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Procedia Engineering 31 (2012) 267 – 273

Procedia Engineering

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International Conference on Advances in Computational Modeling and Simulation

Periodicity and self-similarity of vortex evolution in a double-lid-driven cavity flow

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Abstract

The flow configuration of the two-dimensional low Reynolds number flow in a rectangular cavity with two opposite moving lids and different depth-to-width ratios is investigated. The effects of aspect ratio varying from 0.15 to 6.6 on vortex structure in the cavity were numerated using the differential quadrature method. The critical aspect ratios, streamline patterns and bifurcation diagrams were presented. It is found that the vortex structure distributes in the transverse direction of cavity and the sub-eddy centers gradually merge as aspect ratio increases from 0.15 to 0.7. When the aspect ratio is larger than 0.7, the flow structure unfolds in the longitudinal direction of cavity and the number of vortices gradually increases with the aspect ratio increasing. The evolution of flow pattern exhibits the characteristics of periodicity and self-similarity. The large outer vortices evolve from the growth of new vortices in the middle region of cavity. The flow patterns are always symmetric about the cavity centre at different aspect ratios.

Keywords: rectangular cavity driven flow; vortex structure; differential quadrature method; depth-to-width ratio.

1. Introduction

The flow in a rectangular cavity involves many complex flow phenomena such as different types and scales of vortices, bifurcation, transition and turbulence. These important flow characteristics are closely relevant to a number of practical applications [1, 2]. Meanwhile, this type of flow is also widely used to verify the effectiveness of new and improved numerical solution methods.

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doi:10.1016/j.proeng.2012.01.1023

The two-dimensional steady flow in a rectangular cavity can be driven by one or a pair of translating lids. The cavity flow with a single moving lid has been studied extensively. Relevant conclusions for square cavity flow obtained by Ghia [3] and Schreiber et al [4] were later confirmed and expanded by several other authors. For Re =100, 300, 500 [5], Re =1000, 5000, 10000 [6] and Re =1, 10, 30 [7], the square cavity flow was simulated using different numerical techniques. The numerical solutions showed a significant feature of flow which consists of a primary large vortex and two secondary vortices in the square cavity. The tertiary vortex is formed near the left corner at high Reynolds number. Cheng et al [8] numerated the flow configuration in a rectangular cavity at different aspect ratios and Reynolds numbers.

Investigations about the vortex structure in a double-lid-driven rectangular cavity has been conducted, but to a much lesser extent. Caskell et al [9] analyzed Stokes flow in a square cavity with the lids moving in opposite directions and showed the flow pattern was symmetric about the centre of cavity. Galaktionov et al [10] considered Stokes flow in a rectangular cavity with one cylinder placed in its centre and Young et al [11] considered the same problem but with two cylinders. He et al [12] simulated the vortex structure evolution in a square cavity at different Reynolds numbers.

The aim of this paper is to simulate the vortex evolution of two-dimensional low Reynolds number flow in a rectangular cavity with depth-to-width ratios changing. The flow is driven by tangential movement at the cavity top and bottom lids in opposite directions. The evolution characteristics of the vortex structure which exhibits periodicity and self-similarity is revealed when aspect ratio vary from 0.15 to 6.6.

2. Problem description, solution method and validation

The problem under investigation is schematically in Fig.1. Consider an incompressible fluid with kinematic viscosity μ and density ρ in a rectangular cavity with width L and height H. The flow is induced by two lids moving tangentially in opposite directions with speeds U and U_b. If the length, velocity and pressure are scaled with L , U and $\mu U / L$, then the steady Navier-Stokes and continuity equations can be written as

$$Re\, \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nabla^2 \boldsymbol{u}, \quad \nabla \boldsymbol{u} == 0 \tag{1}$$

where $\boldsymbol{u} = (u, v)$ and p is pressure. The Reynolds number and depth-to-width ratio are denoted respectively by $Re = \rho UL / \mu$ and $A = H / L$.

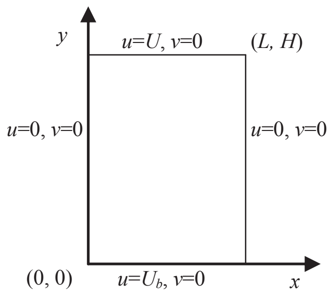
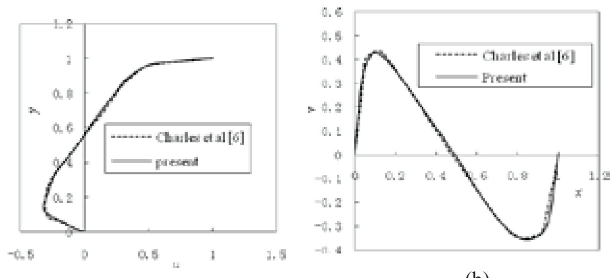


Fig. 1 Schematic of cavity flow



(a) (b)

Fig. 2 Comparison of velocity profile through the geometric centerline of the cavity for A=1, U=1, U_b=0, Re=1000. (a) u-velocity along vertical centerline, (b) v-velocity along horizontal centerline.

Boundary conditions for the velocity field are the lids-driven velocities at the roof-wall and the bottom-wall, and no-slip conditions for velocity components at the two side walls. They are specified as Fig.1.

For the numerical simulation, the polynomial-based differential quadrature method is used to approximate the space partial derivatives. The weighting coefficients is determined based on Lagrangian polynomial as test functions. The non-uniform grid is employed to discrete equations (1). The pressure correction for the iteration of $n+1$ layer is implemented with the SIMPLE algorithm. For details, see [12-14].

In order to check the quality and accuracy of differential quadrature code used to solve two-dimensional incompressible flow in a rectangular cavity, we validate the code first for the problem having benchmark test data. Analysis are carried out in a square cavity with a single moving lid (A=1, U=1, U_b=0) and Re=1000. Fig.2 plots the velocity comparison along the vertical centerline and the horizontal centerline of the cavity. It can be see that the results of the present simulation are found to be in good agreement with the data of Charles-Henri and Mazen [6].

3. Results and discussion

As aspect ratios A increases from 0.15 to 6.6, a sequence of streamline patterns is obtained for U=1, U_b=-1and Re=0.01 as shown in Fig. 3-7. It is observed that all the streamline patterns are always symmetric about the cavity centre at different A.

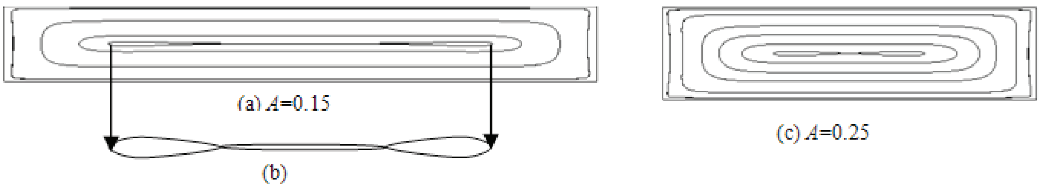


Fig. 3 Streamline pattern one for U=1, U_b=-1, Re=0.01and different A

For the aspect ratio A=0.15, Fig.3(a) shows the streamline patterns with an outer circulation, a separatrix and three sub-eddies plus two saddle points. The sub-eddy centers and saddle points are located on the horizontal centerline of the cavity. The intermediate sub-eddy center coincides with the center of the cavity and two terminal sub-eddies are transversely symmetric. For the sake of clarity, Fig.3 (b) shows an enlargement of the separatrix in Fig.3 (a). As A increases gradually, the intermediate sub-eddy is gradually reduced, and two terminal sub-eddies and saddle points are gradually close to the center of the cavity. At $A_1 \approx 0.167$, two saddle points merge into one in the cavity center and the intermediate sub-eddy disappears. Fig.3(c) where the A=0.25 shows a circulation, a saddle point with a separatrix and two sub-eddies.

At $A_2 \approx 0.315$, two sub-eddies merge and one single large eddy occupies entire cavity. Fig.4 (a), 4(b) and 4(c) respectively show the flow patterns where the A=0.4, A=0.7 and A=0.9. It can be seen from the figures that the large eddy stretches from transverse to longitudinal direction with the A increasing. At $A_3 \approx 0.937$, the center of the cavity becomes a saddle point and two new sub-eddies appear. The centers are located in the vertical centerline of the cavity (see Fig.4 (d) where the A=1.0).

When there is a separatrix through the saddle point, the system's structure is unsteady according to the theory of nonlinear dynamics. Taking aspect ratios $A(A \leqslant 0.937)$ as the bifurcation parameter, Fig.5 illustrates the bifurcation diagram where the x coordinates of stable centers (solid lines) and unstable saddle points (dashed lines) are shown as A is varied. $A_1 \approx 0.167$ and $A_2 \approx 0.315$ are called the critical aspect ratios.

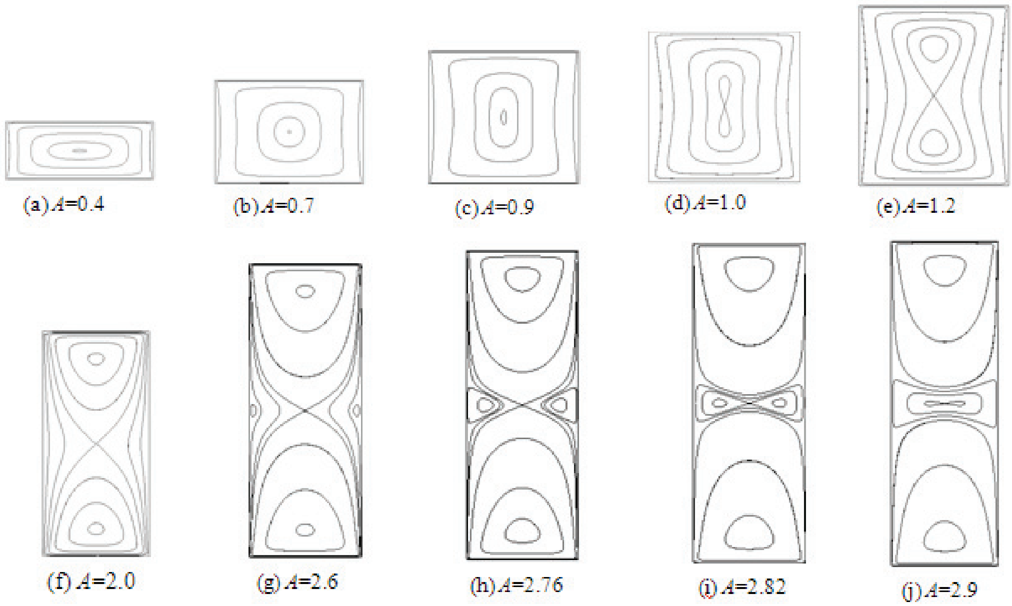


Fig. 4 Streamline pattern two for U=1, U_b=-1, Re=0.01and different A

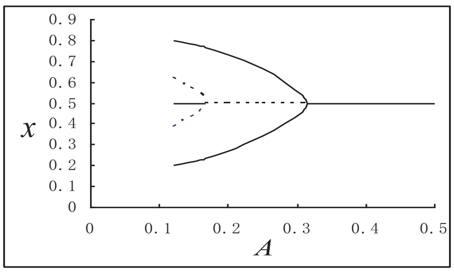


Fig.5 (A, x) *b*ifurcation diagram for U=1, U_b=-1, Re=0.001

With the A increasing, two sub-eddies shown in Fig.4 (d) grow and the centers close to the top and bottom walls. Fig.4 (e) and 4(f) show the streamline pattern where the A=1.2 and A=2.0. At A_4≈2.553, two side eddies are about to emerge in the middle region adjacent to each stationary side wall. Fig.4 (g) where A=2.6 shows a saddle point at the cavity center with a separatrix, two sub-eddies and two isolated side eddies. As A increases, the side eddies expand and approach the center of the cavity (Fig.4 (h) where the A=2.76). At A_5≈2.804, two isolated side eddies meet with each other at the interior saddle point and evolve into a pair of transverse sub-eddies. At critical ratio A_5, there are two pairs of complete sub-eddies within the cavity. As A is increased beyond A_5, the centers of transverse sub-eddies approach the saddle point and the longitudinal sub-eddies are separated into two outer large eddies (Fig.4 (i) where the A=2.82). With the A increasing, the sizes of transverse sub-eddies are gradually decreased. The streamline pattern for A=2.9, which consists of two outer eddies, a middle circulation enclosing two sub-eddies, is shown in Fig.4 (j).

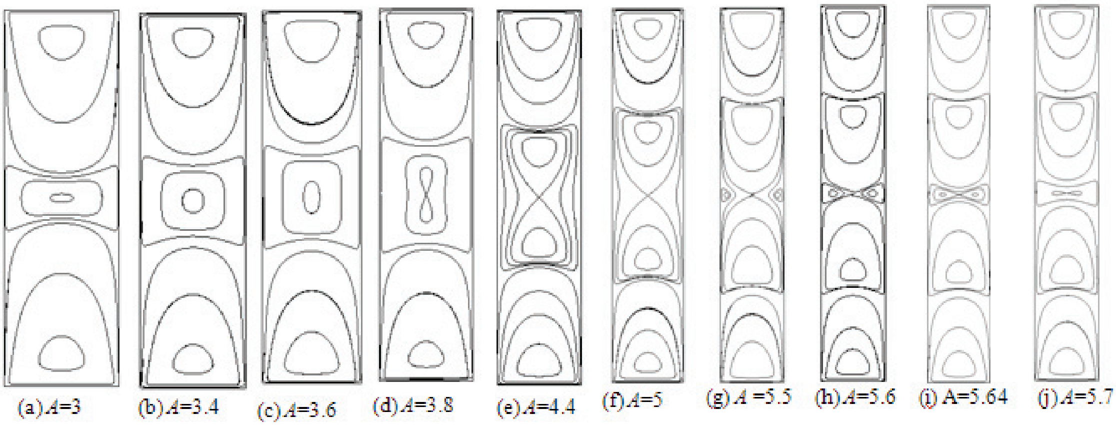


Fig.6 Streamline pattern three for U=1, U_b=-1, Re=0.01and different A

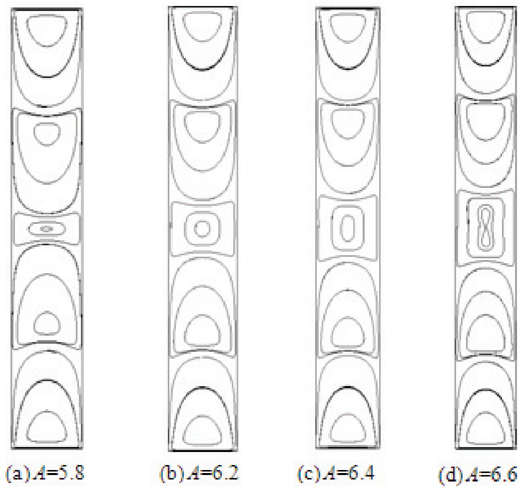


Fig. 7 Streamline pattern four for U=1, U_b=-1, Re=0.01and different A

As A increases, the sub-eddies centers lying on the horizontal centerline of the cavity approach the interior saddle point and coalesce at A_6≈2.926 to form a center. At A_6, the third large is complete between the other two and there are three large eddies now occupying the cavity (Fig.6 (a), 6(b) and 6(c)). Increasing the aspect ratio to A_7≈3.680, the large eddy located in the middle region of the cavity evolves into a pair of sub-eddies with the centers lying on the vertical centerline of the cavity (Fig.6 (d), 6(e) and 6(f)). At A_8≈5.415, a pair of new side eddies is again emerged in the middle region next to each side wall (Fig.6 (g) and 6(h)). At A_9≈5.619, two side eddies touch the saddle point in the cavity center and are turned into transverse sub-eddies. Subsequently, the transverse sub-eddies separate the longitudinal sub-eddies into two outer large eddies. Fig.6 (i) and 6(j) show the streamline pattern which consists of two pairs of outer eddies, a middle circulation enclosing two sub-eddies.

As the aspect ratio is further increased to beyond A_{10}≈5.738, the saddle point in the center of cavity becomes a center and the streamline pattern is constituted by five large eddies (Fig.7 (a), 7(b) and 7(c)). At A_{11}≈6.490, the center becomes a saddle point and a pair of new longitudinal sub-eddies emerges (Fig.7 (d)).

It can be seen from the Fig.4, Fig.6, Fig.7and the above discussion that the vortex structures shown in Fig.4(a) to 4(j) and in the middle region of Fig.6(a) to 6(j) are similar, and those shown in the middle region of Fig.6(a) to 6(d) and Fig.7(a) to 7(d) are also similar. As the aspect ratio is increased from A≈0.315, four main stages of the vortex structure evolution can be summarized as follows:

(1) At the critical aspect ratios A_2=0.315, A_6=2.929 and A_{10}=5.738, the interior substructures such as saddle point, separatrix and sub-eddies disappears and the streamline pattern consists of odd number of large vortex.
(2) At the critical aspect ratios A_3=0.937, A_7=3.680 and A_{11}=6.490, a saddle point emerges in the center of the cavity.
(3) At the critical aspect ratios A_4=2.553, A_8=5.415, two isolated side eddies appear in the middle region adjacent to stationary side walls.
(4) At the critical aspect ratios A_5=2.804, A_9=5.619, the side eddies and the saddle point touch in the center of the cavity.

4. Conclusions

The vortex evolution of the two-dimensional low Reynolds number flow in a rectangular cavity with two opposite moving lids is investigated numerically. The effect of different aspect ratios on the vortex evolution is systematically revealed. Based on the numerical simulation conducted for different A, the following conclusions can be made.

For the low Reynolds number flow in a rectangular cavity with two equal speed and opposite moving lids, the flow patterns are always symmetric about the cavity center at different aspect ratios.

As the aspect ratios A is increased from 0.15 to 0.7, the vortex structure distributes in the transverse direction of cavity, and the sub-eddy centers gradually merge which are followed from three to two sub-eddies and until to one single large eddy.

When the aspect ratio is larger than 0.7, the flow structure unfolds in the longitudinal direction of cavity and the number of vortices gradually increases with the aspect ratio increasing. The evolution of flow pattern exhibits the characteristics of periodicity and self-similarity. In each periodicity, the four main stages of the vortex evolution in the middle region of a rectangular cavity are similar. As the aspect ratio is increasing, it is expected that more outer large vortices will be superimposed near both lids of the cavity.

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