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The numerical solution of singular initial value problems using Chebyshev wavelet collocation method

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ABSTRACT

Wavelet analysis is a recently developed mathematical tool for many problems. In this paper, an efficient and new numerical method is proposed for the numerical solution of singular initial value problems, which is based on collocation points with Chebyshev wavelet. The present method is developed using the Chebyshev wavelet and its operational matrices to obtain higher accuracy. It has been shown here that the present method can be easily implemented and the results obtained are most accurate. Hence the present method has a clear advantage over the classical methods. Numerical order of convergence of the proposed method is calculated. The results show the better accuracy of the proposed method, which is justified through the illustrative examples.

1. Introduction

In recent years, the studies of singular initial value problems in the second order ordinary differential equations (ODEs) have attracted the attention of many mathematicians and physicists. Many methods including numerical and perturbation methods have been used to solve such type of problems. The approximate solutions for these problems were presented by many researchers, for example, Wazwaz [1–3] were used the adomain decomposition method (ADM) and Yildirim and Ozis [4] were used the variational iteration method (VIM).

Nowadays the subject of wavelets has drawn a great deal of attention from mathematical scientists in various disciplines. It is creating a common link between mathematicians, physicists, and engineers. Wavelet theory is a relatively new and an emerging area in mathematical research. It has been applied to a wide range of engineering disciplines; particularly, wavelets are very successfully used in signal analysis for wave form representation and segmentations, time frequency analysis, and fast algorithms for easy implementation. Many families of wavelets have been proposed in the mathematical literature.

Among the different wavelet families, most simple are the Haar wavelets. Haar wavelets have been used by many researchers because of their simplicity and better convergence. Some of the relevant work can be found in [5–12]. The weaker side of using the Haar basis functions for approximating smooth functions is that they are lower in accuracy due to their non-smooth character. To cover this aspect, smooth Chebyshev wavelets [13–15] are considered to get more accurate approximation. Chebyshev wavelet uses Chebyshev polynomials as their basis functions. Chebyshev polynomials and their properties are employed for deriving a general procedure for formation of matrix. Then Chebyshev wavelets expansions along with operational matrices are applied for solving differential equations. Because of their improved smoothness and good interpolating properties, accuracy of Chebyshev wavelets is better than Haar wavelets.

In this paper, the attempt is made to solve singular initial value problems using Chebyshev wavelet collocation method (CWCM). This method consists of reducing the differential equation into a set of algebraic equations by first expanding the Chebyshev wavelets with unknown coefficients. By solving these coefficients, we get the required solution. Here we demonstrate the method by considering the some of the illustrative examples.

The paper is organized as follows; preliminaries of wavelets are given in Section 2. Method of solution is discussed in Section 3. Numerical examples are presented in Section 4. The conclusion of the work is drawn in Section 5.

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2. Wavelet preliminaries

2.1. Wavelets

Recently, wavelets have been applied extensively for signal processing in communications and physics research, and have proved to be a wonderful mathematical tool. Wavelets can be used for algebraic manipulations in the system of equations obtained which leads to better resulting system. Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously, we have the following family of continuous wavelets;

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right),\ a, b \in R, a \neq 0 \tag{2.1}$$

The best way to understand wavelets is through a multi-resolution analysis. Given a function $f \in L_2(R)$ a multi-resolution analysis (MRA) of $L_2(R)$ produces a sequence of subspaces $V_j, V_{j+1}, \ldots,$ such that the projections of f onto these spaces give finer and finer approximations of the function f as $j \to \infty$.

A multi-resolution analysis of $L_2(R)$ is defined as a sequence of closed subspaces $V_j \subset L_2(R), j \in Z$ with the following properties

(i) $\ldots \subset V_{-1} \subset V_0 \subset V_1 \subset \ldots$.
(ii) The spaces V_j satisfy $\cup_{j\in Z} V_j$ is dense in $L_2(R)$ and $\cap_{j\in Z} V_j = 0$.
(iii) If $f(t) \in V_0, f(2^j t) \in V_j$, i.e. the spaces V_j are scaled versions of the central space V_0.
(iv) If $f(t) \in V_0, f(2^j t - m) \in V_j$ i.e. all the V_j are invariant under translation.
(v) There exists $\phi \in V_0$ such that $\phi(t-m); m \in Z$ is a Riesz basis in V_0.

The space V_j is used to approximate general functions by defining appropriate projection of these functions onto these spaces. Since the union of all the V_j is dense in $L_2(R)$, so it guarantees that any function in $L_2(R)$ can be approximated arbitrarily close by such projections. As an example the space $\{V_j, j \in Z\}$ can be defined like

$$V_j = W_j \oplus V_{j-1} = W_{j-1} \oplus W_{j-2} \oplus V_{j-2} = \ldots = \bigoplus_{j=1}^{J+1} W_j \oplus V_0$$

For each j the space W_j serves as the orthogonal complement of V_j in V_{j+1}. The space W_j include all the functions in V_{j+1} that are orthogonal to all those in V_j under some chosen inner product. The set of functions which form basis for the space W_j are called wavelets.

2.2. Chebyshev wavelets and operational matrix of integration

Here we presented a family of wavelets, called Chebyshev wavelets, which are derived from Chebyshev polynomials. For any positive integer k, the Chebyshev wavelets family is defined on the interval [0,1) [13] as follows;

$$C_{n,m}(t) = \begin{cases} \frac{\alpha_m 2^{k/2}}{\sqrt{\pi}} T_m(2^{k+1}t - 2n + 1), & \text{for } \frac{n-1}{2^k} \leqslant t < \frac{n}{2^k} \\ 0, & \text{Otherwise} \end{cases} \tag{2.2}$$

where $n = 1, 2, \ldots, 2^k$ and $m = 0, 1, \ldots, M-1$, M is the maximum order of the Chebyshev polynomial and $\alpha_m = \begin{cases} \sqrt{2}, & m = 0 \\ 2, & \text{Otherwise} \end{cases}$. Here $T_m(t)$ are the well known Chebyshev polynomials of order m. Chebyshev polynomials can be calculated recursively with the help of the following equations;

$$T_0(t) = 1,\ T_1(t) = t,\ T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t),\ m = 1, 2, 3, \ldots$$

Equivalently, for any positive integer k, the Chebyshev wavelets family is defined as fallows;

$$C(t) = C_i(t) = \begin{cases} \frac{\alpha_m 2^{k/2}}{\sqrt{\pi}} T_m(2^{k+1}t - 2n + 1), & \text{for } \frac{n-1}{2^k} \leqslant t < \frac{n}{2^k} \\ 0, & \text{Otherwise} \end{cases} \tag{2.3}$$

where $i = n + 2^k m$. By varying the values of i with respect to the collocation points $t_j = \frac{j-0.5}{N}$, $j = 1, 2, \ldots, N$, we get the Chebyshev matrix of order $N \times N$, where $N = 2^k M$ and Chebyshev polynomials used in the approximation are of degree less than M.

The integration of Chebyshev wavelets is given as

$$\int_0^t C(t)dt = PC(t) = P_1 \tag{2.4}$$

$$\int_0^t PC(t)dt = P^2 C(t) = P_2 \tag{2.5}$$

and in general

$$\int_0^t P^{n-1} C(t)dt = P^n C(t) = P_n,\ n > 0 \tag{2.6}$$

where P is the $N \times N$ operational matrix for integration and is given as

$$P = \begin{pmatrix} D & U & U & \cdots & U \\ 0 & D & U & \cdots & U \\ 0 & 0 & \ddots & \ddots & U \\ \vdots & \vdots & \ddots & D & U \\ 0 & 0 & \cdots & 0 & D \end{pmatrix}$$

where U and D are $M \times M$ matrices given by

$$U = \frac{\sqrt{2}}{2^k} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ -\frac{1}{3} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ -\frac{1}{15} & 0 & 0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ -\frac{1}{M(M-2)} & 0 & 0 & \cdots & 0 \end{pmatrix}$$

and

$$D = \frac{1}{2^k} \begin{pmatrix} \frac{1}{2} & \frac{1}{2\sqrt{2}} & 0 & 0 & \cdots & 0 & 0 & 0 \\ -\frac{1}{4\sqrt{2}} & 0 & \frac{1}{8} & 0 & \cdots & 0 & 0 & 0 \\ -\frac{1}{3\sqrt{2}} & -\frac{1}{4} & 0 & \frac{1}{12} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ -\frac{1}{2\sqrt{2}(M-1)(M-3)} & 0 & 0 & 0 & \cdots & -\frac{1}{4(M-3)} & 0 & \frac{1}{4(M-1)} \\ -\frac{1}{2\sqrt{2}(M)(M-2)} & 0 & 0 & 0 & \cdots & 0 & -\frac{1}{4(M-3)} & 0 \end{pmatrix}$$

3. Method of solution

Consider the singular initial value problem of the form

$$u''(t) + \frac{c}{t}u'(t) + f(u(t)) = g(t) \tag{3.1}$$

subject to the initial condition $u(0) = \alpha, u'(0) = \beta$ (3.2)

where $c, \alpha \& \beta$ are real constants, $f(u(t))$ is a real valued function and $g(t)$ is nonhomogeneous term. The proposed method is follows as

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Step 1: Let us assume that

$$u''(t) = \sum_{i=1}^{N} a_i C_i(t) \tag{3.3}$$

where a_i's, $i = 1, 2, \ldots, N$ are Chebyshev wavelet coefficients to be determined.

Step 2: Integrating (3.3) twice with respect to the given condition (3.2) we get,

$$u'(t) = \beta + \sum_{i=1}^{N} a_i P_{1i}(t) \tag{3.4}$$

and

$$u(t) = \alpha + \beta t + \sum_{i=1}^{N} a_i P_{2i}(t) \tag{3.5}$$

Step 3: Substituting the values of (3.3)–(3.5) in (3.1) then we get,

$$\sum_{i=1}^{N} a_i C_i(t) + \frac{c}{t}\left(\beta + \sum_{i=1}^{N} a_i P_{1i}(t)\right) + f\left(\alpha + \beta t + \sum_{i=1}^{N} a_i P_{2i}(t)\right) = g(t) \tag{3.6}$$

Step 4: Solve (3.6), we obtain Chebyshev wavelet coefficients a_i, substituting these a_i in (3.4) we get the solution of the problem (3.1).

The error will be calculated by using $E = |u_e - u_a|$ and $E_{\max} = \max|u_e - u_a|$, where $u_e \& u_a$ are exact and approximate solutions respectively.

The convergence analysis of the Chebyshev wavelets is given through the following Lemma,

Lemma. Assume that the $u(t) \in L_2(R)$ with the bounded first derivative on (0, 1), then the error norm at kth level satisfies the following inequality $\|e_k(t)\| \leqslant A2^{-(3/2)(N/2)}$, where $A = \sqrt{\frac{K}{7}}C$ is some real constant.

Proof. The error at kth level may be defined as,

$$|e_k(t)| = |u(t) - u_k(t)| = \left|\sum_{i=N+1}^{\infty} a_i C_i(t)\right|$$

where

$$u_k(t) = \sum_{i=1}^{N=2^{k+1}} a_i C_i(t)$$

$$\|e_k(t)\|^2 = \int_{-\infty}^{\infty}\left\langle \sum_{i=N+1}^{\infty} a_i C_i(t), \sum_{l=N+1}^{\infty} a_l C_l(t)\right\rangle dt$$

$$= \sum_{i=N+1}^{\infty}\sum_{l=N+1}^{\infty} a_i a_l \int_{-\infty}^{\infty} C_i(t) C_l(t) dt$$

$$\|e_k(t)\|^2 \leqslant \sum_{i=N+1}^{\infty} |a_i|^2$$

But

$$|a_i| \leqslant C2^{-\frac{3i}{2}} \max|u'(t)|$$

where

$$C = \int_0^1 |tC_2(t)| dt \text{ and } t \in \left(\frac{n-1}{2^k}, \frac{n}{2^k}\right)$$

Then

$$\|e_k(t)\|^2 \leqslant \sum_{i=N+1}^{\infty} KC^2 2^{-3i}$$

where $|u'(t)| \leqslant K \forall t \in (0,1)$ where K is positive constant.

$$\|e_k(t)\|^2 \leqslant KC^2 \frac{1}{7} 2^{-3(N/2)}$$

$$\|e_k(t)\| \leqslant \sqrt{\frac{K}{7}} C2^{-(3/2)(N/2)}$$

$\|e_k(t)\| \leqslant A2^{-(3/2)(N/2)}$, where $A = \sqrt{\frac{K}{7}}C$ is some real constant. □

From the above lemma, the error bound is inversely proportional to the level of the resolution of the Chebyshev wavelets. This ensures that the convergence of the Chebyshev wavelets approximation by increasing the level of resolution.

Rate of convergence $R_c(N)$:

The rate of convergence is defined as $R_c(N) = \frac{\log(E_{\max}(N/2)/E_{\max}(N))}{\log 2}$.

4. Numerical examples

In this section, we consider some of the singular initial value problems to demonstrate the applicability of the proposed method.

Table 1
Comparison of numerical solutions with exact solution for $N = 16$ of Problem 1.

$t(= 1/32)$	VIM	HWCM	CWCM	Exact
1	1.00097703	1.00097815	1.00026748	1.00097703
3	1.00882779	1.00882642	1.00811104	1.00882779
5	1.02471452	1.02470820	1.02398686	1.02471452
7	1.04901493	1.04900127	1.04826932	1.04901493
9	1.08231423	1.08229096	1.08154578	1.08231423
11	1.12542873	1.12539373	1.12462986	1.12542873
13	1.17943916	1.17939066	1.17860099	1.17943918
15	1.24573589	1.24567263	1.24485199	1.24573605
17	1.32607839	1.32600014	1.32211963	1.32607912
19	1.42267242	1.42258103	1.41799363	1.42267522
21	1.53826926	1.53817116	1.53288963	1.53827869
23	1.67629260	1.67620433	1.67019824	1.67632108
25	1.84099996	1.84095995	1.83416112	1.84107853
27	2.03768719	2.03777990	2.03008186	2.03788817
29	2.27294665	2.27334961	2.26460332	2.27342854
31	2.55499169	2.55606378	2.54607106	2.55608441

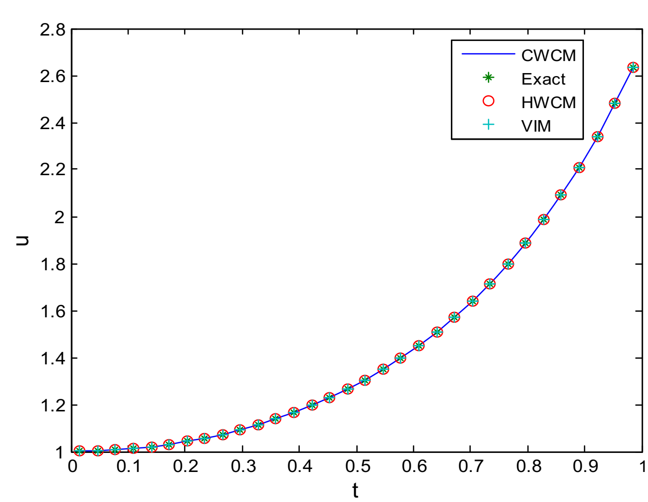

Figure 1. Comparison of numerical solutions with exact solution for $N = 32$ of Problem 1.

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Table 2
Error analysis of Problem 1.

M	k	N	E_{max} (VIM)	E_{max} (HWCM)	E_{max} (CWCM)	Rate of convergence R_C (N)		
						VIM	HWCM	CWCM
8	1	16	1.0927 e−03	1.1858 e−04	1.0013 e−02	–	–	–
8	2	32	1.3304 e−03	3.1418 e−05	8.0366 e−04	0.2840	1.9162	3.6391
8	3	64	1.4664 e−03	7.9647 e−06	5.6995 e−05	−0.1404	1.9799	3.8177
8	4	128	1.5391 e−03	1.9977 e−06	3.8107 e−06	−0.0698	1.9953	3.9027
8	5	256	1.5767 e−03	4.9990 e−07	2.4673 e−07	−0.0348	1.9986	3.9491
8	6	512	1.5958 e−03	1.2500 e−07	1.5702 e−08	−0.0174	1.9997	3.9739

Table 3
Comparison of numerical solutions with exact solution for $N = 16$ of Problem 2.

$t(= 1/32)$	ADM	HWCM	CWCM	Exact
1	0.99983725	0.99983726	0.99982934	0.99983724
3	0.99853644	0.99853577	0.99852790	0.99853579
5	0.99594092	0.99593584	0.99592807	0.99593595
7	0.99206288	0.99204356	0.99203595	0.99204379
9	0.98692059	0.98686804	0.98686064	0.98686845
11	0.98053837	0.98042140	0.98041426	0.98042201
13	0.97294656	0.97271871	0.97271189	0.97271958
15	0.96418146	0.96377797	0.96377151	0.96377913
17	0.95428530	0.95362005	0.95361549	0.95362156
19	0.94330619	0.94226865	0.94226471	0.94227053
21	0.93129800	0.92975019	0.92974688	0.92975249
23	0.91832033	0.91609379	0.91609114	0.91609655
25	0.90443844	0.90133115	0.90132918	0.90133441
27	0.88972309	0.88549648	0.88549523	0.88550028
29	0.87425050	0.86862640	0.86862590	0.86863078
31	0.85810221	0.85075984	0.85076013	0.85076484

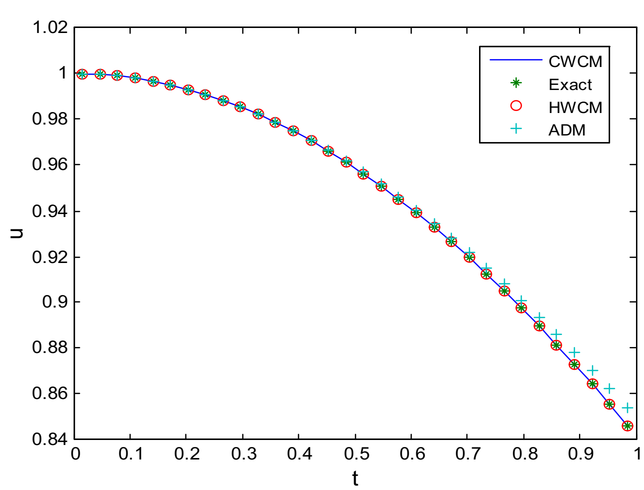

Figure 2. Comparison of numerical solutions with exact solution for $N = 32$ of Problem 2.

Table 5
Comparison of numerical solutions with exact solution for $N = 16$ of Problem 3.

$t(= 1/32)$	ADM	HWCM	CWCM	Exact
1	0	−0.00004853	−0.00100612	−0.00002956
3	0	−0.00064069	−0.00172326	−0.00074672
5	0	−0.00310421	−0.00419508	−0.00321865
7	0	−0.00803492	−0.00915397	−0.00817775
9	0	−0.01583280	−0.01696609	−0.01599025
11	0	−0.02649727	−0.02763139	−0.02665615
13	0	−0.03966067	−0.04078360	−0.03980922
15	0	−0.05459072	−0.05569027	−0.05471706
17	0	−0.07018880	−0.07125271	−0.07028102
19	0	−0.08499002	−0.08600604	−0.08503627
21	0	−0.09716330	−0.09811914	−0.09715175
23	0	−0.10451129	−0.10539472	−0.10443019
25	0	−0.10447044	−0.10526925	−0.10430812
27	0	−0.09411095	−0.09481301	−0.09385585
29	0	−0.07013679	−0.07073004	−0.06977748
31	−0.02841102	−0.02888567	−0.02935821	−0.02841091

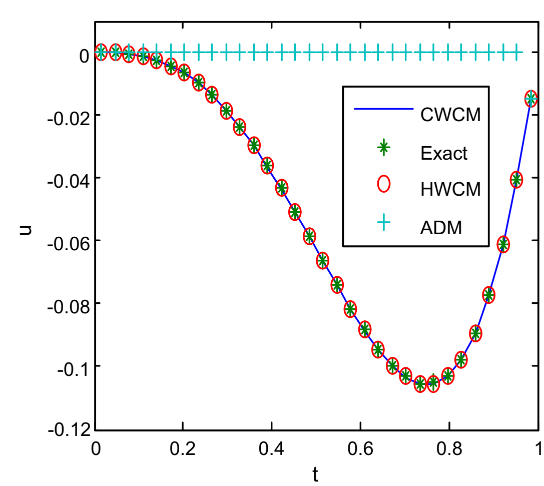

Figure 3. Comparison of numerical solutions with exact solution for $N = 32$ of Problem 3.

Table 4
Error analysis of Problem 2.

M	k	N	E_{max} (ADM)	E_{max} (HWCM)	E_{max} (CWCM)	Rate of convergence R_C (N)		
						ADM	HWCM	CWCM
8	1	16	7.3373 e−03	5.0012 e−06	7.9071 e−06	–	–	–
8	2	32	7.8221 e−03	1.2932 e−06	5.0500 e−07	0.0923	1.9513	3.9688
8	3	64	8.0733 e−03	3.2854 e−07	3.1732 e−08	0.0456	1.9768	3.9923
8	4	128	8.2012 e−03	8.2783 e−08	1.9859 e−09	0.0227	1.9887	3.9981
8	5	256	8.2657 e−03	2.0776 e−08	1.2416 e−10	0.0113	1.9944	3.9995
8	6	512	8.2981 e−03	5.2040 e−09	7.7608 e−12	0.0056	1.9972	3.9999

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Table 6
Error analysis of Problem 3.

M	k	N	E_{max} (ADM)	E_{max} (HWCM)	E_{max} (CWCM)	Rate of convergence R_C (N)		
						ADM	HWCM	CWCM
8	1	16	1.0443 e−01	4.7475 e−04	9.7656 e−04	–	–	–
8	2	32	1.0520 e−01	1.2685 e−04	6.1035 e−05	0.0106	1.9040	4.0000
8	3	64	1.0540 e−01	3.2718 e−05	3.8147 e−06	0.0027	1.9550	4.0000
8	4	128	1.0545 e−01	8.3041 e−06	2.3842 e−07	0.0006	1.9782	4.0000
8	5	256	1.0546 e−01	2.0915 e−06	1.4901 e−08	0.0001	1.9893	4.0000
8	6	512	1.0546 e−01	5.2482 e−07	9.3132 e−10	0	1.9946	4.0000

Table 7
Comparison of numerical solutions with exact solution for $N = 16$ of Problem 4.

$t(= 1/32)$	VIM	HWCM	CWCM	Exact
1	0.00100708	0.00103759	0.00100708	0.00100708
3	0.00961303	0.00955808	0.00961303	0.00961303
5	0.02822875	0.02809547	0.02822875	0.02822875
7	0.05831909	0.05811939	0.05831909	0.05831909
9	0.10134887	0.10108593	0.10134887	0.10134887
11	0.15878296	0.15845846	0.15878295	0.15878295
13	0.23208619	0.23170138	0.23208618	0.23208618
15	0.32272343	0.32227940	0.32272338	0.32272338
17	0.43215955	0.43165735	0.43215942	0.43215942
19	0.56185944	0.56130010	0.56185913	0.56185913
21	0.71328807	0.71267258	0.71328735	0.71328735
23	0.88791045	0.88723969	0.88790893	0.88790893
25	1.08719173	1.08646636	1.08718872	1.08718872
27	1.31259723	1.31181752	1.31259155	1.31259155
29	1.56559252	1.56475809	1.56558227	1.56558227
31	1.84764352	1.84675301	1.84762573	1.84762573

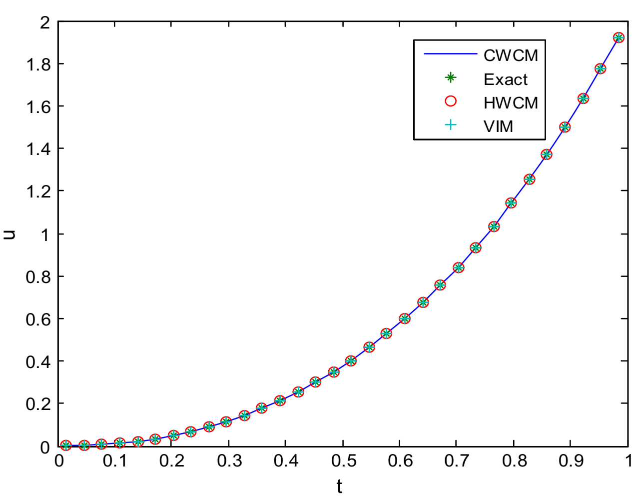

Figure 4. Comparison of numerical solutions with exact solution for $N = 32$ of Problem 4.

Problem 1. First consider the equation of the type [4],

$$u''(t) + \frac{2}{t}u'(t) - 2(2t^2 + 3)u(t) = 0 \tag{4.1}$$

with the initial conditions $u(0) = 1, u'(0) = 0$.

As per the method explained in Section 3, we obtain the solution (CWCM) of the problem (4.1) and is presented in comparison with exact solution $u(t) = e^{t^2}$, Haar wavelet collocation method (HWCM) solution (as the method explained in [11,12]) and VIM solution in Table 1 for $N = 16$ ($M = 8$ & $k = 1$) & Fig. 1 for $N = 32$ ($M = 8$ & $k = 2$). The error analysis for higher values of N is given in Table 2.

Problem 2. Next, consider the equation of the form [3],

$$u''(t) + \frac{2}{t}u'(t) + u(t) = 0 \tag{4.2}$$

with the initial conditions $u(0) = 1, u'(0) = 0$.

Using the method explained in Section 3, we get the CWCM solution and is presented in comparison with ADM, HWCM solution and exact solution $u(t) = \frac{\sin t}{t}$ in Table 3 for $N = 16$ ($M = 8$ & $k = 1$) & Fig. 2 for $N = 32$ ($M = 8$ & $k = 2$). The error analysis for higher values of N is given in Table 4.

Problem 3. Now, consider the non homogeneuos equation of the type [2],

$$u''(t) + \frac{8}{t}u'(t) + tu(t) = t^5 - t^4 + 44t^2 - 30t \tag{4.3}$$

with the initial conditions $u(0) = 0, u'(0) = 0$.

As in the previous examples, we obtained the CWCM solution and is presented in comparison with exact solution $u(t) = t^4 - t^3$, ADM and HWCM solution in Table 5 for $N = 16$ ($M = 8$ & $k = 1$) & Fig. 3 for $N = 32$ ($M = 8$ & $k = 2$). The error analysis for higher values of N is given in Table 6.

Problem 4. Finally consider another non homogeneuos equation of the form [4],

$$u''(t) + \frac{2}{t}u'(t) + u(t) = t^3 + t^2 + 12t + 6 \tag{4.4}$$

with the initial conditions $u(0) = 0, u'(0) = 0$.

Table 8
Error analysis of Problem 4.

M	k	N	E_{max} (VIM)	E_{max} (HWCM)	E_{max} (CWCM)	Rate of convergence R_C (N)		
						VIM	HWCM	CWCM
8	1	16	1.7795 e−05	8.7272 e−04	1.7764 e−15	–	–	–
8	2	32	2.0316 e−05	2.2143 e−04	8.8816 e−16	0.1911	1.9787	1.0001
8	3	64	2.1691 e−05	5.5744 e−05	4.4409 e−16	0.0945	1.9900	1.0000
8	4	128	2.2409 e−05	1.3982 e−05	6.6613 e−16	0.0470	1.9952	−0.5850
8	5	256	2.2776 e−05	3.5015 e−06	6.6613 e−16	0.0234	1.9975	0
8	6	512	2.2961 e−05	8.7609 e−07	8.8818 e−16	0.0117	1.9988	−0.4150

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As in the previous examples, we get the CWCM solution and is presented in comparison with HWCM, VIM solution and exact solution $u(t) = t^2 + t^3$ in Table 7 for $N = 16$ ($M = 8$ & $k = 2$) & Fig. 4 for $N = 32$ ($M = 8$ & $k = 3$). The error analysis for higher values of N is given in Table 8.

5. Conclusion

A Chebyshev wavelet collocation method has been proposed for the numerical solution of singular initial value problems. The performance of CWCM is superior to the HWCM and other classical methods for example VIM & ADM which is justified through the illustrative problems. Superior accuracy is attained in the case of CWCM over the other methods. The main advantage of this method is its simplicity and small computation costs.

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