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CFT(4) partition functions and the heat kernel on AdS(5)

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ABSTRACT

We consider four-dimensional CFTs which admit a large-N expansion, and whose spectrum contains states whose conformal dimensions do not scale with N. We explicitly reorganise the partition function obtained by exponentiating the one-particle partition function of these states into a heat kernel form for the dual string spectrum on AdS(5). On very general grounds, the heat kernel answer can be expressed in terms of a convolution of the one-particle partition function of the light states in the four-dimensional CFT.

1. Introduction

Heat kernel methods have of late played an important role in extracting out quantum effects in gravitational physics. Such applications include the extraction of leading quantum corrections to black hole entropy [1–4], the asymptotic symmetries of gravitational theories [5–9], precision tests of AdS/CFT [10], as well as the many more applications extensively reviewed in [11]. This is essentially because the heat kernel method powerfully captures the leading quantum properties of a given theory. In many cases (especially quantum gravity) while the full quantum theory is poorly understood, these leading properties are potentially tractable.

In this Letter, we shall briefly describe some progress in bringing heat kernel methods to bear on another significant arena, the string theory sigma model on AdS. As is well known, this sigma model is presently quite intractable at the quantum level. In this light, one potential starting point to gain a foothold on the sigma model could be to use the AdS/CFT correspondence [12–14]. In particular, to take the spectrum of the CFT dual to the AdS string and to reproduce the planar CFT partition function in terms of quadratic fluctuations of the dual fields in AdS.[1] Typically, these quadratic fluctuations would arrange themselves into determinants of the Laplacian acting over fields of varying spin. This would essentially provide us with a first-quantised description of the particles that form the string spectrum. More ambitiously, one could attempt to reconstruct the full vacuum amplitude (the torus string amplitude with no vertex operator insertions) in AdS by interpreting the heat kernel proper time in terms of the modulus of the torus worldsheet [18].[2]

In this Letter, we consider CFTs which admit a large-N expansion and whose spectrum contains states whose energies do not scale with N. We exclusively focus on such states, whose bulk description is in terms of weakly coupled particles. This subset of the CFT_4 partition function is reorganised into quadratic fluctuations of particles in AdS_5, thereby performing the first of the above two tasks.

2. The heat kernel for AdS(5): A review

In this section, we shall review the results of [23] for the heat kernel of the Laplacian acting over tensor fields on AdS. For a spin-S particle moving on a spacetime manifold $\mathcal{M}$,

$$\ln \mathcal{Z}^{(S)} \simeq \ln\det\left(-\nabla^2_{(S)}\right) = \operatorname{Tr}\ln\left(-\nabla^2_{(S)}\right) = -\int_0^\infty \frac{dt}{t}\operatorname{Tr} e^{t\nabla^2_{(S)}} \equiv -\int_0^\infty \frac{dt}{t} K^{(S)}(t), \tag{2.1}$$

where the trace of the Laplacian is taken over both the spin and the spacetime indices. The determinants of the Laplacian were explicitly evaluated for the symmetric, transverse-traceless (STT) fields in Euclidean AdS in [23] by the heat kernel method. This exploited the fact that AdS spaces can be realised as cosets of Lie

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[1] For a free planar CFT defined on $S^3 \otimes S^1$ this partition function is very explicitly known by counting the spectrum of gauge invariant operators [15,16], see also [17].

[2] There are good reasons to expect this approach to be a fruitful one. In particular, it has been proposed that the general relation between the heat kernel proper time and the closed string moduli is closely connected to the phenomenon of gauge-string duality [19–21]. Encouragingly, this overall approach based on the heat kernel method has also previously been successful for string theory in flat space [22]. We thank Rajesh Gopakumar for helpful discussions and correspondence about these points.

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groups (see for example [24–28,18]). We shall briefly recollect the main results, specialising to the case of AdS_5.

Firstly, Euclidean AdS_5 is the symmetric space $SO(5,1)/SO(5)$. The spin of a field over AdS_5 is given by the unitary irreducible representation (UIR) of the isotropy group SO(5) that is carried by the field. UIRs of SO(5) are labelled by the array

$$S=(s_1,s_2) \quad \text{s.t.} \quad s_1\geqslant s_2\geqslant 0. \tag{2.2}$$

We shall also need UIRs of $SO(5,1)$, which are labelled by the array

$$R=(i\lambda,m_1,m_2), \quad m_1\geqslant |m_2|, \tag{2.3}$$

where R contains S if [29,28]

$$s_1\geqslant m_1\geqslant s_2\geqslant |m_2|. \tag{2.4}$$

The m's and s's must all simultaneously be integers or half-integers. Further, the m's define an SO(4) UIR $\vec{m}=(m_1,m_2)$, and its conjugate representation $\check{\vec{m}}=(m_1,-m_2)$. With these ingredients, the heat kernel of a spin-S particle on (a quotient of) AdS_5 is given by

$$K^{(S)}(\gamma,t)=\frac{\beta}{2\pi}\sum_{k\in\mathbb{Z}}\sum_{\vec{m}}\int_0^\infty d\lambda\, \chi_{\lambda,\vec{m}}(\gamma^k)e^{tE_R^{(S)}}, \tag{2.5}$$

where $\chi_{\lambda,\vec{m}}$ is the Harish–Chandra character in the principal series of $SO(5,1)$, which has been evaluated [30] to be

$$\chi_{\lambda,\vec{m}}(\beta,\phi_1,\phi_2) = \frac{e^{-i\beta\lambda}\chi_{\vec{m}}^{SO(4)}(\phi_1,\phi_2)+e^{i\beta\lambda}\chi_{\check{\vec{m}}}^{SO(4)}(\phi_1,\phi_2)}{e^{-2\beta}\prod_{i=1}^2|e^{\beta}-e^{i\phi_i}|^2}, \tag{2.6}$$

the eigenvalue of the Laplacian $E_R^{(S)}$ is a function of λ

$$E_R^{(S)}=-\big(\lambda^2+C_2(S)-C_2(\vec{m})+2^2\big), \tag{2.7}$$

and γ denotes the quotient of AdS_5 which corresponds to turning on a temperature β along with angular momentum chemical potentials ϕ_1,ϕ_2 along the SO(4) Cartans. C_2 denotes the quadratic Casimir of the appropriate SO group. The sum over $\vec{m}$ is the sum over all values of m admitted by the branching rules (2.4).

For STT tensors, the branching rules reduce to

$$m_1=s, \qquad m_2=0, \tag{2.8}$$

where s is the rank of the tensor. The partition function of a massless spin-s particle was evaluated with these inputs in [9]. In particular, it was found that (see Eq. (2.29) of [9])

$$\log\mathcal{Z}^{(s)}=\sum_{m=1}^\infty\frac{1}{m}\frac{e^{-m\beta(s+2)}}{|1-e^{-m(\beta-i\phi_1)}|^2|1-e^{-m(\beta-i\phi_2)}|^2} \times\big[\chi_{\frac{s}{2}}(m\alpha_1)\chi_{\frac{s}{2}}(m\alpha_2)-\chi_{\frac{s-1}{2}}(m\alpha_1)\chi_{\frac{s-1}{2}}(m\alpha_2)e^{-m\beta}\big], \tag{2.9}$$

where we have expressed the SO(4) character $\chi_{(s,0)}^{SO(4)}(m\phi_1,m\phi_2)$ as a product of SU(2) characters $\chi_{\frac{s}{2}}(m\alpha_1)\chi_{\frac{s}{2}}(m\alpha_2)$, where $\alpha_1=\phi_1+\phi_2$, and $\alpha_2=\phi_1-\phi_2$.

The reader will recognise (2.9) as the expression for the multiparticle partition function in terms of the one-particle partition function $\mathcal{Y}$, where $\mathcal{Y}$ is the $SO(4,2)$ character evaluated over the short representation $[s+2,\frac{s}{2},\frac{s}{2}]$ [31,32]. See [33,34] for a classification of unitary representations of the conformal algebra. Using the AdS/CFT correspondence [12–14], if a CFT partition function contains a character of the representation $[s+2,\frac{s}{2},\frac{s}{2}]$, there must be bulk degrees of freedom giving rise to one-loop determinants over STT fields, as reviewed above. For example, given a long primary $[\Delta,\frac{s}{2},\frac{s}{2}]$ in the CFT, one can infer the presence of quadratic fluctuations in AdS giving rise to a one-loop determinant of the operator $-\nabla^2+m^2$, where $m^2=(\Delta-2)^2-s-4$. The corresponding heat kernel is given by

$$K^{[\Delta,\frac{s}{2},\frac{s}{2}]}(\gamma,t) = \frac{\beta}{\sqrt{\pi t}}\sum_{k=1}^\infty\frac{\chi_{\frac{s}{2}}(k\alpha_1)\chi_{\frac{s}{2}}(k\alpha_2)}{e^{-2k\beta}|e^{k\beta}-e^{ik\frac{(\alpha_1+\alpha_2)}{2}}|^2|e^{k\beta}-e^{ik\frac{(\alpha_2-\alpha_1)}{2}}|^2} \times e^{-t(\Delta-2)^2}e^{-\frac{k^2\beta^2}{4t}} \tag{2.10}$$

for the dual bulk fluctuations. It may be verified by doing the t integral as in [23] that this gives rise to the expected partition function. This forms the basis of the analysis of Section 3.

3. The heat kernel for mixed symmetry fields

In this section, we will compute the heat kernel for the AdS_5 degrees of freedom that correspond to primaries of mixed symmetry in the CFT. These correspond to representations S of SO(5) where $s_2\neq 0$, i.e.

$$S=(s_1,s_2) \quad \text{s.t.} \quad s_1\geqslant s_2>0. \tag{3.11}$$

The main ingredient of this calculation will be the tensors for which some of the inequalities in the branching rules (2.4) get saturated. In particular, that

$$m_1=s_1, \qquad |m_2|=s_2. \tag{3.12}$$

The eigenvalues of the Laplacian, for such fields are now given by

$$E_R^S=-\big(\lambda^2+s_1+s_2+4\big), \qquad R=(i\lambda,s_1,\pm s_2). \tag{3.13}$$

The heat kernel for the Laplacian acting over such fields is then given by

$$K^{(j_1,j_2)}(\gamma,t) = \frac{\beta}{\sqrt{\pi t}}\sum_{k=1}^\infty\frac{(\chi_{j_1}\chi_{j_2}+\chi_{j_2}\chi_{j_1})(k\alpha_1,k\alpha_2)}{e^{-2k\beta}|e^{k\beta}-e^{ik\frac{(\alpha_1+\alpha_2)}{2}}|^2|e^{k\beta}-e^{ik\frac{(\alpha_2-\alpha_1)}{2}}|^2} \times e^{-t(2j_1+4)}e^{-\frac{k^2\beta^2}{4t}}, \tag{3.14}$$

where

$$\chi_{j_1}\chi_{j_2}(k\alpha_1,k\alpha_2)\equiv\chi_{j_1}(k\alpha_1)\chi_{j_2}(k\alpha_2). \tag{3.15}$$

For later convenience, we have expressed the answer in terms of the $SU(2)\otimes SU(2)$ characters rather than the SO(4) ones. The precise dictionary is

$$\chi_{(s_1,s_2)}^{SO(4)}(\phi_1,\phi_2)=\chi_{j_1}(\alpha_1)\chi_{j_2}(\alpha_2), \tag{3.16}$$

where

$$j_1=\frac{s_1+s_2}{2}, \quad \alpha_1=\phi_1-\phi_2, \qquad j_2=\frac{s_1-s_2}{2}, \quad \alpha_2=\phi_1+\phi_2. \tag{3.17}$$

We shall now use (3.14) as a building block for the bulk contributions that correspond to mixed symmetry primaries in the boundary CFT.

Consider first a long $SO(4,2)$ representation of highest weight $[\Delta,j_1,j_2]\oplus[\Delta,j_2,j_1]$. This primary is dual to a massive field in the bulk. The corresponding heat kernel is given by

$$
\begin{aligned}
&K^{[\Delta,j_1,j_2]}(\gamma,t)\\
&\quad=\frac{\beta}{\sqrt{\pi t}}\sum_{k=1}^{\infty}\frac{(\chi_{j_1}\chi_{j_2}+\chi_{j_2}\chi_{j_1})(k\alpha_1,k\alpha_2)}{e^{-2k\beta}|e^{k\beta}-e^{ik\frac{(\alpha_1+\alpha_2)}{2}}|^2|e^{k\beta}-e^{ik\frac{(\alpha_2-\alpha_1)}{2}}|^2}\\
&\qquad\times e^{-t(\Delta-2)^2}e^{-\frac{k^2\beta^2}{4t}}.
\end{aligned}
\tag{3.18}
$$

This corresponds to the heat kernel of the operator $-\nabla^2+m^2$ evaluated on the tensor fields (3.12), where $m^2+2j_1+4=(\Delta-2)^2$. Equivalently,

$$
\log\mathcal{Z}_{[\Delta,j_1,j_2]\oplus[\Delta,j_2,j_1]}=\frac{1}{2}\int_0^{\infty}\frac{dt}{t}K^{[\Delta,j_1,j_2]}. \tag{3.19}
$$

Finally, we consider massless fields. These correspond to short representations of the conformal group with highest weight $[j_1+j_2+2,j_1,j_2]$. We can similarly show that the appropriate heat kernel is

$$
\left(K^{[j_1+j_2+2,j_1,j_2]}-K^{[j_1+j_2+3,j_1-\frac{1}{2},j_2-\frac{1}{2}]}\right)(\gamma,t), \tag{3.20}
$$

in the notation of (3.18).

4. From the CFT partition function to the AdS heat kernel

We now organise the full CFT partition function into the form of a heat kernel in AdS_5 for a theory which has only long representations in its spectrum. Remarkably, it turns out that the final answer (4.31) thus obtained is unchanged when short multiplets are included. This is outlined below.

Suppose the CFT has operators with quantum numbers $[\Delta,j_1,j_2]$ appearing $N_{[\Delta,j_1,j_2]}$ times. The one-particle partition function is a sum of SO(4, 2) characters evaluated over the modules generated by these primaries.

$$
\mathcal{Y}(q,a,b)=\sum_{\Delta,j_1,j_2}N_{[\Delta,j_1,j_2]}\frac{q^{\Delta}\chi_{j_1}(a)\chi_{j_2}(b)}{\prod_{i=1}^4(1-qx_i)}. \tag{4.21}
$$

The (multi-particle) partition function of the theory is then obtained by exponentiating the one-particle partition function.

$$
\log\mathcal{Z}=\sum_{k=1}^{\infty}\frac{1}{k}\sum_{\Delta,j_1,j_2}N_{[\Delta,j_1,j_2]}\chi_{[\Delta,j_1,j_2]}\left(q^k,a^k,b^k\right). \tag{4.22}
$$

We remind the reader that we focus only on the states whose conformal dimension does not scale with N. Only for these is the idea of a 'multiparticle' collection sensible. We have introduced notation

$$
q=e^{-\beta},\qquad a=e^{i\alpha_1},\qquad b=e^{i\alpha_2}. \tag{4.23}
$$

Shortly we will also define

$$
x_1=\sqrt{ab},\qquad x_2=\sqrt{a\bar{b}},\qquad x_3=\sqrt{\bar{a}b},\qquad x_4=\sqrt{\bar{a}\bar{b}}. \tag{4.24}
$$

In what follows, it is useful to treat symmetric and mixed-symmetric tensors on a different footing, i.e. sum up the $j_1=j_2$ and $j_1\neq j_2$ contributions separately. We then have

$$
\begin{aligned}
\log\mathcal{Z}&=\sum_{k=1}^{\infty}\frac{1}{k}\sum_{\Delta,j}N_{[\Delta,j,j]}\chi_{[\Delta,j,j]}\\
&\quad+\sum_{k=1}^{\infty}\frac{1}{k}\cdot\frac{1}{2}\sideset{}{'}\sum_{\Delta,j_1,j_2}N_{[\Delta,j_1,j_2]}(\chi_{[\Delta,j_1,j_2]}+\chi_{[\Delta,j_2,j_1]}).
\end{aligned}
\tag{4.25}
$$

The prime over the second sum reminds us that in this sum, $j_1\neq j_2$. The factor of half in the second term is from the fact that this sum counts each (j_1,j_2) pair twice. The dependence on (q^k,a^k,b^k) is implicit. We have imposed the condition that $N_{[\Delta,j_1,j_2]}=N_{[\Delta,j_2,j_1]}$ to club terms together in the second sum.

We will now reinterpret, as per (3.18), each Verma module character above as arising from a heat kernel in AdS_5. Using (3.18) and (3.19), we have

$$
\begin{aligned}
\log\mathcal{Z}=\frac{1}{2}\int_0^{\infty}\frac{dt}{t}\Bigg(&\sum_{\Delta,j}N_{[\Delta,j,j]}K^{[\Delta,j,j]}\\
&+\frac{1}{2}\sideset{}{'}\sum_{\Delta,j_1,j_2}N_{[\Delta,j_1,j_2]}K^{[\Delta,j_1,j_2]}\Bigg).
\end{aligned}
\tag{4.26}
$$

We will now evaluate the sums over Δ,j_1,j_2. To do so, the following identity is useful

$$
e^{-t(\Delta-2)^2}=\sqrt{\frac{1}{4\pi t}}\int_{-\infty}^{\infty}dy\,e^{-\frac{y^2}{4t}+iy(\Delta-2)}. \tag{4.27}
$$

The heat kernel formulae in our new notations are

$$
\begin{aligned}
&K^{[\Delta,j,j]}(\gamma,t)\\
&\quad=\sum_{k=1}^{\infty}\frac{\beta}{2\pi t}\int_{-\infty}^{\infty}dy\,e^{-\frac{y^2+k^2\beta^2}{4t}}e^{-2iy}\frac{q^{2k}\chi_j(a^k)\chi_j(b^k)}{\prod_{i=1}^4(1-q^kx_i^k)}e^{i\Delta y},\\
&K^{[\Delta,j_1,j_2]}(\gamma,t)=\sum_{k=1}^{\infty}\frac{\beta}{2\pi t}\int_{-\infty}^{\infty}dy\,e^{-\frac{y^2+k^2\beta^2}{4t}}e^{-2iy}\\
&\qquad\times\frac{q^{2k}(\chi_{j_1}(a^k)\chi_{j_2}(b^k)+\chi_{j_2}(a^k)\chi_{j_1}(b^k))}{\prod_{i=1}^4(1-q^kx_i^k)}e^{i\Delta y}.
\end{aligned}
\tag{4.28}
$$

We will now use these expressions to evaluate (4.26). As is apparent, most of (4.28) does not depend on Δ,j_1,j_2 and factors out of the sum. The sum that we essentially have to evaluate is

$$
\begin{aligned}
&\sum_{\Delta,j}N_{[\Delta,j,j]}e^{i\Delta y}\chi_j(a^k)\chi_j(b^k)\\
&\qquad+\frac{1}{2}\sideset{}{'}\sum_{\Delta,j_1,j_2}N_{[\Delta,j_1,j_2]}e^{i\Delta y}\left(\chi_{j_1}(a^k)\chi_{j_2}(b^k)+\chi_{j_2}(a^k)\chi_{j_1}(b^k)\right)\\
&\quad=\sum_{\Delta,j_1,j_2}N_{[\Delta,j_1,j_2]}e^{i\Delta y}\chi_{j_1}(a^k)\chi_{j_2}(b^k).
\end{aligned}
\tag{4.29}
$$

We can now use the definition (4.21) (replacing q by e^{iy}) to write this as

$$
\mathcal{Y}(e^{iy},a^k,b^k)\prod_{i=1}^4\left(1-e^{iy}x_i^k\right). \tag{4.30}
$$

We therefore find that

$$
\begin{aligned}
\log\mathcal{Z}=\sum_{k=1}^{\infty}\int_0^{\infty}dt\,\frac{\beta}{4\pi t^2}\int_{-\infty}^{\infty}dy\,e^{-\frac{y^2+k^2\beta^2}{4t}}e^{-2iy}q^{2k}\mathcal{Y}\left(e^{iy},a^k,b^k\right)\\
\times\prod_{i=1}^4\frac{(1-e^{iy}x_i^k)}{(1-q^kx_i^k)}.
\end{aligned}
\tag{4.31}
$$

This is an expression for the multi-particle partition function of the AdS_5 theory in terms of the heat kernel time t and the single-particle partition function of its dual CFT. We remind the reader that the single-particle partition function $\mathcal{Y}$ has been very explicitly computed for a free, planar CFT by enumerating the gauge invariant operators in the CFT spectrum [15,16].[3]

Before concluding, we outline how the formula (4.31) extends to the case where the spectrum of the theory has short multiplets of SO(4, 2). For definiteness, consider the case of a theory having a short multiplet $[2m+2, m, m]$ that appears $N_{\{m\}}$ times, another short multiplet $[\ell_1+\ell_2+2, \ell_1, \ell_2] \oplus [\ell_1+\ell_2+2, \ell_2, \ell_1]$ that appears $N_{\{\ell\}}$ times. All other multiplets are long. Given this spectrum

$$\begin{aligned}\log \mathcal{Z} = {} & \sum_{k=1}^{\infty} \frac{1}{k}\Bigg({\sum_{\Delta,j}}' N_{[\Delta,j,j]}\chi_{[\Delta,j,j]} \\ & + N_{\{m\}}\big(\chi_{[2m+2,m,m]} - \chi_{[2m+3,m-\frac{1}{2},m-\frac{1}{2}]}\big)\Bigg) \\ & + \sum_{k=1}^{\infty} \frac{1}{k}\Bigg({\sum_{\Delta,j_1,j_2}}' N_{[\Delta,j_1,j_2]}\chi_{[\Delta,j_1,j_2]} \\ & + N_{\{\ell\}}\big(\chi_{[\ell_1+\ell_2+2,\ell_1,\ell_2]} - \chi_{[\ell_1+\ell_2+3,\ell_1-\frac{1}{2},\ell_2-\frac{1}{2}]} \\ & + \chi_{[\ell_1+\ell_2+2,\ell_2,\ell_1]} - \chi_{[\ell_1+\ell_2+3,\ell_2-\frac{1}{2},\ell_1-\frac{1}{2}]}\big)\Bigg). \end{aligned} \tag{4.32}$$

The prime in the first term is to indicate that the short representation $[2m+2, m, m]$ is not summed over. The double-prime on the second sum is to indicate that the short representation involving the ℓs as well as the terms where $j_1 = j_2$ are not summed in this. We can then show, as above, that the final result for $\log \mathcal{Z}$ is still given by (4.31).

5. Conclusions

In this note, we reinterpreted the partition function of a free CFT on $S^3 \otimes S^1$ in terms of the heat kernel corresponding to a free string in AdS_5. The answer was expressed as a convolution of the one-particle partition function of the dual CFT. This would correspond to the one-loop string path integral in AdS_5 *after* the level matching condition is imposed. How this answer might be extended to recover the full string path integral before level matching is an interesting question. This is work in progress.

Finally, we mention that the results of [23] extend to arbitrary dimensional hyperboloids, though the odd-dimensional case is perhaps the nicest. Corresponding expressions for the characters of the conformal group are also available [32]. Therefore, the analysis presented here extends straightforwardly to AdS/CFT dualities in other dimensions as well.

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[3] Taking the free limit in the CFT side corresponds to the limit in which the string theory on AdS becomes tensionless. This is of course a very non-trivial limit of string theory about which much remains to be understood. However, from the AdS/CFT correspondence, the spectrum of string theory should still match with the spectrum of conformal primaries of the CFT, and our analysis would still be valid. At generic values of the coupling, the spectrum of string theory is also implicitly known through TBA, see [35] for a review. We thank Arkady Tseytlin for discussions regarding these points.

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