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Cosmological energy in a thermo-horizon and the first law

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Abstract

We consider a cosmological horizon, named thermo-horizon, to which are associated a temperature and an entropy of Bekenstein–Hawking and which obeys the first law for an energy flow calculated through the corresponding limit surface. We point out a contradiction between the first law and the definition of the total energy contained inside the horizon. This contradiction is removed when the first law is replaced by a Gibbs' equation for a vacuum-like component associated to the event horizon.

1. Introduction

The generalization of the thermodynamics of black holes (BH) to cosmological horizons represents an important take to understand different issues in cosmology such as the nature of the dark energy (DE) in relation with the problems of the cosmological constant (CC) and of the vacuum energy, the acceleration of the present universe, the coincidence problem and the early inflation.

This generalization was first introduced for de Sitter spacetime [1]. Thereafter, it was tentatively extended to quasi-de Sitter FRW spacetimes in different frameworks (see [2–6]). In an interesting approach, Bousso [7,8] considers the flow of energy through the horizon as a null surface. He interprets the variation of the entropy of the horizon through the variation of its surface as the response of the horizon to the flux of energy, in the same way as the "first law" of the BH.

Following this approach, several authors (see for example [9,10]) have estimated that the apparent horizon (a.h.) is the only limit surface (excluding other horizons such as the event horizon (e.h.)) having coherent thermodynamical properties to address problems such as the nature of the DE.

Our main goal is to shed some light on the contradiction between the amount of energy calculated from the first law as defined in [7,8] and the definition of the energy contained inside the horizon, independently of the choice of the thermo-horizon (t.h.).

We restrict our study to a spatially flat FRW spacetime, which is the starting point of other studies (non-spatially flat spacetimes, cases with interactions, ...).

After a brief review of the definition of a t.h. in a Q-space introduced in [7,8], we show that any t.h. obeys the second law (Section 2). In Section 3, we present the contradiction between the amount of energy derived from the first law and the definition of the energy inside the horizon. We then show that this contradiction is resolved in a thermodynamical model for a DE [4,5] based on the e.h. (Section 4).

2. Definition of a thermo-horizon

In a spatially flat FRW spacetime

$$ds^2 = -dt^2 + a(t)^2\left(dr^2 + r^2\, d\Omega^2\right), \tag{1}$$

the dynamical evolution of the scale factor $a(t)$ is given for a perfect fluid with energy density ρ and pressure P by

$$\left(\frac{\dot a}{a}\right)^2 = H^2 = \chi\frac{\rho}{3}, \tag{2}$$

$$\frac{\ddot a}{a} = -\frac{\chi}{6}(\rho + 3P), \tag{3}$$

where $\chi = 8\pi$ is the Einstein constant, with $G = 1$ and $c = 1$. The equation of state (EoS) ω of the fluid is given by $P = \omega\rho$

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and we introduce the parameter $\varepsilon = \frac{3}{2}(1+\omega)$. In the following, we restrict our study to the Q-space [8], namely accelerated universes, for which $0 < \varepsilon < 1$. Extending the reasoning of [8], we consider an horizon (null surface) with a given radius L. According to the first law, the flow of energy through this surface is given by

$$-\dot{E} = 4\pi L^2 \rho(1+\omega) = T\dot{S}. \tag{4}$$

We assume that we can associate a temperature T and an entropy S to the dynamical horizon of radius L, given by the relations of Bekenstein–Hawking for a BH or a de Sitter horizon

$$T = \frac{1}{2\pi L} \quad \text{and} \quad S = \pi L^2. \tag{5}$$

Any horizon of radius L with a temperature and a entropy given by (5) and which obeys the first law (4) is called thermo-horizon (t.h.).

Using (5), we obtain directly $T\dot{S} = \dot{L}$ and Eq. (4) becomes

$$\varepsilon L^2 H^2 = \dot{L}. \tag{6}$$

With our notations, Eq. (3) is given by

$$\left(\dot{\frac{1}{H}}\right) = \varepsilon, \tag{7}$$

and the first law (6) rewrites

$$\dot{H} = \left(\dot{\frac{1}{L}}\right). \tag{8}$$

After integration, this equation leads to

$$HL - 1 = CL, \tag{9}$$

where C is a constant. Eq. (9) establishes a general relation between the t.h. L and the a.h. $R_A = \frac{1}{H}$ which is satisfied by any thermo-horizon of radius L without restriction on ε (in particular without assuming $\varepsilon = \text{const}$). With the constant C, this relation is more general than Eq. (28) of [8].

If L is the a.h., then $L = \frac{1}{H} = R_A$, implying $C = 0$. Conversely, only $C = 0$ leads to $L = \frac{1}{H}$. Therefore the a.h. obeys the first law (4) if and only if $C = 0$. This special case only is considered by [8].

More generally, any horizon L defined by (9) with a temperature and an entropy given by (5) is a t.h. and it obeys the first law (4).

Eq. (6) can be rewritten with the help of (9)

$$\dot{L} = \varepsilon(1+CL)^2. \tag{10}$$

Any t.h. verifies this equation. Using Eq. (10), L is strictly increasing in the Q-space (accelerated universe) where $0 < \varepsilon < 1$. The same result can be derived for the entropy S given by (5).

3. Energy in a thermo-horizon and the first law

On one side, the total amount of energy contained inside the a.h. for a spatially flat FW spacetime is (e.g. [9] before Eq. (20))

$$E = \rho\frac{4\pi}{3}R_A^3 = \frac{R_A}{2}. \tag{11}$$

Let us remark that in [10] this relation is used for a non-spatially flat FRW spacetime, albeit no more valid in this case.

On the other side, using (4) and (5), the first law applied to the a.h. considered as a t.h. leads to

$$-\dot{E} = \dot{R}_A, \tag{12}$$

where $-\dot{E}$ is the total amount of energy crossing the a.h. by unit of time. According to the conservation of the energy, this amount of energy is equal to the variation of the total energy (11) per unit time, $\dot{E} = \frac{\dot{R}_A}{2}$. This result is in contradiction with (12) except when R_A is constant, which corresponds to a de Sitter spacetime where the a.h. identifies with the e.h.

This result is not restricted to the a.h. and can be extended to any t.h. Using (7), the left hand side of (4) becomes for a t.h. of radius L,

$$-\dot{E} = L^2 H^2 \varepsilon = -L^2 \dot{H}, \tag{13}$$

while the total energy inside the horizon is

$$E = \frac{1}{2}H^2 L^3. \tag{14}$$

Differentiating (14) and equating with (13), we obtain with (8)

$$\frac{3}{2}(HL)^2 - HL + 1 = 0, \tag{15}$$

where $\dot{H} \neq 0$ has been assumed. No real root can be found for this equation. In particular, $L = \frac{1}{H}$ is not a solution. Therefore, the above contradiction can only be removed for $\dot{H} = 0$, namely for a de Sitter spacetime.

4. Thermodynamical model of the event horizon for the dark energy

The preceding results are independent of the underlying model for the DE. They depend only on the assumptions of the existence of a temperature and an entropy associated to a t.h. through the relation (5) and of the validity of the extension of the first law of BHs (4) to cosmological t.h. With these assumptions, we obtain (9) (assuming $C = 0$), as demonstrated by [7,8] and by [9], in Sections II-A and II-B.

In Section II-B of [9], the authors consider only the specific model for the DE developed in [6]. Let us emphasize that their results can be obtained independently of any model for the DE (see Section 2) because the demonstration involves only the density of the total energy ρ. Consequently, the reasoning developed in [9] cannot question the validity of the model assumed for the component DE and in particular the approach proposed in [6]. The first law (4) is a relation between the density of energy ρ and the entropy. It does not involve the density of energy of the DE ρ_Λ and therefore cannot be used to discuss or refute its expression.

In [9], the authors emphasize the apparent discrepancy between the horizon chosen as IR cut-off in the expression of ρ_Λ in the holographic model of Li [6], which is the e.h. r,

$$\chi\rho_\Lambda = \frac{3c^2}{r^2}, \tag{16}$$

and the t.h. L which must be the a.h. R_A in order to satisfy the first law. They suggest that the e.h. r should be identical to the t.h. L obeying the first law. So, they implicitly assume that in any holographic model for ρ_Λ, i.e. $\chi\rho_\Lambda = 3/l^2$, we have to choose for cut-off l the same horizon as the t.h. L obeying the first law.

Let us discuss the full consequences of the assumption $l = L$. On one hand, by setting $L = l = r$ in the expression of the flux of energy in the first law (4), the first law is no more satisfied (see Section 3). On the other hand, if we choose $L = l = R_A$, we obtain for the holographic model:

$$\chi\rho_\Lambda = \frac{3}{R_A^2} = 3H^2 \tag{17}$$

which with (2) implies $\rho = \rho_\Lambda$, excluding any other contribution to the total energy (dark matter (DM), dust, radiation, ...) in contradiction with the present observations where the DM takes a non-negligible part (about 1/3) of the energy of the universe.

Two points of view can be followed to solve this dilemma:

(i) First, we can choose to preserve the first law (4) and propose an holographic model of the form (17) albeit not compatible with the observations [6,12].
(ii) Secondly, we can consider the holographic model of the DE (16) compatible with the present observations (acceleration and EoS today $\omega_\Lambda \simeq -1$ [5,6,11]) and modify the first law (4) to be compatible with the chosen model for the DE.

Because compatible with the observational features, the second alternative is more reasonable. This leads naturally to question the first law (4) which seems to fail because, as seen in Section 3, it contradicts the definition (11) of the total energy in the horizon. The first law must be modified in order to include the DE through a model linking the DE with the chosen horizon. To achieve this goal, we propose a Gibbs' equation describing the thermodynamics of the component DE instead of the first law (4). This approach was introduced in the model [4,5], where a DE component with an energy density of type (16) (with $c = 1$) and an EoS of vacuum-type were considered

$$\frac{\Lambda}{\chi} = \rho_\Lambda = -P_\Lambda = 3/\chi r^2, \tag{18}$$

with r the radius of the e.h. In this model, the Gibbs' equation relative to this component DE is given at the specific level by

$$T_\Lambda\, ds_\Lambda = d\varepsilon_\Lambda + P_\Lambda\, dv_\Lambda, \tag{19}$$

where $\varepsilon_\Lambda \equiv \frac{\Lambda}{n_\Lambda \chi}$ is the specific energy, $v_\Lambda \equiv \frac{1}{n_\Lambda}$ the specific volume and n_Λ the number density. With the EoS (18), Eq. (19) becomes

$$\frac{\dot{\Lambda}}{n_\Lambda} = \chi T_\Lambda \dot{S}_\Lambda = -\dot{r}, \tag{20}$$

with $T_\Lambda = \frac{1}{2\pi r}$. After integration, we obtain for the specific entropy

$$S_\Lambda = -\pi r^2 + K, \tag{21}$$

where K is a constant. In [4,5] the local equation of the conservation of the energy $u_\beta \nabla_\alpha T^{\alpha\beta} = 0$ is used instead of the expression of the global energy E and without assuming any peculiar expression for the entropy. In these articles, we used local thermodynamic equilibrium which seems to be more accurate to general relativity because in particular the Einstein equations involve densities. This necessity was also remarked by other authors (e.g. [13, Section 1, p. 5390] and [14]).

Considering the global energy of the DE component inside the horizon as in [9,10], $E_\Lambda = \rho_\Lambda \frac{4\pi}{3} r^3 = \frac{r}{2}$, the Gibbs' equation associated to this component becomes

$$dE_\Lambda = T_\Lambda\, dS_\Lambda - P_\Lambda\, dV_\Lambda \quad \text{or} \quad \dot{E}_\Lambda = T_\Lambda \dot{S}_\Lambda - P_\Lambda \dot{V}_\Lambda. \tag{22}$$

With (18) and (21), this equation leads to $\dot{E}_\Lambda = \frac{\dot{r}}{2}$, in full agreement with the previous definition of $E_\Lambda = \frac{r}{2}$.

The previous results show that the contradiction described in Section 3 is essentially related to the mainly questionable assumption of the validity of the expression of the static entropy $S = \pi L^2$ of the BH when extended to the cosmological horizon [7,8]. On the contrary, our expression (21) of S_Λ is not postulated but deduced from the Gibbs' equation for the model (18) of the DE [4,5], which is supported by the holographic approach [6].

By comparison of the two equations (5) and (21), let us first note the difference of sign in the expression of the entropy. The difference between the situations considered explains this difference. In the first case (BH), the observer is outside the limit-surface of the BH and loses information, while in the second case (cosmological case), the observer is inside the limit-surface.

Secondly, let us remark the presence of a pressure term $-P_\Lambda\, dV_\Lambda$ in the Gibbs' equation (22) which does not appear in the first law (4). In the present case, this term does not vanish because the pressure of the DE P_Λ is defined (albeit negative, see the equation of state (18)) and because the volume varies ($dV_\Lambda \neq 0$) due to the "response" of the surface to the flow of energy [7]. This pressure term is responsible for the sign "minus" in the expression of the entropy Eq. (21), obtained by integration of Eq. (20). This point relates the two preceding remarks and strengthens the consistency of our thermodynamical approach which is compatible with the conservation of the energy. From Eq. (3), we obtain that the universe can be accelerated only if the strong energy condition $\rho + 3P \geqslant 0$ is violated, which is only fulfilled for a negative pressure $P_\Lambda < 0$ (because $P_m = 0$ for the dust). The existence of such a pressure term in the first law was recently considered and discussed in another contexts by [13,14] and [15].

5. Conclusion

By a general demonstration independent of the underlying model for the DE, we show in this article a contradiction (except in de Sitter) between the first law introduced in [7,8] for the thermo-horizon that leads to $-\dot{E} = \dot{R}_A$, and the definition of the total energy in the horizon $E = \frac{R_A}{2}$. To solve this contradiction, we propose to replace the first law by a Gibbs' equation for the DE component, which is naturally associated to the e.h. in the model [4,5], later supported by an holographic model [6] in an independent approach.

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