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Relaxing a large cosmological constant

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ABSTRACT

The cosmological constant (CC) problem is the biggest enigma of theoretical physics ever. In recent times, it has been rephrased as the dark energy (DE) problem in order to encompass a wider spectrum of possibilities. It is, in any case, a polyhedric puzzle with many faces, including the cosmic coincidence problem, i.e. why the density of matter ρ_m is presently so close to the CC density ρ_Λ. However, the oldest, toughest and most intriguing face of this polyhedron is the big CC problem, namely why the measured value of ρ_Λ at present is so small as compared to any typical density scale existing in high energy physics, especially taking into account the many phase transitions that our Universe has undergone since the early times, including inflation. In this Letter, we propose to extend the field equations of General Relativity by including a class of invariant terms that automatically relax the value of the CC irrespective of the initial size of the vacuum energy in the early epochs. We show that, at late times, the Universe enters an eternal de Sitter stage mimicking a tiny positive cosmological constant. Thus, these models could be able to solve the big CC problem without fine-tuning and have also a bearing on the cosmic coincidence problem. Remarkably, they mimic the ΛCDM model to a large extent, but they still leave some characteristic imprints that should be testable in the next generation of experiments.

1. Introduction

High Energy Physics is described by quantum field theory (QFT) and string theory. Unfortunately, these theoretical descriptions are plagued by large hierarchies of energy scales associated to the existence of many possible vacua. Such situation is at the root of the old and difficult CC problem [1], i.e., the formidable task of trying to understand the enormous ratio between the theoretical computation of the vacuum energy density and its observed value, $\rho_\Lambda^0 \sim 10^{-47}\ \text{GeV}^4$, obtained from modern cosmological data [2]. The extremal possibility occurs when the Planck mass $M_P \sim 10^{19}$ GeV is used as the fundamental scale; then the ratio M_P^4/ρ_Λ^0 becomes $\sim 10^{123}$. One may think that physics at the Planck scale is not well under control and that this enormous ratio might be fictitious. However, consider the more modest scale $v = 2M_W/g \simeq 250$ GeV of the electroweak Standard Model (SM) of Particle Physics (the experimentally most successful QFT known to date), where M_W and g are the $W^\pm$ boson mass and $SU(2)$ gauge coupling, respectively. In this case, that ratio reads $|\langle V\rangle|/\rho_\Lambda^0 \gtrsim 10^{55}$, where $\langle V\rangle = -(1/8)M_H^2 v^2 < 0$ is the vacuum energy (i.e. the expectation value of the Higgs potential) and $M_H \gtrsim 114.4$ GeV is the lower bound on the Higgs boson mass. Although one may envisage the possibility that there is a cancelation between the various theoretical contributions to the physical CC (including the bare value), this has never been considered a realistic option owing to the enormous fine-tuning that it entails (which, in addition, must be corrected order by order in perturbation theory).

In this Letter, we discuss a dynamical mechanism that protects the Universe from any initial CC of arbitrary magnitude $|\rho_\Lambda^i| \gg \rho_\Lambda^0$, which could emerge, for instance, from quantum zero-point energy (contributing roughly $\sim m^4$ for any mass m), phase transitions ($\rho_\Lambda^i = \langle V\rangle$) or even vacuum energy at the end of inflation. We admit that $\rho_\Lambda = \rho_\Lambda(t)$ (with $\rho_\Lambda(t_i) = \rho_\Lambda^i$, $\rho_\Lambda(t_0) = \rho_\Lambda^0$) can actually be an effective quantity evolving with time.

Phenomenological models with variable ρ_Λ have been considered in many places in the literature and from different perspectives, see e.g. [3]. At the same time, models with variable CC with a closer relation to fundamental aspects of QFT have also been proposed [4–7]. In all these cases, the effective quantity $\rho_\Lambda = \rho_\Lambda(t)$ still has an equation of state (EOS) $p_\Lambda = -\rho_\Lambda$ and, in this sense, it can be called a CC term.

The basic framework of our proposal is the generalized class of ΛXCDM models introduced in [8], in which there is a fixed or variable ρ_Λ term together with an additional "effective" component X (in general *not* related to a fundamental, e.g. scalar, field).

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This particular class of variable CC models is especially significant in that they could cure the cosmic coincidence problem [8] in full consistency with cosmological perturbations [9]. Here we present a generalization of these models that might even cure the old ("big") CC problem [1]. Recently, in [10] a model along these lines was introduced with a DE density ρ_D and an inhomogeneous EOS $p_D = \omega\rho_D - \beta H^{-\alpha}$ which includes a term proportional to the negative power of the Hubble rate H. This additional term becomes sufficiently large to compensate an initial ρ_Λ^i when this is about to dominate the universe and forces it eventually into a final de Sitter era with a small CC. For recent related work on relaxation mechanisms, see e.g. [11–13]. In a different vein, the CC problem can also be addressed in quantum cosmology models of inflation, through the idea of multiuniverses [14] and the application of anthropic considerations [1].

Let us recall that, historically, most of the models addressing the relaxation of the CC were based on dynamical adjustment mechanisms involving scalar field potentials [15]. In the present work, the relaxation mechanism that we propose is also dynamical, it does not require any fine-tuning and, as noted, it does not depend in general on scalar fields. To be more precise, the model we present here is a ΛXCDM relaxation model of the CC, which includes also matter and radiation eras. We study the two possibilities $\rho_\Lambda^i < 0$ and $\rho_\Lambda^i > 0$, with arbitrary value. For $\rho_\Lambda^i < 0$, our scenario avoids the big crunch at early times and allows the cosmos to evolve starting from a radiation regime with subsequent matter and de Sitter eras like the standard ΛCDM model. Finally, let us emphasize that our method to tackle the CC problem is formulated directly at the level of the (generalized) field equations, rather than from an effective action functional. In this sense, we follow the historical path of Einstein's derivation of the original field equations. At the moment, a version of our model with an action functional is not available, but its efficiency at the level of the field equations is truly remarkable, as we will show. In this sense, its phenomenological success may constitute a first significant step in the way of finding a solution of the difficult CC problem.

The present Letter is organized as follows. In Section 2 we present the basic setup of our model. In Section 3 we present a toy model of the CC relaxation mechanism which helps to understand the basic idea behind our proposal, although it is still too simple to describe our Universe. Only in Section 4 we present a first realistic version of the full relaxation mechanism and we perform a numerical analysis of it. In Section 5, we discuss in more detail some aspects and implications of our model. Finally, in the last section we draw our conclusions.

2. The setup

We start from the generalized Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi}{M_P^2}\left(T_{\mu\nu}^m + T_{\mu\nu}^X + g_{\mu\nu}\rho_{\Lambda,\mathrm{eff}}\right), \tag{1}$$

where $T_{\mu\nu}^m$ is the energy–momentum tensor of ordinary matter – including the energy densities of radiation (ρ_r) and baryons (ρ_b). Furthermore, $T_{\mu\nu}^X$ describes the X component (ρ_X), interacting with the effective CC term $g_{\mu\nu}\rho_{\Lambda,\mathrm{eff}}$ in such a way that the total density of the dark sector, $\rho_D = \rho_{\Lambda,\mathrm{eff}} + \rho_X$, is covariantly conserved (in accordance with the Bianchi identity). The effective CC density $\rho_{\Lambda,\mathrm{eff}}$ is given by

$$\rho_{\Lambda,\mathrm{eff}} = \rho_\Lambda^i + \rho_{\mathrm{inv}}. \tag{2}$$

Here, ρ_Λ^i is an arbitrarily large initial (and constant) cosmological term, and $\rho_{\mathrm{inv}} = \rho_{\mathrm{inv}}(R, S, T)$ is some function of the general coordinate invariant terms

$$\begin{aligned} R &\equiv R_{\mu\nu} g^{\mu\nu} = 6H^2(1-q), \\ S &\equiv R_{\mu\nu} R^{\mu\nu} = 12H^4\left[\left(\frac{1}{2} - q\right)^2 + \frac{3}{4}\right], \\ T &\equiv R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 12H^4\left(1+q^2\right), \end{aligned} \tag{3}$$

which we have evaluated in the flat Friedmann–Robertson–Walker (FRW) metric in terms of the expansion rate $H = \dot{a}/a$, and the deceleration parameter $q = -\ddot{a}a/\dot{a}^2 = -\dot{H}/H^2 - 1$. We find it useful to write the structure of ρ_{inv} in the form

$$\rho_{\mathrm{inv}} = \frac{\beta}{f}, \tag{4}$$

where β is a dimension 6 parameter and $f = f(R, S, T)$ is a dimension 2 function of the aforementioned invariants. This form is particularly convenient since the function f must grow at high energies and hence the vacuum energy is ultraviolet safe, i.e. in the early Universe $\rho_{\mathrm{inv}} \to 0$ and $\rho_{\Lambda,\mathrm{eff}} \to \rho_\Lambda^i$, where ρ_Λ^i is arbitrarily large but finite.

The generalized field equations (1) fall into the metric-based category of extensions of General Relativity. However, at this point the following observation is in order. In the literature, the extensions of Einstein's field equations are usually of a restricted class, namely those that can be derived from effective gravitational actions of the form

$$\Gamma = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{16\pi} R + F(R, S, T)\right]. \tag{5}$$

This class of models may be called the "$F(R, S, T)$-theories" as they are characterized by an arbitrary (albeit sufficiently differentiable) local function F of the invariants defined in Eqs. (3), usually some polynomial of these invariants. Work along these lines has been put forward e.g. in [16]. The particular subclass of models in which the function F depends only on R, or "$F(R)$-theories", is well known and has been subject of major interest [17,18].

However, as advertised in the introduction, in this work we formulate the relaxation mechanism directly in terms of the generalized field equations (1), without investigating at the moment the eventual connection with an appropriate effective action. The reason is, basically, because we aim at maximal simplicity at the moment. To be sure, after many years of unsuccessful attempts, the CC problem has revealed itself as one of the most difficult problems (if not the most difficult one) of all theoretical physics; and we should not naively expect to shoot squarely at it and hope to hit the jackpot at the first trial, so to speak. In this sense, if we can find a way to solve, or at least to significantly improve, the problem directly at the level of the field equations, we might then find ourselves in a truly vantage point to subsequently attempt solving the CC problem at the level of some generalized form of the effective action of gravity.

All in all, let us warn the reader that the connection between the two approaches (viz. the one based on the field equations and the functional one) is, if existent, non-trivial. In fact, we note that the presumed action behind the field equations (1) need not be of the local form (5), and in general we cannot exclude that it may involve some complicated contribution from non-local terms. These terms, however involved they might be, are nevertheless welcome and have been advocated in the recent literature as a possible solution to the dark energy problem from different perspectives [19]. In the present work, we wish to put aside the discussion of these terms, and, for that matter, all issues related to the hypothetical action functional behind our field equations. Instead, we want to exclusively concentrate on the phenomenological possibilities that our framework can provide on relaxing the effective CC term (2) depending on the choice of the function f. In particular, if the initial ρ_Λ^i is a very large cosmological constant associated to a strong phase transition (e.g. some GUT phase transition triggering the process of inflation, the zero point energy of some field, or just the electroweak vacuum energy of the SM), the late time behavior of $\rho_{\Lambda,\mathrm{eff}}$ can be sufficiently tamed (without fine-tuning) so as to be perfectly acceptable by the known cosmological data.

The main aim of the present approach is thus of practical nature; if the CC problem can be efficiently tackled at the level of the field equations to start with (something that, to the best of our knowledge, has never been accomplished before), it should fit the bill as it can be already a crucial first step in the path to solve the CC problem – namely, before unleashing a more formal (and, predictably, even more difficult) theoretical assault to it at the effective action level. We leave this part of the investigation for future work, and we concentrate here on the potential phenomenological benefits of assuming a set of generalized field equations of the form (1).

3. Toy model

Let us first illustrate the mechanism for a universe with only radiation and with f equal to just the Ricci scalar R, so that Eq. (2) becomes

$$\rho_{\Lambda,\mathrm{eff}} = \rho_\Lambda^i + \frac{\beta}{6H^2(1-q)}. \tag{6}$$

To start with, we consider the case $\rho_\Lambda^i < 0$ (as in the SM case) and take $\beta > 0$. Let us also assume a spatially flat Universe. At late times, the de Sitter regime is realized and the deceleration approaches the value $q \to -1$ while the Hubble rate becomes constant and very small, $H \to H_*$ (of order of the current rate H_0). The vacuum energy density takes on the tiny observed value $\rho_{\Lambda,\mathrm{eff}} \to \rho_\Lambda^* \simeq \rho_\Lambda^0$, which is related to H_* by the Friedmann equation: $3M_P^2 H_*^2 = 8\pi\rho_\Lambda^*$. Since the initial value $|\rho_\Lambda^i|$ is assumed to be much larger than ρ_Λ^0, the final Hubble rate is approximately given by

$$H_*^2 \approx -\frac{\beta}{12\rho_\Lambda^i}. \tag{7}$$

We observe that the large $|\rho_\Lambda^i|$ is responsible for the small final value of H_*, provided the parameter β has a suitable order of magnitude and without any need of fine-tuning because this relation does not include differences between large numbers (cf. [20] for a discussion on fine-tuning issues). Moreover, this solution is stable. Indeed, the driving of H^2 to small values by the large and negative $\rho_\Lambda^i < 0$ becomes compensated by the positive second term in $\rho_{\Lambda,\mathrm{eff}}$, which grows as H decreases. On the other hand, a potential instability caused by a growing H would also be unharmful because it would make the second term decrease, so that $\rho_\Lambda^i < 0$ would stabilize H again.

At earlier times, $H \gg H_*$ and the relaxation of ρ_Λ originates from the $(1-q)$ factor in the function f, namely the negative ρ_Λ^i drives dynamically the deceleration parameter q to larger values until q becomes very close to 1, which corresponds to radiation-like expansion. However, q cannot cross $q = 1$ from below since the (positive) second term in $\rho_{\Lambda,\mathrm{eff}}$ would dominate over ρ_Λ^i and stop the cosmic deceleration before q reaches 1. Summarizing, this simplified model keeps the enormous vacuum energy ρ_Λ^i under control at any time thanks to the relaxation mechanism implemented in the function f in Eq. (4). Furthermore, it provides a reasonable expansion history with radiation-like expansion ($q \simeq 1$, H large) in the past and a stable de Sitter solution ($q = -1$, $H = H_* \lesssim H_0$ tiny) in the future. The transition is smooth and happens when the Hubble rate is sufficiently small to ensure the CC relaxation as explained above.

4. Full relaxation model

The simple model discussed above is able to handle the large negative term ρ_Λ^i without abrupt changes in the expansion history. However, the model is unrealistic in that there is no matter era yet, because q will stay around the radiation domination value $q \simeq 1$ until the de Sitter phase starts. Therefore, we have to make sure that the universe goes also through the matter epoch by completing the structure of $\rho_{\Lambda,\mathrm{eff}}$ with a term proportional to $(1/2-q)^{-1}$, which would work like R^{-1} but with $q = 1/2$ as the stabilizing point for the next high H interval. For this purpose, we use the scalar invariants from Eq. (3). A useful expression is to involve not only R but also S, as follows:

$$R^2 - S = 24H^4(2-q)(1/2-q). \tag{8}$$

Notice that this combination is proportional to $(1/2-q)$ and hence allows the relaxation of the vacuum energy in the matter era. Again, this expression alone would be unrealistic because it would enforce the Universe to linger in the matter era and would prevent the existence of a preceding radiation era. However, we can combine the three invariants R, S, T to finally form a much more realistic ansatz for f in Eq. (4), i.e. in such a way that the radiation and matter epochs occur sequentially before the de Sitter universe is eventually reached in the infrared. Indeed, consider the expression

$$\begin{aligned} f &= \frac{R^2 - S}{R} + y \cdot RT \\ &= 4H^2 \frac{(\frac{1}{2}-q)(2-q)}{(1-q)} + y \cdot 72H^6(1-q)\big(1+q^2\big). \end{aligned} \tag{9}$$

We see that f is constructed such that the first term contains only two powers of H, which ensures that the cosmic evolution is reasonably close to that of the ΛCDM model during the matter and subsequent de Sitter stages. The second term contains T to provide a different scaling ($\sim H^6$) in terms of the expansion rate. As a result, the factor $(1-q)$ will dominate over $(1/2-q)$ for large values of H, i.e., during the radiation regime.

The matter–radiation transition ("equality") happens when both terms in f are of the same magnitude and the corresponding time is fixed by the parameter $y \sim H_{\mathrm{eq}}^{-4}$, where $H_{\mathrm{eq}} \sim 10^5 H_0$ is the Hubble rate at equality. The generic behavior of q for the relaxation model under consideration can be seen in Fig. 1. The transitions between different epochs are not as smooth as in the ΛCDM cosmos, although this feature depends on the detailed form of the function f. For our illustrative purposes, the qualitative results given here should be sufficient to appreciate the virtues of this relaxation mechanism. Its main benefit is the complete insensitivity of the universe with respect to an initial cosmological constant ρ_Λ^i of arbitrary magnitude (which in the present example we have chosen to be negative).

Remarkably enough, our construction does not require to fine-tune any of the parameters of the model. Thus, the insurmountable problems associated to the traditional "cancelation" procedure can be solved automatically through this dynamical relaxation mechanism, which is triggered by pure gravitational physics (no scalar fields at all). The current value ρ_Λ^0 of the effective vacuum energy and the corresponding low Hubble rate H_* are fixed only by the magnitude of the parameter β, which is the 6th power of a mass scale M. Since $|\rho_\Lambda^i| \gg \rho_\Lambda^0$, Eqs. (4) and (9) indicate that

$$|\beta| \equiv M^6 = \big|\rho_\Lambda^i\big| \cdot f \sim \big|\rho_\Lambda^i\big| H_*^2, \tag{10}$$

where $H_* \simeq H_0 \sim 10^{-42}$ GeV. Remarkably, M can be of the order of a typical Particle Physics scale; if e.g. the initial vacuum energy

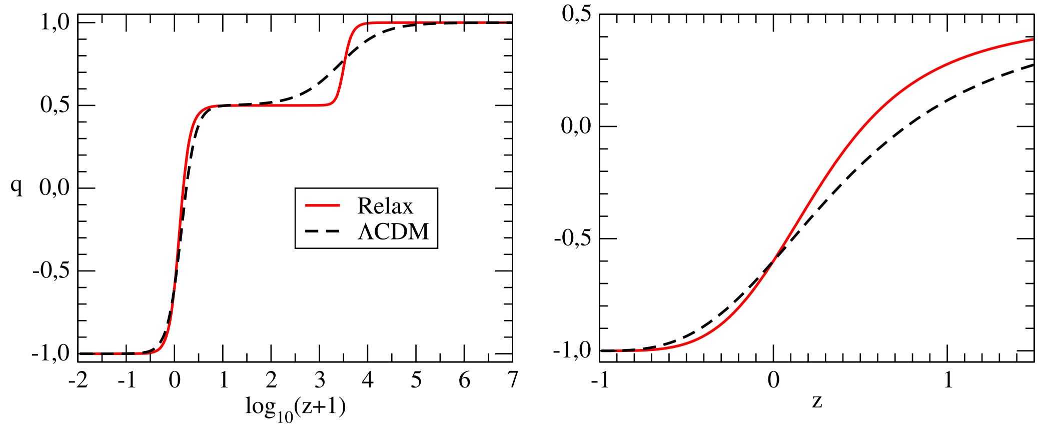

Fig. 1. General behavior of the deceleration parameter q in the relaxation CC model (2) (solid) and in the ΛCDM model (dashed) as a function of the cosmological redshift z. For any initial ρ_Λ^i, the universe goes through a radiation-dominated epoch ($q=1$), a matter-dominated epoch ($q=1/2$) and, eventually, into a final de Sitter phase ($q=-1$) with $\rho_{\Lambda,\mathrm{eff}} \simeq \rho_\Lambda^0 \ll |\rho_\Lambda^i|$. Concrete parameters in this plot are as in Fig. 2.

is $\sim M_P^4$, then $M \lesssim 100$ MeV (i.e., of the order of the characteristic QCD scale Λ_{QCD} where the lowest phase transition occurs in the SM). The above relation (10) can be rephrased in another suggestive way. Since $q \simeq -1$ in the eventual de Sitter regime – which starts approximately near our time – we find that the current value of the CC (which is of the order of the asymptotic value $\rho^*_{\Lambda,\mathrm{eff}}$) is roughly given by the appropriate ratio of the two order parameters characterizing the most extreme phase transitions ever occurred in our Universe:

$$\rho_\Lambda^0 \simeq \rho^*_{\Lambda,\mathrm{eff}} \simeq \frac{\Lambda^6_{\mathrm{QCD}}}{100 M_P^2}. \tag{11}$$

So far, the discussion of the CC relaxation was based only on the form of $\rho_{\Lambda,\mathrm{eff}}$ in Eq. (2), and probably it can be implemented in various ways without losing its benefits (e.g. models with inhomogeneous EOS, modified gravity Lagrangian). In the following, we will discuss the concrete dynamics in a ΛXCDM-like setup [8], where the total energy density $\rho_{\mathrm{tot}} = \rho_r + \rho_b + \rho_D$ includes the usual components of the known universe, ρ_r and ρ_b (i.e. radiation and baryons) as well as the extra contributions from the dark sector: $\rho_D = \rho_X + \rho_{\Lambda,\mathrm{eff}}$. The conventional components are covariantly conserved leading to the usual scaling laws $\rho_r \propto a^{-4}$ and $\rho_b \propto a^{-3}$. Since the dark sector does not interact with the conventional components, ρ_D is conserved, too. From the Bianchi identity satisfied by the terms on the l.h.s. of Eq. (1) and the covariant conservation law of ordinary matter ($\nabla^\mu T^m_{\mu\nu} = 0$), the corresponding covariant conservation in the dark sector reads

$$\nabla^\mu \left[T^X_{\mu\nu} + g_{\mu\nu} \rho_{\Lambda,\mathrm{eff}} \right] = 0. \tag{12}$$

Let us assume that X is a pressureless component. Computing the previous expression in the FLRW metric, it boils down to

$$\dot\rho_{\Lambda,\mathrm{eff}} + \dot\rho_X + 3H\rho_X = 0. \tag{13}$$

This equation shows that the two components of ρ_D are actually interacting. The fact that the EOS parameter of X is taken to be $\omega_X = 0$ (i.e. pressureless) is because X can then mimic (and can be referred to as) dark matter (DM). In this sense, the energy density of the dark sector, ρ_D, can be thought of as being the sum of the DM and DE (interacting) energy densities, where the DE one is, in turn, the sum of the true cosmological constant ρ_Λ^i and the effective gravitational component ρ_{inv}, i.e. Eq. (2). Thus, we have all the necessary ingredients to implement realistically our Universe within the relaxation model.

The basic dynamical equations read:

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = H_0^2 \frac{\rho_{\mathrm{tot}}}{\rho_c^0} = \frac{H_0^2}{\rho_c^0}(\rho_r + \rho_b + \rho_X + \rho_{\Lambda,\mathrm{eff}}), \tag{14}$$

$$\begin{aligned} qH^2 &= -\frac{\ddot a}{a} = \frac{1}{2}\frac{H_0^2}{\rho_c^0}\sum_n \rho_n(1+3\omega_n) \\ &= \frac{H_0^2}{\rho_c^0}\left(\rho_r + \frac{1}{2}\rho_b + \frac{1}{2}\rho_X - \rho_{\Lambda,\mathrm{eff}}\right), \end{aligned} \tag{15}$$

where $\rho_c^0 \equiv \rho_{\mathrm{tot}}(t_0) = 3H_0^2 M_P^2/(8\pi)$ is the current critical energy density.

The various EOS parameters for $n = r, b, X, \Lambda$ are $\omega_r = 1/3$ (accounting for photons and light neutrinos) and $\omega_b = \omega_X = 0$, $\omega_\Lambda = -1$. Using these equations, in Fig. 1 we plot the numerical solution for $q(z)$ and in Fig. 2 we solve for the normalized densities $\Omega_n(z) \equiv \rho_n(z)/\rho_{\mathrm{tot}}(z)$, where we have assumed an initial cosmological constant $\rho_\Lambda^i = -10^{40}\rho_c^0$. This should suffice to illustrate the great efficiency of this relaxation mechanism. Since $y \simeq 10^{21} H_0^{-4}$, Eq. (10) yields $\beta \sim (10^{-3}\ \mathrm{eV})^6$. Notice that in the presence of several phase transitions, the value of $\beta \equiv M^6$ is fixed by the strongest one. We have seen above that $M \lesssim \Lambda_{\mathrm{QCD}} \simeq 100$ MeV for all transitions below the Planck scale.

The following two points are in order concerning the role played by the X component. On the one hand, let us note that we have taken X as representing the full dark matter (DM) content of the Universe. This is a possibility, which we have chosen for definiteness in this presentation of the model, in part for simplicity and also because, then, the quantity $\rho_D = \rho_X + \rho_{\Lambda,\mathrm{eff}}$ provides a kind of economical unification of the DM and DE parts into an overall, self-conserved, dark sector. However, this ansatz must be further elaborated. In particular, it must be confronted with the structure formation data from the analysis of cosmic perturbations in this model [21]. Another possibility would be, for instance, that the "conventional DM" is contained in what we have called the ordinary energy–momentum tensor in Eq. (1). In this alternative scenario, the total matter content is covariantly conserved, and X appears as a kind of additional entity in the DE sector, which would interact with the effective CC, and whose only purpose is to make the total DE covariantly conserved. Since we still have $\omega_X = 0$, the entity X looks now more as a new form of DM that is integrated into the DE.

On the other hand, once the dynamics of $\rho_{\Lambda,\mathrm{eff}}$ is fixed, in this case through (2) and (9), the evolution of the component X, in

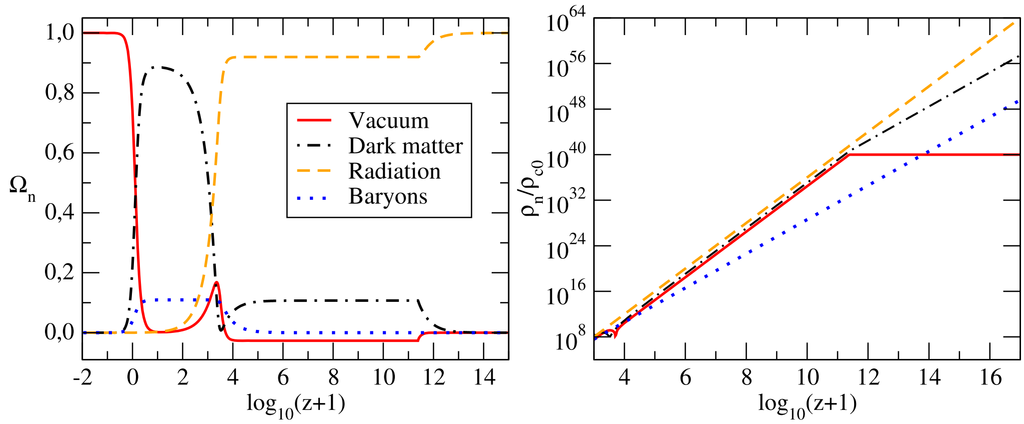

Fig. 2. *Left*: Normalized energy densities $\Omega_n(z) = \rho_n(z)/\rho_{tot}(z)$ for the CC, dark matter, radiation and baryons. The initial CC is $\rho^i_\Lambda = -10^{40}\rho^0_c$. Parameters at $z = 0$ read: $\Omega^0_\Lambda = 0.73$, $\Omega^0_r = 10^{-4}$, $\Omega^0_b = 0.04$, $q_0 = -0.6$. The eventual de Sitter regime is $\rho_{tot} \to \rho^*_\Lambda \simeq \rho^0_c$. *Right*: Absolute energy densities ρ_n/ρ^0_c of the CC, dark matter, radiation and baryons. Note that $\rho_{\Lambda,\text{eff}} < 0$ and the plot shows $|\rho_{\Lambda,\text{eff}}|$.

whatever of the two options discussed above, is completely determined by the local covariant conservation law (13). This implies that X cannot be generally assimilated to a scalar field, because a dynamical scalar field with some particular potential has its own dynamics. In this sense, X is to be viewed as an effective entity within the generalized field equations. In the particular option that we have analyzed in Fig. 2, it is supposed to mimic all effects associated to a real DM substratum.

5. Discussion

Let us now further describe the different stages of cosmic evolution in this framework. First, in the matter era, the total energy density is dominated by the DM component ρ_X rather than by the vacuum energy $\rho_{\Lambda,\text{eff}}$. Due to $q \simeq 1/2$ this can be understood easily by eliminating ρ_X from Eqs. (14), (15),

$$H^2\left(q - \frac{1}{2}\right) = \frac{H_0^2}{\rho_c^0}\left(\frac{1}{2}\rho_r - \frac{3}{2}\rho_{\Lambda,\text{eff}}\right) \ll H^2 \approx \frac{H_0^2}{\rho_c^0}(\rho_X + \rho_b). \tag{16}$$

Therefore, this epoch behaves very similar to the ΛCDM matter era. Finally, the vacuum component becomes dominant at very late times and the Universe smoothly enters the eternal de Sitter regime with a very small positive $\rho_{\Lambda,\text{eff}} \simeq \rho^0_\Lambda \ll \rho^i_\Lambda$. In both eras, the relaxation model does not deviate much from the ΛCDM model, and the sign of the large initial vacuum density ρ^i_Λ is not relevant.

Significant deviations from standard cosmology emerge in the radiation era, because there is no constraint that enforces $\rho_{\Lambda,\text{eff}}$ to be negligible. In fact, the exact behavior of ρ_X and $\rho_{\Lambda,\text{eff}}$ depends on initial conditions and the details of f. Nevertheless, in that epoch (for which $q \simeq 1$ and ρ_b is negligible), subtraction of Eqs. (14), (15) leads to

$$\frac{R}{6H_0^2} = \frac{H^2}{H_0^2}(1-q) = \frac{1}{\rho_c^0}\left(\frac{1}{2}\rho_X + 2\rho_{\Lambda,\text{eff}}\right). \tag{17}$$

Moreover, the second term in the function f in Eq. (9) is dominant in the radiation era (by construction). Thus, while radiation dominates we have

$$\rho_{\Lambda,\text{eff}} = \rho^i_\Lambda + \frac{\beta}{y \cdot 72H^6(1-q)(q^2+1)} = \rho^i_\Lambda + \frac{\beta}{y \cdot 24H^4}\cdot\frac{1}{R}, \tag{18}$$

and by eliminating the Ricci scalar R from the two previous equations, we obtain

$$\rho_{\Lambda,\text{eff}} = \rho^i_\Lambda + \frac{\gamma}{(\frac{1}{2}\rho_X + 2\rho_{\Lambda,\text{eff}})}. \tag{19}$$

With $\beta \sim -\rho^i_\Lambda H_0^2$, $y = (H_{eq})^{-4}$ and the Hubble rate $H_{eq} \sim 10^5 H_0$ at the radiation–matter transition we find that the variable

$$\gamma = \rho_c^0 \cdot \frac{\beta}{y \cdot 24H^4 \cdot 6H_0^2} \approx \rho_c^0 \cdot \frac{-\rho^i_\Lambda}{144}\cdot\left(\frac{H_{eq}}{H}\right)^4 \tag{20}$$

becomes subdominant for very large Hubble rates $H \gg H_{eq}$. Eq. (19) has two solutions for $\rho_{\Lambda,\text{eff}}$,

$$\rho_\pm = \frac{1}{8}\left(4\rho^i_\Lambda - \rho_X \pm \sqrt{32\gamma + (4\rho^i_\Lambda + \rho_X)^2}\right), \tag{21}$$

and the physical one has to be compatible with $|\rho_{\Lambda,\text{eff}}| \ll |\rho^i_\Lambda|$ at late times. Also, ρ_X is taken to be positive.

For negative initial vacuum energy $\rho^i_\Lambda < 0$ the following limits exist. At very high redshift, where $\rho_X \gg -4\rho^i_\Lambda$, the effective vacuum energy acts like a true (and subdominant) constant,

$$\rho_+ \simeq \frac{1}{8}\left(4\rho^i_\Lambda - \rho_X + |4\rho^i_\Lambda + \rho_X|\right) = \rho^i_\Lambda. \tag{22}$$

Thus, the universe evolves in a standard way, and the X component redshifts like non-interacting dust. At smaller redshift, in the range when $\sqrt{\gamma} \ll \rho_X \ll -4\rho^i_\Lambda$ holds, we find a (temporary) tracking regime:

$$\rho_+ \simeq \frac{1}{8}\left(4\rho^i_\Lambda - \rho_X - (4\rho^i_\Lambda + \rho_X)\right) = -\frac{1}{4}\rho_X. \tag{23}$$

In this regime, $\rho_{\Lambda,\text{eff}}$ not only tracks the energy density of the X component, but also that of radiation ρ_r. Indeed, in view of (23), the conservation equation (13) takes on the form

$$\dot\rho_i + 4H\rho_i \simeq 0 \quad (\text{for both } \rho_i = \rho_X, \rho_{\Lambda,\text{eff}}) \tag{24}$$

during this regime. Finally, for $\rho_X \ll \sqrt{\gamma}, |\rho^i_\Lambda|$ the relaxation of the CC becomes obvious by

$$\rho_+ \simeq \frac{1}{8}\left(4\rho^i_\Lambda - \rho_X + |4\rho^i_\Lambda + \rho_X| + \frac{32\gamma}{2\cdot 4|\rho^i_\Lambda|} + \mathcal{O}\left((\rho^i_\Lambda)^{-2}\right)\right) \ll |\rho^i_\Lambda|. \tag{25}$$

Note that the analytical discussion is nicely supported by the numerical results shown in Fig. 2.

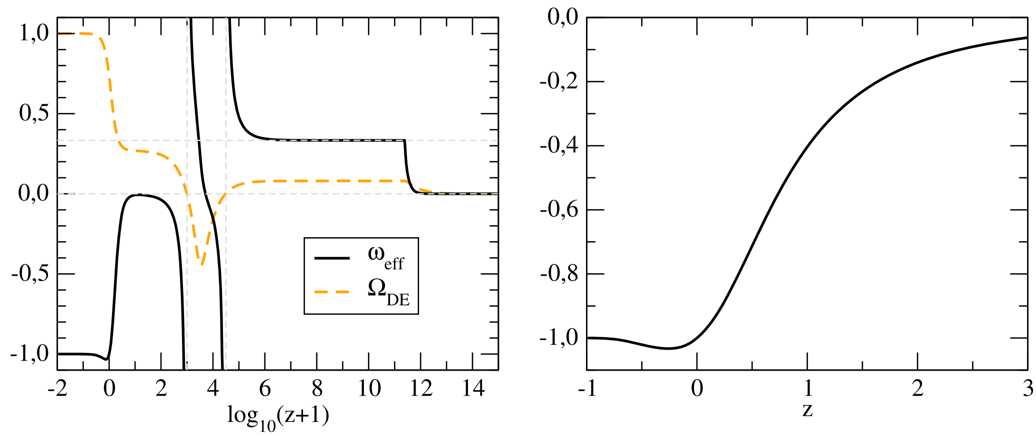

Fig. 3. Effective equation of state ω_{eff} and relative effective DE density $\Omega_{\text{DE}} = \rho_{\text{DE}}/\rho_{\text{tot}}$. Parameters are as in Fig. 2.

This relaxation regime also exists for positive initial $\rho^i_\Lambda > 0$, when $\rho_X \ll \sqrt{|\gamma|}, \rho^i_\Lambda$,

$$\rho_- \simeq \frac{1}{8}\left(4\rho^i_\Lambda - \rho_X - \left(4\rho^i_\Lambda + \rho_X\right) - \frac{32\gamma}{2\cdot 4\rho^i_\Lambda} + \mathcal{O}\left(\left(\rho^i_\Lambda\right)^{-2}\right)\right) \ll \rho^i_\Lambda. \tag{26}$$

Whereas for higher redshift, when $\rho_X \gg \sqrt{|\gamma|}$, we also find a tracking regime, $\rho_- \simeq -\frac{1}{4}\rho_X$. However, differently from the $\rho^i_\Lambda < 0$ case, this tracking behavior is persistent even for $\rho_X \gg \rho^i_\Lambda$. Consequently, at the end of reheating, the energy densities of radiation, dark matter and dark energy could be of the same order of magnitude.

Because of the tracking relation $\rho_X \simeq -4\rho_{\Lambda,\text{eff}} \propto \rho_r$, we should care about bounds from nucleosynthesis. At that time ($z \lesssim 10^9$), we have in our example $\Omega_D = \Omega_\Lambda + \Omega_X \simeq 3\Omega_X/4 \approx 0.08$ versus $\Omega_r \simeq 0.9$ (cf. Fig. 2), and so the ratio $\Omega_D/\Omega_r \lesssim 10\%$ is safe for standard Big Bang nucleosynthesis (similarly as in [8]). On the other hand, this model offers also the possibility to solve the coincidence problem, in that ρ_X, ρ_r and $\rho_{\Lambda,\text{eff}}$ are not very different during the tracking regime until the matter–radiation transition (cf. Fig. 2). Note also that in comparison to ΛCDM, much more dark matter is allowed before the relaxation regime thereby weakening the constraints on dark matter properties.

Finally, we discuss the effective EOS ω_{eff}, which is a useful tool for comparing interacting DE models with non-interacting ones. According to [22], ω_{eff} is given by the EOS of a self-conserved DE component ρ_{DE} in an universe with the same Hubble rate $H(z)$ and total energy density ρ_{tot} as the relaxation model. Within this effective description, DM obeys the usual scaling law of dust $\tilde\rho_X \propto (z+1)^3$ since the interaction with DE is absent. Thus,

$$\omega_{\text{eff}} = -1 + \frac{1+z}{3}\frac{1}{\rho_{\text{DE}}(z)}\frac{d\rho_{\text{DE}}(z)}{dz}, \tag{27}$$

with $\rho_{\text{DE}} = \rho_{\text{tot}} - \tilde\rho_X - \rho_r - \rho_b$. Since ω_{eff} is more accessible for observations at low redshift, we magnify this range in the second plot of Fig. 3. In the relaxation regime, ω_{eff} follows mostly the EOS of the dominant component. Notice that Ω_{DE} is well defined everywhere despite the behavior of ω_{eff}.

6. Conclusions

In this Letter, we have addressed the old cosmological constant problem, i.e. the difficult problem of relaxing the value of the cosmological vacuum energy. The necessity of tempering this value ably and plausibly is absolutely crucial for a realistic cosmological evolution from the early times till today. Indeed, the vacuum energy of the early Universe is expected to be huge in Particle Physics standards, since the expansion history must drive through a series of phase transitions of diverse nature; in particular, it goes through a process of fast inflation (presumably associated to some Grand Unified Theory) and also through the spontaneous breaking of the electroweak symmetry. Finally, it undergoes the more modest chiral symmetry breaking transition, which occurs at the milder scale $\Lambda_{\text{QCD}} = \mathcal{O}(0.1)$ GeV and is connected to the quark–gluon–hadron transition. Even if the latter would have been the only phase transition ever occurred, and the associated vacuum energy density would still persist, it would be a disaster for our Universe. The reason is that the value of the cosmological constant associated to that energy density would have accelerated the expansion history to the point of preventing the formation of any of the structures that we now see in our cosmos, as they would have been ripped off by the fast expansion rate during the first stages of formation. However, being the Standard Model (SM) of Particle Physics such an extraordinary successful theoretical and experimental framework, we must conclude that both of its important contributions to the vacuum energy (associated to the electroweak and strong interactions) must have been relaxed very fast after the corresponding phase transitions occurred; namely, sufficiently fast as to insure not only the possibility to form structures in the late Universe, but also to leave the nucleosynthesis process fully unscathed after the first minute of expansion.

In this Letter, we have tackled a possible cure to this longstanding problem; we have proposed a dynamical relaxation mechanism of the vacuum energy that operates at the level of the generalized gravitational field equations. Our relaxation mechanism achieves this goal without fine-tuning. Apart from the ordinary baryonic matter, which in our model stays covariantly conserved, we assume that the dark sector is made out of an effective cosmological term $\rho_{\Lambda,\text{eff}}$ and another dynamical component X exchanging energy with it. The cosmological term of our model is actually an effective one, in the sense that it is defined as the sum of an arbitrarily large cosmological constant, ρ^i_Λ, and a particular combination of curvature invariants, ρ_{inv}, such that the sum behaves as an overall CC term $\rho_{\Lambda,\text{eff}}(t) = \rho^i_\Lambda + \rho_{\text{inv}}(t)$, but one that evolves with time. As we have said, the component X interacts with the variable $\rho_{\Lambda,\text{eff}}$, but the total density of the dark sector, $\rho_D = \rho_X + \rho_{\Lambda,\text{eff}}$, is covariantly conserved. This construction fits into the class of the so-called ΛXCDM models existing in the literature [8]. Interestingly enough, in the present context, the X component can be interpreted as the dark matter (DM) and the contribution ρ_{inv} can be viewed as a (dynamical) dark energy component that adds up to the traditional cosmological constant term. However, such dynamical DE component has nothing to do with scalar fields as it is of purely gravitational origin. Therefore, the total energy density of the dark sector splits into the sum of the various components DM, CC and DE, namely $\rho_D = \rho_X + \rho_\Lambda^i + \rho_{\rm inv}$, whereas the ordinary (baryonic) matter does not interact at all with the dark sector and remains safely conserved.

The evolution of this ΛXCDM universe keeps the total DE subdominant during the radiation and matter epochs, and only at late times it leads to a tiny effective CC whose smallness is a direct consequence of the large initial vacuum energy, ρ_Λ^i, rather than to a severe fine-tuning involving ugly cancelation of large terms. The resulting cosmos can transit from a fast early inflationary Universe, then drive through the standard radiation and matter dominated epochs and, eventually, ends up in an extremely slow de Sitter phase; in fact, a model of Universe very close to the one suggested by the modern cosmological data [2]. Finally, we obtained new insights into the coincidence problem, as we observed an interesting tracking behavior in the radiation era. A more detailed exposition of our approach discussing the universality of the CC relaxation and the corresponding analysis of the cosmological perturbations will be presented elsewhere [21].

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