A hybrid metaheuristic for the distance-constrained capacitated vehicle routing problem

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Abstract
The delivery of goods is a crucial operational process, which is at the heart of the supply chain and logistic fields. Numerous companies have known a significant drop of their running costs due to the efficient coping with distribution networks. Therefore, several studies in operational research area are interested in modeling and solving transportation problems, especially vehicle routing problems (VRPs). Due to the NP-Hardness of VRPs, various state-of-the-art metaheuristics were developed to generate near-optimal solutions in a reasonable computational time. Against this background, we study in this paper the capacitated vehicle routing problem with distance constraints (DCVRP) consisting in deriving the most favorable vehicle pathways that minimize the vehicles’ traveled distances subject to system requirements. The set of vehicles, based on a central depot, has a maximum weight capacity and can travel up to an allowed maximum distance. To tackle the DCVRP, we formulate it as an integer-programming problem and propose a hybrid swarm-based metaheuristic, named PSO-VNS, which integrates a variable neighborhood search within the particle swarm optimization. Results conducted on benchmark instances show that the proposed PSO-VNS approach is highly competitive, compared to existing solution approaches, and converges to promising solutions.

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Keywords: Vehicle routing problem, Particle swarm optimization, Variable neighborhood search, integer linear programming;

1. Introduction

The delivery of goods to customers is considered to be one of the most challenging activities in logistic sectors. It has a major effect on the overall costs of industrial firms as well as on the environmental resources. Transportation and distribution problems are generally modeled as vehicle routing problems (VRPs) since they consist in minimizing the overall cost while satisfying routing constraints and customers’ orderings. The VRP is concerned with finding a set of vehicle routes to serve known customers’ demands from a single depot at minimum cost (Ren et al., 2010). By involving additional requirements on routes construction, various VRP variants are to be addressed. The CVRP, firstly evoked by Dantzig and Ramser (1959), has received much attention since it models a wide range of applications as fuel consumption optimization (Xiao et al., 2012) and school bus routing problems (Riera-Ledesma & Salazar-Gonzalez, 2012). The CVRP consists in determining several vehicle routes with...
minimum cost for serving a set of customers, whose geographical coordinates and demands are known in advance (Chen et al. (2010), Szeto et al. (2011) and Marinakis (2012)). One variant of the CVRP of particular importance is the DCVRP, where both maximum weight and maximum distance constraints are imposed, i.e. for each vehicle; a prefixed threshold of run distance is to be respected. Despite its great importance, the DCVRP has been evoked only in few studies such as Li et al. (1992), Nagy & Salhi (2005), Cheang et al. (2012) and Almoustafa et al. (2013).

For solving CVRP variants, the scientific community mainly focused on metaheuristic methods for solving the CVRP variants, we can refer to genetic algorithms (Nazif & Lee, 2012), greedy randomized adaptive search procedure (Marinakis, 2012), tabu search (Jin et al., 2012), local search (Ke & Feng, 2013), ant colony optimization (Ting & Chen, 2013) have been proposed.

The main contribution of this paper is to propose a hybrid swarm-based metaheuristic (PSO-VNS) that integrates the PSO and the VNS in order to cope with the DCVRP. An experimental design is conducted to show the competitiveness of the PSO-VNS algorithm compared to state-of-the art solution approaches.

The remainder of this paper is organized as follows. In the second section we state DCVRP and its mathematical formulation. In the third section, we detail the proposed PSO-VNS. The computational experiments and their interpretations are announced in the fourth section.

2. Distance-constrained capacitated vehicle routing problem

The DCVRP is defined as an undirected graph $G = (V, E)$ where a node $j \in V$ corresponds to a customer and an edge $e \in E$ expresses a directed route between a pair of customers. Let $n$ be the number of customers and a central depot denoted as 0. Each order distinguished by a weight $w_j$ and a volume $v_j$ ($j \in \{1,2,3,\ldots,n\}$) is to be delivered to its corresponding customer by a vehicle $k$. Every vehicle from the heterogenous fleet is characterized by a maximum volume $v_{\text{max}}$, a capacity weight in the range $[w_{\text{min}},w_{\text{max}}]$ and can travel a distance in $[d_{\text{min}},d_{\text{max}}]$. The addressed DCVRP aims to minimize the travel cost or the number of employed vehicles while fulfilling structural constraints such as visiting each customer once, path continuity, sub tour elimination, weight constraints, distance constraints and circuit requirements.

We state in Table 1 the symbols used throughout this paper for the mathematical model of the DCVRP.

### Table 1. Glossary of mathematical symbols

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$n$</td>
<td>The total number of vehicles</td>
</tr>
<tr>
<td>$m$</td>
<td>The total number of customers</td>
</tr>
<tr>
<td>$V$</td>
<td>The set of vertices: $V = {0,...,m}$ where 0 refers to the depot</td>
</tr>
<tr>
<td>$E$</td>
<td>The set of edges: $E = {(i,j):i,j \in V}$</td>
</tr>
<tr>
<td>$w_j$</td>
<td>The order's weight of customer $j$</td>
</tr>
<tr>
<td>$v_j$</td>
<td>The cost of traveling from customer $i$ to $j$ of vehicle $k$</td>
</tr>
<tr>
<td>$d_{ij}$</td>
<td>The distance between customers $i$ and $j$</td>
</tr>
<tr>
<td>$v_{\text{max}}$</td>
<td>The maximum volume for vehicle $k$</td>
</tr>
<tr>
<td>$[w_{\text{min}},w_{\text{max}}]$</td>
<td>The range of weight capacity supported by vehicle $k$</td>
</tr>
<tr>
<td>$[d_{\text{min}},d_{\text{max}}]$</td>
<td>The range of the total distance that vehicle $k$ can travel</td>
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<table>
<thead>
<tr>
<th>Variables</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$x_{ij}^k$</td>
<td>$\begin{cases} 1 &amp; \text{if the arc } (i,j) \text{ is traversed by the vehicle } k \ 0 &amp; \text{elsewhere} \end{cases}$</td>
</tr>
<tr>
<td>$y^k$</td>
<td>$\begin{cases} 1 &amp; \text{if the vehicle } k \text{ is used} \ 0 &amp; \text{elsewhere} \end{cases}$</td>
</tr>
<tr>
<td>$X = (x_{11}^1,x_{12}^1,...,x_{mn}^m,y^1,y^2,...,y^m)$</td>
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The DCVRP mathematical model is proposed as follow:

$$\text{Min } F(X) = \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij}^k \times x_{ij}^k$$

(1)
Update the optimum particle; Update the optimum solution of each particle; Evaluate the new fitness function of each particle; Calculate the new position of each particle using (10); Calculate the velocity of each particle using (9);

Following, the outline of the proposed algorithm is stated:

**Algorithm 1:** Hybrid PSO-VNS algorithm for the DCVRP

**Data:** Initialize PSO parameters
**Result:** Best particle

Initialize the initial swarm using Clark & Wright heuristic; Evaluate the fitness function of each particle; Explore neighborhood using VNS (Algorithm 2); Keep optimum particle of the whole swarm; Keep optimum solution of each particle;

while t < T do
  Calculate the velocity of each particle using (9);
  Calculate the new position of each particle using (10);
  Evaluate the new fitness function of each particle;
  Explore neighborhood using VNS (Algorithm 2);
  Update the optimum solution of each particle;
  Update the optimum particle;

The objective function (1) consists in minimizing the total routes’ cost. The constraint (2) and (3) are expressing that each travel should start and end at the depot. Constraints (4) guarantee that each customer is served by exactly one vehicle. The set of requirements (5) is the typical flow conservation equation that ensures the continuity of each vehicle route. The sub tour elimination is presented by equations (6). The maximum capacity of a vehicle as well as its traveled distance belongs to an allowed range, as reported respectively in equations (7) and (8).

3. Hybrid swarm-based metaheuristic (PSO-VNS)

The proposed hybrid swarm-based approach (PSO-VNS) integrates VNS within the PSO approach. The PSO-VNS algorithm is driven by the social behavior of a bird flock and can be viewed as a population-based stochastic optimization algorithm. The expanding neighborhood search is performed using the VNS heuristic that includes 2-opt and 3-opt moves (Lin, 1965). In this approach, a particle status on the d-dimensional search space is characterized by its velocity $v^{t}_{pn}$ and its position $x^{t}_{pn}$ that are iteratively updated by the following equations:

$$v^{t+1}_{pn} = w^{t}v^{t}_{pn} + c_{1}\times r_{1}(b^{t}_{pn} - x^{t}_{pn}) + c_{2}\times r_{2}(g^{t}_{pn} - x^{t}_{pn})$$

$$x^{t+1}_{pn} = x^{t}_{pn} + v^{t+1}_{pn}$$

Where $t$ is the iteration counter ($t = 1, ... , T$); $w$ is the inertia weight; $r_{1}$ and $r_{2}$ are two random variables uniformly distributed in $[0, 1]$. The acceleration coefficients, referred to as $c_{1}$ and $c_{2}$, control how far a particle will move in a single iteration. $b^{t}_{pn}$ is the previous particle’s best position and $g^{t}_{pn}$ is the best position among the swarm. In the following, the outline of the proposed algorithm is stated:

**Algorithm 2:** Integrated VNS algorithm

**Data:** Particles
**Result:** Best particle

Solution improves
while
  Solution improves
    Apply the 2-OPT algorithm
  Apply the 3-OPT algorithm
4. Experimental results

In this section, we report the experimental results generated by the proposed PSO-VNS to handle the DCVRP. Since there is no dataset for the DCVRP, the experiments were conducted on the CVRP benchmarking problems at www.branchandcut.org. We choose to test the performance of the PSO-VNS algorithm using Augerat benchmark (Set A). The implementation of the PSO-VNS was coded in JAVA language and executed on a computer with a 8.00 GB RAM and an Intel Core i5, 2.50 GHz CPU. In order to evaluate the PSO-VNS, its performance is compared to two stochastic-based approaches (Christiansen & Lysgaard, 2007 and Goodson et al., 2012) that were tested on the same set of instances. Then, we construct a DCVRP dataset inspired from the CVRP benchmark by including the maximum distance value $[d_{\text{min}}^k, d_{\text{max}}^k] \in [0,800]$. Experimental results are shown in table 2 where $C$ is the number of customers, $V$ designates the number of vehicles, and Best reports the best-known solution. In the CVRP column of table 2, we show the Cost and $GAP_1$ for each method, where $GAP_1$ ($GAP_1 = \frac{\text{Cost}_{\text{PSO-VNS}} - \text{Best}}{\text{Best}} \times 100$) expressed as the percentage between Cost and Best. In the DCVRP column, we show the Cost and $GAP_2$ for each instance, where $GAP_2$ ($GAP_2 = \frac{\text{Cost}_{\text{PSO-VNS}} - \text{Cost}_{\text{Best}}}{\text{Cost}_{\text{Best}}} \times 100$) is the relative percentage of derivation from $C^*$. 

| CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance | CVRP Instance |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $A-n32-k5$    | 32            | 5             | 784           | $A-n35-k5$    | 33            | 5             | 661           | $A-n35-k6$    | 33            | 6             | 742           | $A-n34-k5$    | 34            | 5             | 778           | $A-n36-k5$    | 36            | 5             | 799           |                |
| $A-n37-k5$    | 37            | 5             | 669           | $A-n37-k6$    | 37            | 6             | 949           | $A-n38-k5$    | 38            | 5             | 730           | $A-n39-k5$    | 39            | 5             | 822           | $A-n39-k6$    | 39            | 6             | 831           |                |
| $A-n44-k5$    | 44            | 7             | 937           | $A-n45-k6$    | 45            | 6             | 944           | $A-n45-k7$    | 45            | 7             | 1146          | $A-n46-k5$    | 46            | 7             | 914           | $A-n46-k6$    | 46            | 7             | 1073          |                |
| $A-n48-k7$    | 48            | 7             | 1073          |                |               |               |               |                |               |               |               |                |               |               |               |                |               |               |                |

Table 2 shows that the PSO-VNS could provide reasonably good solutions in testing the CVRP benchmark. For the fifteen problems our approach outperforms in all cases the simulated annealing approach (Goodson et al., 2012) and the branch and price approach (Christiansen & Lysgaard, 2007). As it can be seen, the deviation between the PSO-VNS solution and the best known solution is always under 5.52% while it is up to 10.37% and 12.45% for the other methods. Moreover, in table 2, we can show that distance constraints have a significant effect on the cost, i.e. in the DCVRP column the solution value ($\text{Cost}^{**}$) increases compared to the CVRP solution ($C^*$). $GAP_2$ Shows that the DCVRP findings are very close to CVRP results thus the distance requirements have minor effects in the solution quality. For instance, for the $A-n33-k6$ instance the DCVRP solution is within 0.04% deviation from the CVRP solution. In summary, we can say that the impact of introducing distance constraints on the solution cost amounts to 0.96%.

5. Conclusion

In this paper, a hybrid swarm-based approach is proposed to cope with the distance-constrained capacitated vehicle routing problem. This NP-hard optimization problem is about deriving the most favorable vehicle pathways that minimize the vehicles’ traveled distances subject to system requirements. Computational results show that the
PSO-VNS hybrid metaheuristic is more effective than other state-of-the-art methods for solving the CVRP. By involving distance constraints, the PSO-VNS is also proven to be efficient in handling the DCVRP. This can be an incentive for applying the PSO-VNS for other problem variants as the open VRP.

References


