



ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Physics Letters B 575 (2003) 279–289

PHYSICS LETTERS B

www.elsevier.com/locate/physletb

Neutrino properties from Yukawa structure

A. Ibarra^a, G.G. Ross^{a,b}

^a *Theory Group, CERN, 1211 Geneva 23, Switzerland*

^b *Department of Physics, Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK*

Received 5 August 2003; accepted 16 September 2003

Editor: G.F. Giudice

Abstract

We discuss the implications for lepton mixing and CP violation of structure in the lepton mass matrices, for the case that neutrino masses are generated by the see-saw mechanism with an hierarchical structure for the Majorana masses. For a particularly interesting case with enhanced symmetry in which the lepton Dirac mass matrices are related to those in the quark sector, the CHOOZ angle is near the present limit and the CP violating phase relevant to thermal leptogenesis and to $\nu 0\beta\beta$ decay is near maximal.

© 2003 Published by Elsevier B.V. Open access under [CC BY license](https://creativecommons.org/licenses/by/4.0/).

1. Introduction

The origin of the structure observed in quark and lepton masses and mixing angles remains one of the most pressing and interesting questions left unanswered by the Standard Model. The continuing improvement in the measurement of the CKM and MNS matrix elements and the neutrino masses has stimulated a renewed theoretical effort to answer these questions.

In the case of quarks one proposed structure going beyond the Standard Model has proved to be robust, giving a quantitatively accurate prediction for the Cabibbo angle (strictly V_{12}^{CKM}). It follows from the postulate that the up and down quark mass matrices have a simultaneous “texture” zero in the (1, 1) position¹ and that the magnitude of the matrix elements are symmetric for the first two generations [1]. The measured masses and mixing angles are consistent with additional texture zeros [2], although this may require a departure from the symmetric form of the mass matrices [3]. One reason for the interest in texture zeros is that they may indicate the presence of a new family symmetry which require certain matrix elements be anomalously small. Thus identification of texture zeros may be an important step in unravelling the origin of the fermion masses and mixings.

In this Letter we extend the analysis of possible texture zeros to the lepton sector for the case that neutrino masses are given by the see-saw mechanism [4]. In analogy with the quark case we consider the predictions

E-mail address: alejandro.ibarra@cern.ch (A. Ibarra).

¹ A texture zero does not imply a matrix element is absolutely zero, but only that it is small enough so that it does not significantly affect the masses and mixing angles.

resulting from a symmetric form for the magnitudes of the Dirac mass matrix elements together with texture zeros. Of particular interest is the case of simultaneous zeros in the (1, 1) position. If this proves to be the case it would be a strong indication of a symmetry between the up and the down quarks and the charged lepton and neutrino sectors, respectively. For the case that the Majorana mass matrix does not contribute significantly to lepton mixing we obtain predictions for the CHOOZ mixing angle and for the CP violating phases. If the neutrino Majorana mass does contribute significantly to mixing these predictions may be viewed as indicative to the magnitude of these parameters barring what would seem to be an unnatural cancellation between the contribution of the Dirac and Majorana sectors. We also consider the implications further restrictions on the form of the lepton mass matrices. The analysis is done in the context that the mass of one of the Majorana neutrinos is anomalously large [5]. This case includes the possibilities that there is sequential right hand neutrino dominance [6] that offers an attractive way of explaining near bi-maximal neutrino mixing in the case that the quark and neutrino Dirac mass matrices are related [7,8].

The Letter is organised as follows. In Section 2 we review a general parameterisation for the effective light neutrino masses for the case of the see-saw mechanism that is useful in studying the implications of texture zeros. We discuss the constraints on this parameterisation coming from texture zeros, from a symmetric form of the magnitudes of the mass matrix elements and from the case that one of the Majorana neutrinos is anomalously large. In Section 4 we apply this parameterisation to derive general constraints on neutrino mixing and CP violation and consider the implications for leptogenesis. Section 5 summarizes the results.

2. Parameterisation of the see-saw mechanism

We consider the case of three generations of left-handed $SU(2)$ doublet neutrinos, $\nu_{L,i}$, and three generations of right-handed Standard Model singlet neutrinos, $\nu_{R,i}$. The Lagrangian responsible for lepton masses has the form

$$L_{\text{Mass}}^l = \nu_R^{cT} \cdot Y_\nu^D \cdot \nu_L \langle H^0 \rangle + l_R^{cT} Y_l^D \cdot l_L \langle \bar{H}^0 \rangle - \frac{1}{2} \nu_R^{cT} \cdot M_\nu^M \cdot \nu_R^c, \quad (1)$$

where Y_ν^D , Y_l^D are the matrices of Yukawa couplings which give rise to the neutrino and charged lepton Dirac mass matrices respectively and M_ν^M is the neutrino Majorana mass matrix. We are interested in studying the implications of simultaneous zeros in Y_ν^D and Y_l^D for observable quantities, masses and mixing angles and CP violating phases. For the case of quarks and charged leptons it is easy to do this because the Yukawa couplings are directly related to the mass matrices. For neutrinos, however, the existence of the Majorana masses complicates the connection between the Dirac Yukawa couplings and the neutrino observables. The light neutrino mass matrix, \mathcal{M} , is given by the see-saw form

$$\mathcal{M} = Y_\nu^{DT} \cdot M_\nu^{M-1} \cdot Y_\nu^D. \quad (2)$$

Sometimes it is convenient to use an alternative form for the see-saw formula, expressing Y_ν^D in terms of the neutrino mass eigenvalues, mixing angles and CP violation [9]. In the basis in which the Majorana mass matrix, M_ν^M , is diagonal the parameterisation has the form

$$Y_\nu^D = D_{\sqrt{M}} \cdot R \cdot D_{\sqrt{m}} \cdot W^\dagger / \langle H^0 \rangle, \quad (3)$$

where $D_{\sqrt{M}}$ is the diagonal matrix of the square roots of the eigenvalues of M_ν^M , $D_{\sqrt{m}}$ is the diagonal matrix of the roots of the physical masses, m_i , of the light neutrinos, W is the neutrino mixing matrix, and R is an orthogonal matrix which parameterises the residual freedom in Y_ν^D once the other parameters are fixed. It is parameterised by

3 complex “mixing” angles.²

$$R = \begin{pmatrix} \sin \theta_2 \sin \theta_3 & \cos \theta_1 \cos \theta_3 + \sin \theta_1 \cos \theta_2 \sin \theta_3 & \sin \theta_1 \cos \theta_3 - \cos \theta_1 \sin \theta_3 \\ \sin \theta_2 \cos \theta_3 & -\cos \theta_1 \sin \theta_3 + \sin \theta_1 \cos \theta_2 \cos \theta_3 & -\sin \theta_1 \sin \theta_3 - \cos \theta_1 \cos \theta_3 \\ \cos \theta_2 & \sin \theta_1 \sin \theta_2 & -\cos \theta_1 \sin \theta_2 \end{pmatrix}, \quad (4)$$

where $\theta_1, \theta_2, \theta_3$ are arbitrary complex angles. These, together with the three Majorana masses, the three light neutrino masses, the three mixing angles and three phases of W make up the eighteen real parameters needed to specify Y_ν^D . With this form it is straightforward to study the implications of a zero in Y_ν^D for the physical measureables.

In our study of texture zeros we will be interested in simultaneous texture zeros in Y_ν^D and Y_l^D . Of course this is basis dependent as a zero in one basis will not in general remain zero after a rotation. In this sense the appearance of simultaneous texture zeros specifies the “texture zero” basis. The idea is that there is some dynamical reason, such as a family symmetry, which generates the texture zero structure. For the case of a family symmetry the “texture zero” basis is just the current quark basis, defined as the one in which the fermion states are eigenstates of the family symmetry group. In the phenomenological analysis of texture zeros this basis choice is taken into account by modifying the parameterisation so that the charged lepton mass matrix is not diagonal. In this case it is the combination $U_l^\dagger W$ that should be identified with the MNS matrix, where U_l is the unitary matrix needed to diagonalise the charged lepton mass matrix, starting from the texture basis.

It is instructive to determine how many free parameters are left in R when Y_ν^D is constrained in various ways. If any element of Y_ν^D is zero, there is a reduction of two complex parameters needed to specify Y_ν^D and a corresponding reduction of the parameters in R . For more than 3 texture zeros there will be relations between measurable quantities.³ However, depending on the position of the texture zero, there may be predictions for fewer texture zeros.

For the case that Y_ν^D is symmetric (or hermitian or has off diagonal elements antisymmetric) the number of real parameters needed to specify it are reduced to 12, so in this case R is completely determined. This does not lead to any relations between measurable quantities but if, in addition, there is a texture zero there will be such relations (this is the analogue to the GST relation in the quark sector).

For the case one of the Majorana masses, $M_{\nu,3}^M$, is anomalously heavy the Standard Model singlet component, $\nu_{R,3}$, does not play a role in determining the two heaviest of the light neutrino eigenstates. Following from Eq. (2) we see that in this case the couplings $(Y_\nu^D)_{3j}, j = 1, \dots, 3$ do not contribute to the light masses and mixing angles. There is also a reduction in the number of parameters needed to specify R . Following from the condition that $Y_\nu^D W$ is finite as $M_{\nu,3}^M \rightarrow \infty$, we see that in this limit $R_{3j} \propto \sqrt{1/M_{\nu,3}^M}, j = 2, 3$ and $R_{ij} \leq O(1), i, j = 1, \dots, 3$. Inserting these constraints in Eq. (4) we find the form of R is given by

$$R = \begin{pmatrix} \propto \sqrt{1/M_{\nu,3}^M} & \cos z & \pm \sin z \\ \propto \sqrt{1/M_{\nu,3}^M} & -\sin z & \pm \cos z \\ \sim 1 & \propto \sqrt{1/M_{\nu,3}^M} & \propto \sqrt{1/M_{\nu,3}^M} \end{pmatrix}, \quad (5)$$

where $z = \theta_3 - \theta_1$. This \pm refer to a reflection ambiguity. In practice we can work with the positive sign only and absorb this ambiguity in the unknown phases specified below. The Yukawa couplings $(Y_\nu^D)_{ij}, i = 1, 2, j = 1, \dots, 3$ are thus given in terms of z alone in the limit $M_{\nu,3}^M \rightarrow \infty$. If we require the (1, 2) block be symmetric, antisymmetric or hermitian, z will be determined and for 1 texture zero there will be relations between measureables. Alternatively more than 1 texture zero will give relations even if the (1, 2) and (2, 1) matrix elements are not related.

² Up to reflections which can be absorbed in the unknown phases discussed below.

³ We include the Majorana mass eigenvalues amongst our “measureables” and also the mixing angles in W ; of course it is necessary to discuss the lepton sector to relate W to U_{MNS} .

3. The charged lepton mass matrix

The MNS matrix is given by $U_l^\dagger W$ and has a contribution coming from the matrix U_l which diagonalises the charged lepton mass matrix. The latter has to reproduce the hierarchical structure of lepton masses and this may place constraints on the magnitude of the charged lepton mixing angles. Let us consider the case the lepton mass matrix is symmetric and that, like the quarks, the hierarchy of lepton masses is due to an hierarchical structure in the matrix elements and not due to a cancellation between different contributions. This is what is expected if there is an underlying grand unified symmetry relating leptons to quarks. Moreover a cancellation between different contributions to lepton masses seems very difficult to reconcile with an underlying family symmetry as it requires non-trivial relations between different matrix elements which are difficult to arrange even in the context of non-Abelian family symmetry. With this constraint it is easy to limit $(U_l)_{23}$, because $(M_l)_{23}^2 \leq m_\mu m_\tau$, giving

$$|(U_l)_{23}| \leq \sqrt{\frac{m_\mu}{m_\tau}}. \quad (6)$$

Similarly one obtains a bound on $(U_l)_{12}$ from the constraint that $(M_l)_{12}^2 < m_e m_\mu$ which follows from taking the determinant of the mass matrix. This in turn implies

$$|(U_l)_{12}| \leq \sqrt{\frac{m_e}{m_\mu}} \quad (7)$$

with equality occurring if there is a texture zero in the (1, 1) position.

The constraint on $(M_l)_{12}^2$ also leads to the constraint $|(U_l)_{13}(U_l)_{23}| \leq \sqrt{m_e m_\mu}/m_\tau$. If $|(U_l)_{23}| = \sqrt{m_\mu/m_\tau}$, which occurs when there is a texture zero in the (2, 2) position, we have the bound $|(U_l)_{13}| \leq \sqrt{m_e/m_\tau}$. If, however, $|(U_l)_{23}| \ll \sqrt{m_\mu/m_\tau}$ we have $(M_l)_{22} = m_\mu$ and then from the determinant we have $(M_l)_{13}^2 \leq m_e m_\tau$ which again gives

$$|(U_l)_{13}| \leq \sqrt{\frac{m_e}{m_\tau}}. \quad (8)$$

In practice the magnitudes of $(U_l)_{23}$ and $(U_l)_{13}$ are so small that they do not affect the mixing coming from the neutrino sector. However $(U_l)_{12}$ close to the upper bound given in Eq. (6) does give a significant contribution to the CHOOZ angle. Its effect is considered below.

The discussion above relies on a symmetric structure relating the magnitudes of the charged lepton mass matrix elements. If we relax this condition there is no constraint on the magnitude of the matrix elements of U_l . In this case the contributions to the MNS matrix coming from the neutrino sector should be considered as an indication of the lower bound on the MNS matrix elements, assuming there is no delicate cancellation between the contributions of U_l and W .

We turn now to a determination of the relations that follow for various form of the Yukawa couplings.

4. Structure of the MNS matrix

4.1. Symmetric Yukawa couplings and a single texture zero in Y_ν^D

4.1.1. (1, 1) texture zero

We first consider in detail how the analysis proceeds for the case the texture zero is in the (1, 1) position and both Y_ν^D and Y_l^D are symmetric. In the analogous case in the quark sector a (1, 1) texture zero leads to the remarkably successful GST relation [1], so this case is particularly interesting for, if it leads to a phenomenologically realistic prediction, it may indicate a connection between quarks and leptons.

As discussed above we are interested in the case $M_{1,2}/M_3 \ll m_2/m_3$ and the Majorana mass matrix, M_ν^M is diagonal and real. We include the CP violating phases in U_{MNS} , i.e., we write it in the form

$$U = V \cdot \text{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1), \tag{9}$$

where ϕ and ϕ' are the CP violating phases and V has the form of the CKM matrix. In this case a symmetric structure in the Dirac neutrino mass matrices and a texture zero will lead to a relation between measurable parameters.

Following from Eq. (3) the condition $(Y_\nu^D)_{11} = 0$ gives⁴

$$\tan z = -\sqrt{\frac{m_2}{m_3}} \frac{W_{12}^*}{W_{13}^*}, \tag{10}$$

where W is the matrix acting on the left-handed neutrino states needed to diagonalise the Dirac neutrino mass matrix. To express this in terms of U_{MNS} we use the constraints of Eqs. (6)–(8) to determine W . There is a residual phase ambiguity because the basis in which the MNS matrix has the standard form can be different from the “symmetry” basis in which the texture zero appears. This corresponds to the simultaneous redefinition of the phase of the left- and right-handed states such that the Dirac structure is invariant (the change in the Majorana matrix is absorbed in a redefinition of ϕ and ϕ' in Eq. (9)). With this we have $W = U_l P U_{\text{MNS}}$ where $P = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$.

From the symmetric constraint $(Y_\nu^D)_{12} = (Y_\nu^D)_{21}$ one obtains

$$\sqrt{\frac{M_1}{M_2}} = \frac{-\tan z \sqrt{m_2} W_{12}^* + \sqrt{m_3} W_{13}^*}{\sqrt{m_2} W_{22}^* + \tan z \sqrt{m_3} W_{23}^*}.$$

Substituting for $\tan z$ leads to the relation

$$W_{13}^{*2} + \frac{m_2}{m_3} W_{12}^{*2} = -\sqrt{\frac{M_1}{M_2}} \sqrt{\frac{m_2}{m_3}} W_{31} \det W^*, \tag{11}$$

where $\det W = e^{i\beta}$. We choose the phases of the right handed charged leptons such that U_l is real in the (1, 2) block. Then in leading order we have $W_{ij} \simeq e^{i\alpha_i} U_{ij}$ except for

$$W_{13} \simeq e^{i\alpha_1} U_{13} + e^{i\alpha_2} (U_l)_{12} U_{23}, \tag{12}$$

where we have written $U_{\text{MNS}} = U$. In Eq. (12) we have dropped terms involving the roots of ratios of lepton masses relative to unity. Using Eq. (12) in Eq. (11) gives

$$U_{13} \equiv |U_{13}| e^{i\delta} = -e^{i(\alpha_2 - \alpha_1)} (U_l)_{12} U_{23} \pm \sqrt{-\frac{m_2}{m_3} U_{12}^2 - \frac{M_1 m_2}{M_2 m_3} U_{31} e^{-i(\beta + 2\alpha_1)}}. \tag{13}$$

For the case of a (1, 1) texture zero in $(Y_l^D)_{11}$ we have $(U_l)_{12} = \sqrt{m_e/m_\mu}$. For the case of a texture zero in $(Y_l^D)_{12}$, $(U_l)_{12} = 0$. Other possibilities for a lepton texture zero or no texture zero at all give $(U_l)_{12} \leq \sqrt{m_e/m_\mu}$.

The implications of Eq. (13) for the CHOOZ angle are shown in Fig. 1 for the case $(U_l)_{12} = 0$ and $\sqrt{m_e/m_\mu}$, respectively.⁵ In these plots we have chosen a random distribution of the unknown phases β, α_i . One may see there is a clustering of values within a small range with the CHOOZ angle near the current bound, $\sin \theta_{13} < 0.24$ at 3σ . This implies that, barring an unnatural cancellation between terms, we expect a large CHOOZ angle, in the range

⁴ Here and in what follows we do not include the ambiguity due to the square roots as they can be absorbed in the unknown phases.

⁵ This and subsequent plots are made using the best fit points for the masses and mixing angles of [10].

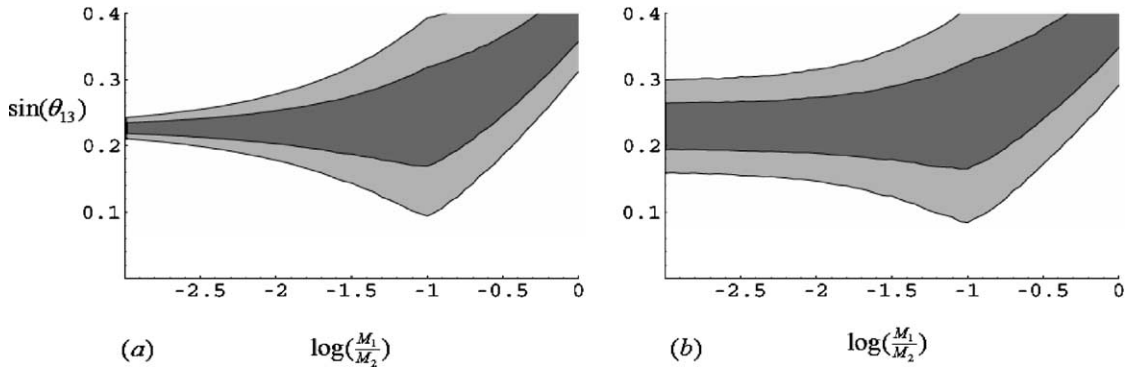


Fig. 1. The CHOOZ angle from a (1, 1) texture zero for the limiting cases of a simultaneous texture zero in the charged lepton mass matrix in the (a) (1, 2) and (b) (1, 1) positions.

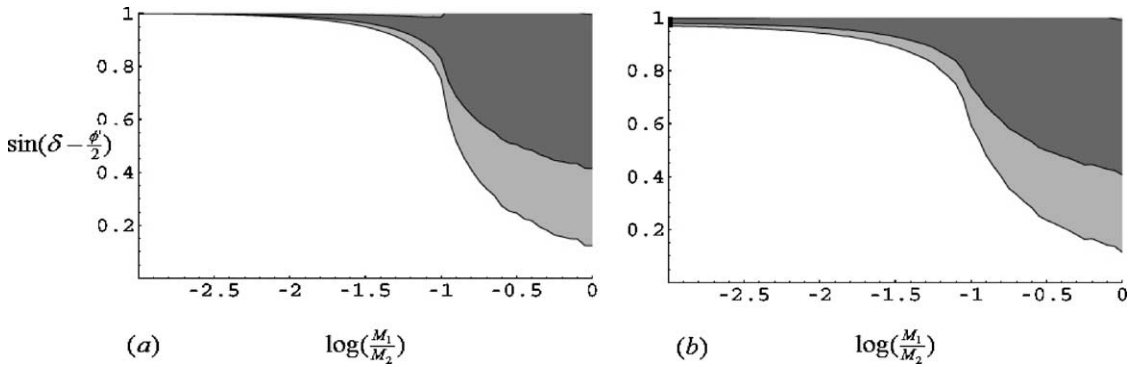


Fig. 2. The $\nu 0\beta\beta$ CP violating phase from a (1, 1) texture zero for the limiting cases of a simultaneous texture zero in the charged lepton mass matrix in the (a) (1, 2) and (b) (1, 1) positions.

that would make the long baseline neutrino factory searches for CP violation feasible. To quantify this we have determined the range of the CHOOZ angle which includes 95% of the points, giving $\sin \theta_{13} > 0.1$ over the whole range of M_1/M_2 .

In Fig. 2 we plot the distribution for the CP violating phase combination $\sin(\delta - \phi'/2)$. This is the CP violating phase relevant to neutrinoless double beta decay. We see that $\sin(\delta - \phi'/2)$ clusters near its maximal value. In this case the 95% cutoff implies $\sin(\delta - \phi'/2) > 0.4$.

Finally we determine the implications of our results for thermal leptogenesis, assuming that the lightest Majorana state dominates [11]. In this case the asymmetry is given by

$$\epsilon \simeq -\frac{3}{8\pi} \frac{M_1}{v^2} \frac{\text{Im}(\sin^2 z m_2^2 + \cos^2 z m_3^2)}{m_2 |\sin^2 z| + m_3 |\cos^2 z|} = -\frac{3}{8\pi} \frac{M_1}{v^2} \frac{(m_3^2 - m_2^2) \text{Im}(\cos^2 z)}{m_2 |\sin^2 z| + m_3 |\cos^2 z|}.$$

Since $|\epsilon_{\text{max}}| = \frac{3}{8\pi} \frac{M_1 m_3}{(H^0)^2}$ [12], we have

$$\frac{\epsilon}{|\epsilon_{\text{max}}|} \simeq -\frac{\text{Im}(\cos^2 z)}{|\cos^2 z| + \frac{m_2}{m_3} |\sin^2 z|}.$$

Note that ϵ depends only on $\tan z$. The dependence of $\tan z$ on low energy phases may be read from Eq. (10) showing which combination is relevant for leptogenesis. The magnitude of $\epsilon/\epsilon_{\text{max}}$ is plotted in Fig. 3. Note that, if

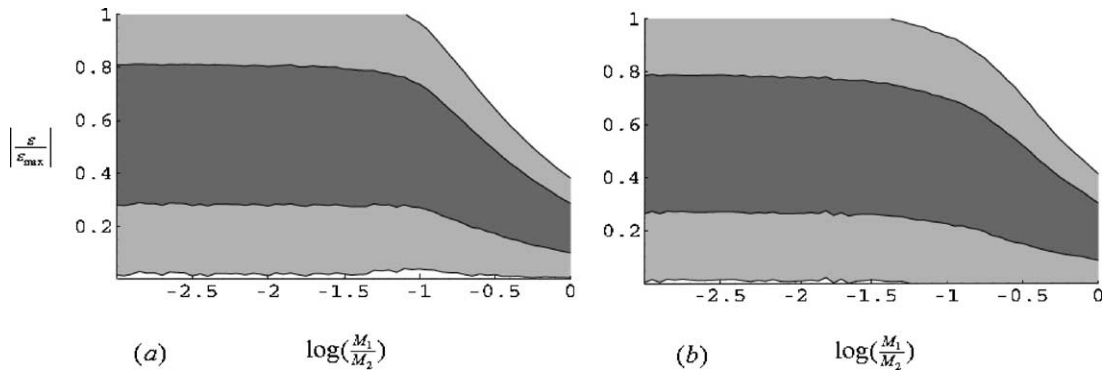


Fig. 3. The CP asymmetry compared to the maximal value in thermal leptogenesis from a (1, 1) texture zero for the limiting cases of a simultaneous texture zero in the charged lepton mass matrix in the (a) (1, 2) and (b) (1, 1) positions.

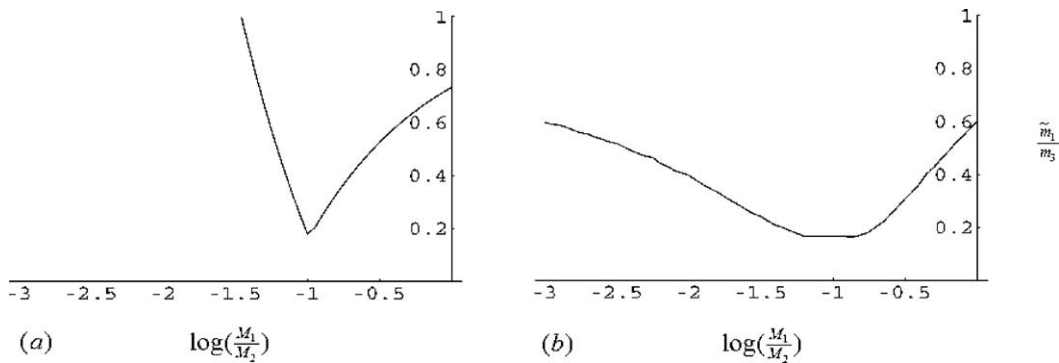


Fig. 4. A plot of the lower bound of \tilde{m}_1/m_3 versus $\log M_1/M_2$ for the case of a (1, 1) texture zero.

we ignore the lepton contribution, a (1, 1) texture zero with an hierarchical Majorana mass spectrum has the same value for the CP violating phase in double beta decay as the CP violating phase determining the lepton asymmetry in leptogenesis [13]. This explains the correlation seen between the plots of Fig. 3, although note that in Fig. 3(b) a significant lepton contribution has been added.

Whether this asymmetry can lead to the observed baryon asymmetry depends on the subsequent washout. This is characterised by the parameter \tilde{m}_1 [14]. It is given by

$$\tilde{m}_1 = m_2 |\sin^2 z| + m_3 |\cos^2 z|.$$

For the case of a (1, 1) texture zero the value of \tilde{m}_1 is given in Fig. 4. Only in the restricted region $\log M_1/M_2 \simeq -1$ is $\tilde{m}_1 \simeq m_2$. Elsewhere $\tilde{m}_1 > m_2$ and the washout will reduce the baryon asymmetry below the observed value unless M_1 is very large [15]. In the case of SUGRA this implies a reheat temperature above the gravitino abundance bound implying that in this case thermal leptogenesis cannot work. However in other supersymmetry breaking mediation scenarios, such as gauge mediation, the gravitino is much lighter and a heavier M_1 is consistent with the gravitino bound.

4.1.2. A single texture zero in the (1, 2), (1, 3), (2, 2) or (2, 3) positions

It is straightforward to apply the analysis just discussed to the other possible positions for a single texture zero in the Dirac neutrino matrix. The results are presented in Table 1. Note that, unlike the case for a (1, 1) texture zero, the prediction for $\tan z$ is in terms of the measured large MNS matrix elements. As a result one obtains a definite

Table 1

The constraints following from a symmetric mass matrix and a single texture zero. χ is 1 for a (1, 1) texture zero in the charged lepton sector and 0 for a (1, 2) texture zero. If there is no lepton texture zero χ lies between these limiting cases. c_{12} is $\cos(\theta_{12})$

Texture zero	$\tan z$	ϵ/ϵ_{\max}	\tilde{m}_1	MNS relation
(1, 1)	$\sqrt{\frac{m_2}{m_3}} \frac{W_{12}^*}{W_{13}^*}$	see text	$> m_2$	$U_{13} = -\chi e^{i(\alpha_2 - \alpha_1)} \sqrt{\frac{m_e}{m_\mu}} U_{23} \pm \sqrt{-\frac{m_2}{m_3} U_{12}^2 + \sqrt{\frac{M_1 m_2}{M_2 m_3}} U_{31}} e^{-i(\beta + 2\alpha_1)}$
(1, 2)	$\sqrt{\frac{m_2}{m_3}} \frac{U_{22}^*}{U_{23}^*}$	$\frac{\sin \phi' m_2}{m_3} c_{12}^2$	$> \frac{m_3}{1 + \frac{m_2}{m_3} c_{12}^2}$	$U_{13} = -\chi e^{i(\alpha_2 - \alpha_1)} \sqrt{\frac{m_e}{m_\mu}} U_{23} - \frac{m_2}{m_3} \frac{U_{12} U_{22}^*}{U_{23}^*}$
(1, 3)	$\sqrt{\frac{m_2}{m_3}} \frac{U_{32}^*}{U_{33}^*}$	$\frac{\sin \phi' m_2}{m_3} c_{12}^2$	$> \frac{m_3}{1 + \frac{m_2}{m_3} c_{12}^2}$	$U_{13} = -\chi e^{i(\alpha_2 - \alpha_1)} \sqrt{\frac{m_e}{m_\mu}} U_{23} - \frac{m_2}{m_3} \frac{U_{12} U_{32}^*}{U_{33}^*} + e^{i(\beta - 2\alpha_1 - \alpha_3)} \sqrt{\frac{M_1}{M_2}} \sqrt{\frac{m_2}{m_3}} \frac{U_{11}^*}{U_{33}^*}$
(2, 2)	$\sqrt{\frac{m_3}{m_2}} \frac{U_{23}^*}{U_{22}^*}$	$-\frac{\sin \phi' c_{12}^2}{1 + c_{12}^2}$	$> \frac{m_2(1 + c_{12}^2)}{1 + \frac{m_2}{m_3} c_{12}^2}$	$U_{31} = e^{i(\beta - 2\alpha_2 - \alpha_3)} \sqrt{\frac{M_1}{M_2}} \sqrt{\frac{m_3}{m_2}} (U_{23}^{*2} + \frac{m_2}{m_3} U_{22}^{*2})$
(2, 3)	$\sqrt{\frac{m_3}{m_2}} \frac{U_{33}^*}{U_{32}^*}$	$-\frac{\sin \phi' c_{12}^2}{1 + c_{12}^2}$	$> \frac{m_2(1 + c_{12}^2)}{1 + \frac{m_2}{m_3} c_{12}^2}$	$U_{21} = e^{i(\beta - 2\alpha_2 - \alpha_3)} \sqrt{\frac{M_1}{M_2}} \sqrt{\frac{m_3}{m_2}} (U_{23}^* U_{33}^* + \frac{m_2}{m_3} U_{22}^* U_{32}^*)$

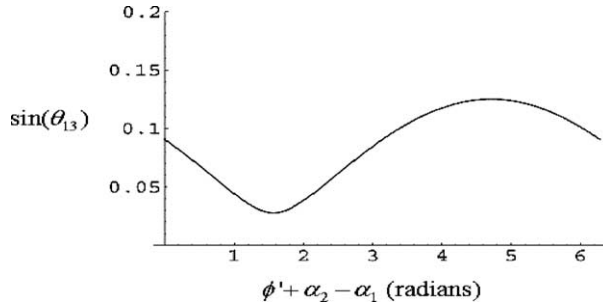


Fig. 5. The CHOOZ angle for the (1, 2) texture zero plotted against the unknown phase.

prediction for leptogenesis which is also given in the Table. For the case of (1, 2) and (1, 3) texture zeros we see that $\tan z$ is suppressed by $\sqrt{m_2/m_3}$ which leads to a m_2/m_3 suppression in ϵ/ϵ_{\max} . The bound on \tilde{m}_1 is much more stringent than in the (1, 1) texture zero case so we expect washout effects to be very efficient. As a result baryogenesis through thermal leptogenesis will not proceed in these cases. For the case of the (2, 2) and (2, 3) texture zeros $\tan z$ is enhanced by $\sqrt{m_3/m_2}$ which leads to a near maximal form for ϵ/ϵ_{\max} . The bound on \tilde{m}_1 is somewhat stronger in this case than in the case of a (1, 1) texture zero but is independent of M_1/M_2 .

For the case of the (1, 2) texture zero the prediction for the CHOOZ angle depends only on unknown phases with the distribution is shown in Fig. 5. For a (1, 3) texture zero the CHOOZ angle also depends on the ratio M_1/M_2 as in the previous cases. This is plotted in Fig. 6. In both cases θ_{13} is predicted to be large, although the 95% lower range is smaller than that found for the (1, 1) texture zero case.

For the case of the (2, 2) and (2, 3) texture zeros one obtains a relation between the large elements of the MNS matrix. From this one may extract a relation between the phases and a prediction for M_1/M_2 . Unfortunately these do not lead to a relation between measurable parameters, although the constraint that $M_1/M_2 \simeq m_2/m_3$ may be of interest in model building.

4.2. The case of two texture zeros

For two texture zeros one obtains a prediction even without imposing the symmetric constraint. There are fifteen ways of assigning two texture zeros to the first two rows of the Dirac neutrino mass matrix (the third row plays no

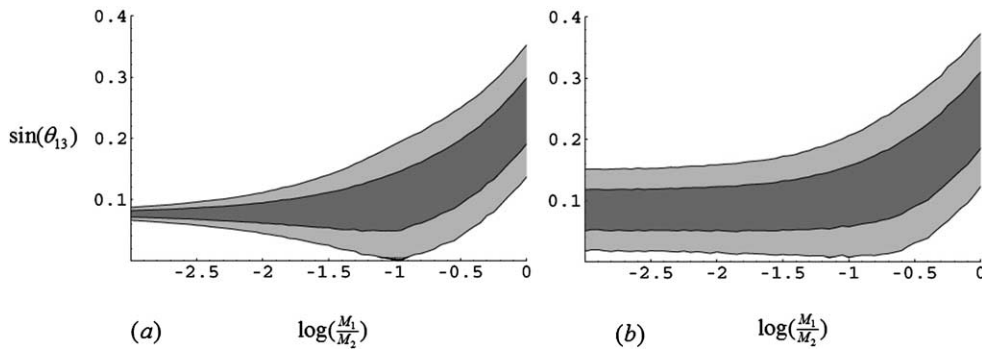


Fig. 6. The CHOOZ angle from a (1, 3) texture zero for the limiting cases of a simultaneous texture zero in the charged lepton mass matrix in the (a) (1, 2) and (b) (1, 1) positions.

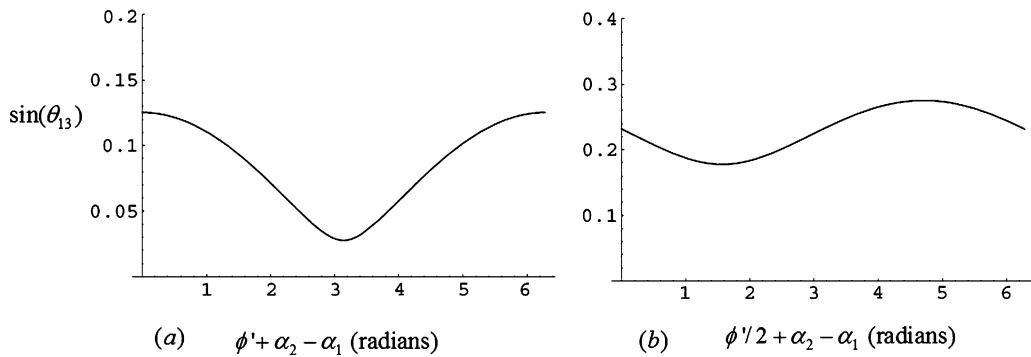


Fig. 7. The prediction for the CHOOZ angle for the two texture zero cases: (a) (1, 1) and (2, 3) or (1, 3) and (2, 1) (b) (1, 1) and (2, 1). The plot is for the $\chi = 1$ case and is plotted against the relative phase between the two terms appearing in Table 2.

role in the case the third Majorana neutrino is anomalously heavy). All but five lead to inconsistent results; below we discuss only the viable choices.

From Table 1 we may readily solve the constraint following from equating the two forms for $\tan z$ that follow from (1, 1) and (2, 2) texture zeros. This gives the prediction for U_{13} given in Table 2. One may see it is identical to the prediction (c.f. Fig. 5) obtained for a single texture zero in the (1, 2) position with the symmetric condition imposed although in this case we have not imposed this condition. If one further imposes the condition that the matrix is symmetrical one also obtains the prediction for U_{13} given in Eq. (13). Equating these results fixes one combination of the phases (which does not lead to new relations between measurable phases) and fixes the ratio $M_1/M_2 \simeq m_2/m_3$. The prediction for ϵ/ϵ_{\max} is as given in Table 1 for the (2, 2) texture zero case.

The remaining possibilities are given in Table 2. The prediction for the CHOOZ angle is approximately the same for the (1, 1) and (2, 3) or the (1, 3) and (2, 1) cases and is shown in Fig. 7(a). The remaining case with a (1, 1) and a (2, 1) texture zero is shown in Fig. 7(b).

For the case of (1, 1) and (2, 3) texture zeros one again needs $M_1/M_2 \simeq m_2/m_3$ if one requires the Dirac mass matrix be symmetric. For the last two cases there is no solution if one additionally imposes the condition the Dirac mass matrix be symmetric. In all cases the prediction for ϵ/ϵ_{\max} is as given in Table 1 for the appropriate texture zero. This follows because the prediction comes from the constraint on $\tan z$ only and does not require the symmetric condition.

Table 2

The constraints following from two texture zeros. Only those cases shown are consistent apart from the (1, 2), (2, 1) case which has already been discussed when considering symmetric textures. Also shown are the additional constraints following from imposing a symmetric structure for the two cases this is consistent. χ is 0 for a (1, 1) texture zero in the charged lepton sector and 0 for a (1, 2) texture zero. For no lepton texture zero χ is between these limiting cases

Texture zero	U_{13}
(1, 1) and (2, 2)	$\pm \frac{m_2}{m_3} \frac{U_{12}U_{22}}{U_{23}} - \chi e^{i(\alpha_2 - \alpha_1)} \sqrt{\frac{m_e}{m_\mu}} U_{23}$
(1, 1) and (2, 3)	$\pm \frac{m_2}{m_3} \frac{U_{12}U_{32}}{U_{33}} - \chi e^{i(\alpha_2 - \alpha_1)} \sqrt{\frac{m_e}{m_\mu}} U_{23}$
(1, 1) and (2, 1)	$\pm \sqrt{\frac{m_2}{m_3}} U_{12} - \chi e^{i(\alpha_2 - \alpha_1)} \sqrt{\frac{m_e}{m_\mu}} U_{23}$
(1, 3) and (2, 1)	$\pm \frac{m_2}{m_3} \frac{U_{12}U_{32}}{U_{33}} - \chi e^{i(\alpha_2 - \alpha_1)} \sqrt{\frac{m_e}{m_\mu}} U_{23}$

5. Summary and conclusions

The combination of the see-saw mechanism, an hierarchical structure for the Majorana mass matrix and a combination of texture zeros and/or a symmetrical form for the moduli of the mass matrix elements leads to relations amongst observable properties of neutrinos. In this Letter we have determined these predictions in a model independent way.

The case of a (1, 1) texture zero is of particular interest because, in the quark sector, it leads to a relation in excellent agreement with experiment. In the neutrino case the equivalent (1, 1) texture zero leads to a prediction for the CHOOZ angle that is close to the present limit and a near maximal CP violating phase relevant to thermal leptogenesis and to $\nu 0\beta\beta$. For a strongly restricted range of the ratio of the lightest to the next lightest Majorana masses thermal leptogenesis can give rise to acceptable baryogenesis while satisfying the gravitino bounds on the reheat temperature in supergravity models. Outside this range an acceptable rate of baryogenesis is only possible if the gravitino constraints are relaxed, for example in theories in which the supersymmetry breaking occurs at a lower scale.

In the case that the texture zero appears in the (1, 2) or (1, 3) positions the CHOOZ angle is still predicted to be large, encouraging for long baseline CP violation studies. However in these cases washout effects after thermal leptogenesis are too efficient to allow for adequate baryogenesis. The case of (2, 2) and (2, 3) texture zeros does not lead to phenomenologically interesting relations. On the other hand there are five viable cases in which two texture zeros can be present. In these cases a large CHOOZ angle is again predicted.

The determination of the parameters involved in the see-saw mechanism is an illdefined problem due to the large number of parameters relative to measureables. The best hope is that the system has a high degree of symmetry, reducing the number of parameters. Our analysis has explored a particularly promising possibility suggested by the structure observed in the quark sector in which the Dirac masses have one (or more) texture zero(s) and the magnitude of the mass matrix elements may be symmetric. In addition we have assumed an hierarchical structure for the Majorana matrix, motivated by the fact this can readily explain the large neutrino mixing angles while having a relation between quark and lepton Dirac masses. Such a structure for the Dirac and Majorana masses can be derived from an underlying family symmetry [16] and, if the resultant predictions for neutrino properties should be confirmed, it would provide strong support for such an underlying symmetry organising the fermion mass matrices.

Acknowledgements

This work was partly supported by the EU network, Physics Across the Present Energy Frontier HPRV-CT-2000-00148.

References

- [1] R. Gatto, G. Sartori, M. Tonin, *Phys. Lett. B* 28 (1968) 128;
R.J. Oakes, *Phys. Lett. B* 29 (1969) 683;
R.J. Oakes, *Phys. Lett. B* 31 (1970) 630, Erratum;
R.J. Oakes, *Phys. Lett. B* 30 (1970) 26.
- [2] P. Ramond, R.G. Roberts, G.G. Ross, *Nucl. Phys. B* 406 (1993) 19.
- [3] R.G. Roberts, A. Romanino, G.G. Ross, L. Velasco-Sevilla, *Nucl. Phys. B* 615 (2001) 358, hep-ph/0104088.
- [4] M. Gell-Mann, P. Ramond, R. Slansky, in: P. Van Nieuwenhuizen, D. Freedman (Eds.), *Proceedings of the Supergravity Stony Brook Workshop*, New York, 1979;
T. Yanagida, in: A. Sawada, A. Sugamoto (Eds.), *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, Japan, 1979;
R.N. Mohapatra, G. Senjanovic, *Phys. Rev. Lett.* 44 (1980) 912;
R.N. Mohapatra, G. Senjanovic, *Phys. Rev. D* 23 (1981) 165;
S.L. Glashow, in: *Proceedings, Quarks and Leptons, Cargese, 1979*, p. 687;
S.L. Glashow, Harvard preprint HUTP-79-A059.
- [5] For related work see P.H. Frampton, S.L. Glashow, T. Yanagida, *Phys. Lett. B* 548 (2002) 119, hep-ph/0208157;
R. Barbieri, T. Hambye, A. Romanino, *JHEP* 0303 (2003) 017, hep-ph/0302118.
- [6] S.F. King, *Phys. Lett. B* 439 (1998) 350, hep-ph/9806440;
S.F. King, *Nucl. Phys. B* 562 (1999) 57, hep-ph/9904210;
S.F. King, *Nucl. Phys. B* 576 (2000) 85, hep-ph/9912492;
G. Altarelli, F. Feruglio, I. Masina, *Phys. Lett. B* 472 (2000) 382, hep-ph/9907532;
A.Y. Smirnov, *Phys. Rev. D* 48 (1993) 3264, hep-ph/9304205.
- [7] S.F. King, G.G. Ross, *Phys. Lett. B* 520 (2001) 243, hep-ph/0108112.
- [8] G.G. Ross, L. Velasco-Sevilla, *Nucl. Phys. B* 653 (2003) 3, hep-ph/0208218.
- [9] J.A. Casas, A. Ibarra, *Nucl. Phys. B* 618 (2001) 171, hep-ph/0103065.
- [10] M.C. Gonzalez-Garcia, C. Pena-Garay, hep-ph/0306001.
- [11] W. Buchmuller, P. Di Bari, M. Plumacher, hep-ph/0302092.
- [12] S. Davidson, A. Ibarra, *Phys. Lett. B* 535 (2002) 25, hep-ph/0202239;
K. Hamaguchi, H. Murayama, T. Yanagida, *Phys. Rev. D* 65 (2002) 043512, hep-ph/0109030.
- [13] S.F. King, hep-ph/0211228;
L. Velasco-Sevilla, *Hierarchical neutrino mass matrices, CP violation and leptogenesis*, Oxford preprint OUTP-03/14P.
- [14] M. Plumacher, *Z. Phys. C* 74 (1997) 549, hep-ph/9604229.
- [15] Similar results have been found recently by P.H. Chankowski, K. Turzyski, hep-ph/0306059.
- [16] S.F. King, G.G. Ross, *Fermion masses and mixing angles from $SU(3)$ family symmetry and unification*, Southampton and CERN preprint, SHEP 03-14, CERN-TH/2003-147.