behaviour on how to work out the examples correctly but also reinforce understanding of what could be changed and what must remain constant in the examples. Information from the profiles of thinking/understanding will then form the basis to incorporate example construction tasks alongside with traditional classroom teaching practices. Examples constructed will be analysed to see whether they have revealed any depth in awareness of the topic. This will be done by examining the number of dimensions varied in the examples. The example construction tasks serve as educating awareness (of dimensions of possible variation in examples) which will produce flexibility in behaviour when doing mathematics.

References


