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# Surface vacuum energy and stresses for a brane in de Sitter spacetime

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## Abstract

Vacuum expectation values of the surface energy–momentum tensor is investigated for a massless scalar field obeying mixed boundary condition on a brane in de Sitter bulk. To generate the corresponding vacuum surface densities we use the conformal relation between de Sitter and Rindler spacetimes.

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## 1. Introduction

de Sitter (dS) spacetime is the maximally symmetric solution of Einstein's equation with a positive cosmological constant. Recent astronomical observations of supernovae and cosmic microwave background [1] indicate that the universe is accelerating and can be well approximated by a world with a positive cosmological constant. If the universe would accelerate indefinitely, the standard cosmology leads to an asymptotic dS universe. de Sitter spacetime plays an important role in the inflationary scenario, where an exponentially expanding approximately dS spacetime is employed to solve a number of problems in standard cosmology. The quantum field theory on dS spacetime is also of considerable interest. In particular, the inhomogeneities generated by fluctuations of a quantum field during inflation provide an attractive mechanism for the structure formation in the universe. Another motivation for investigations of dS based quantum theories is related to the recently proposed holographic duality between quantum gravity on dS spacetime and a quantum field theory living on boundary identified with the timelike infinity of dS spacetime [2].

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The investigation of quantum effects in braneworld models is of considerable phenomenological interest, both in particle physics and in cosmology. The braneworld corresponds to a manifold with dynamical boundaries and all fields which propagate in the bulk will give Casimir-type contributions to the vacuum energy, and as a result to the vacuum forces acting on the branes. In dependence of the type of a field and boundary conditions imposed, these forces can either stabilize or destabilize the braneworld. In addition, the Casimir energy gives a contribution to both the brane and bulk cosmological constants and, hence, has to be taken into account in the self-consistent formulation of the braneworld dynamics. Motivated by these, the role of quantum effects in braneworld scenarios has received a grate deal of attention. For conformally coupled scalar this effect was initially studied in [3] in the context of M-theory, and subsequently in [4–18] for a background Randall–Sundrum geometry. The models with dS and AdS brane, and higher-dimensional brane models are considered as well [11,13–18]. For a conformally coupled bulk scalar the cosmological backreaction of the Casimir energy is investigated in [3,11,13].

As a brane we take 4-dimensional hypersurface which is the conformal image of a plate moving with constant proper acceleration in the Rindler spacetime. We will assume that the field is prepared in the state conformally related to the Fulling–Rindler vacuum in the Rindler spacetime. To generate the vacuum expectation values in dS bulk, we use the conformal relation between dS and Rindler spacetimes [19] and the results from [20] for the corresponding Rindler problem with mixed boundary conditions. Previously this method has been used in [21] to derive the vacuum stress on parallel plates for a scalar field with Dirichlet boundary conditions in de Sitter spacetime and in Ref. [10] to investigate the vacuum characteristics of the Casimir configuration on background of conformally flat brane-world geometries for massless scalar field with Robin boundary conditions on plates.

The present Letter is organized as follows. In the next section the geometry of our problem and the conformal relation between dS and Rindler spacetimes are discussed. The results are presented for the vacuum expectation values of the energy–momentum tensor for a scalar field induced by a plate uniformly accelerated through the Fulling–Rindler vacuum. In Section 3, by using the formula relating the renormalized energy–momentum tensors for conformally related problems in combination with the appropriate coordinate transformation, we derive expressions for the vacuum energy–momentum tensor in dS space. The main results are rementioned and summarized in Section 4.

## 2. Conformal relation between dS and Rindler problems

Consider a conformally coupled massless scalar field  $\varphi(x)$  satisfying the equation

$$(\nabla_l \nabla^l + \zeta R)\varphi(x) = 0, \quad \zeta = \frac{3}{16}, \quad (1)$$

on background of a  $(4 + 1)$ -dimensional dS spacetime. In Eq. (1),  $\nabla_l$  is the operator of the covariant derivative, and  $R$  is the Ricci scalar for the corresponding metric  $g_{ik}$ . In static coordinates  $x^i = (t, r, \theta, \phi)$  dS metric has the form

$$ds_{\text{dS}}^2 = g_{ik} dx^i dx^k = \left(1 - \frac{r^2}{\alpha^2}\right) dt^2 - \frac{dr^2}{1 - \frac{r^2}{\alpha^2}} - r^2 d\Omega_3^2, \quad (2)$$

where  $d\Omega_3^2$  is the line element on the 3-dimensional unit sphere in Euclidean space, and the parameter  $\alpha$  defines the dS curvature radius. Note that  $R = 12/\alpha^2$ . We will assume that the field satisfies the mixed boundary condition

$$(A_s + n^l \nabla_l)\varphi(x) = 0, \quad (3)$$

on the brane, where  $A_s$  is a constant and  $n^l$  is the unit inward normal to the brane. This type of conditions is an extension of Dirichlet and Neumann boundary conditions and appears in a variety of situations, for example, the Casimir effect for massless scalar field with Robin boundary conditions on two parallel plates in de Sitter spacetime

is calculated in [21], the Robin type boundary condition in domain wall formation is investigated in [23]. Mixed boundary conditions naturally arise for scalar and fermion bulk fields in the Randall–Sundrum model [9,10,24]. To make maximum use of the flat spacetime calculations, first of all let us present the dS line element in the form conformally related to the Rindler metric. With this aim we make the coordinate transformation  $x^i \rightarrow x'^i = (\tau, \xi, \mathbf{x}')$ ,  $\mathbf{x}' = (x'^2, x'^3, x'^4)$

$$\begin{aligned} \tau &= \frac{t}{\alpha}, & \xi &= \frac{\sqrt{\alpha^2 - r^2}}{\Omega}, & x'^2 &= \frac{r}{\Omega} \sin \theta \cos \theta_2, \\ x'^3 &= \frac{r}{\Omega} \sin \theta \sin \theta_2 \cos \phi, & x'^4 &= \frac{r}{\Omega} \sin \theta \sin \theta_2 \sin \phi, \end{aligned} \tag{4}$$

with the notation

$$\Omega = 1 - \frac{r}{\alpha} \cos \theta. \tag{5}$$

Under this coordinate transformation the dS line element takes the form

$$ds_{\text{dS}}^2 = g'_{ik} dx'^i dx'^k = \Omega^2 (\xi^2 d\tau^2 - d\xi^2 - d\mathbf{x}'^2). \tag{6}$$

In this form the dS metric is manifestly conformally related to the Rindler spacetime with the line element  $ds_{\text{R}}^2$ :

$$ds_{\text{dS}}^2 = \Omega^2 ds_{\text{R}}^2, \quad ds_{\text{R}}^2 = g_{ik}^{\text{R}} dx'^i dx'^k = \xi^2 d\tau^2 - d\xi^2 - d\mathbf{x}'^2, \quad g'_{ik} = \Omega^2 g_{ik}^{\text{R}}. \tag{7}$$

By using the standard transformation formula for the vacuum expectation values of the energy–momentum tensor in conformally related problems (see, for instance, [22]), we can generate the results for dS spacetime from the corresponding results for the Rindler spacetime. In this Letter as a Rindler counterpart we will take the vacuum surface energy–momentum tensor induced by an infinite plate moving by uniform proper acceleration through the Fulling–Rindler vacuum. We will assume that the plate is located in the right Rindler wedge and has the coordinate  $\xi = a$ . Observe that in coordinates  $x^i$  the boundary  $\xi = a$  is presented by the hypersurface

$$\sqrt{\alpha^2 - r^2} = a \left( 1 - \frac{r}{\alpha} \cos \theta \right), \tag{8}$$

in dS spacetime. In Ref. [25] it was argued that the energy–momentum tensor for a scalar field on manifolds with boundaries in addition to the bulk part contains a contribution located on the boundary. For the boundary  $\partial M_s$  the surface part of the energy–momentum tensor is presented in the form [25]

$$T_{ik}^{(\text{surf})} = \delta(x; \partial M_s) \tau_{ik} \tag{9}$$

with

$$\tau_{ik} = \zeta \varphi^2 K_{ik} - (2\zeta - 1/2) h_{ik} \varphi n^l \nabla_l \varphi, \tag{10}$$

and the “one-sided” delta-function  $\delta(x; \partial M_s)$  locates this tensor on  $\partial M_s$ . In Eq. (10),  $K_{ik}$  is the extrinsic curvature tensor of the boundary  $\partial M_s$  and  $h_{ik}$  is the corresponding induced metric. Let  $\{\varphi_\alpha(x), \varphi_\alpha^*(x)\}$  be a complete set of positive and negative frequency solutions to the field equation (1), obeying boundary condition (3). Here  $\alpha$  denotes a set of quantum numbers specifying the solution. By expanding the field operator over the eigenfunctions  $\varphi_\alpha(x)$ , using the standard commutation rules and the definition of the vacuum state, for the vacuum expectation value of the surface energy–momentum tensor one finds

$$\langle 0 | T_{ik}^{(\text{surf})} | 0 \rangle = \delta(x; \partial M_s) \langle 0 | \tau_{ik} | 0 \rangle, \quad \langle 0 | \tau_{ik} | 0 \rangle = \sum_{\alpha} \tau_{ik} \{ \varphi_\alpha(x), \varphi_\alpha^*(x) \}, \tag{11}$$

where  $|0\rangle$  is the amplitude for the corresponding vacuum state, and the bilinear form  $\tau_{ik} \{ \varphi, \psi \}$  on the right of the second formula is determined by the classical energy–momentum tensor (10). The surface energy–momentum

tensor has a diagonal structure:

$$\langle 0 | \tau_i^k | 0 \rangle = \text{diag}(\varepsilon, 0, -p, -p, -p), \quad (12)$$

with the surface energy density  $\varepsilon$  and stress  $p$ , and the equation of state

$$\varepsilon = - \left[ 1 + \frac{2\zeta}{A(4\zeta - 1)} \right] p, \quad A = aA_s. \quad (13)$$

Here we take a plane boundary with coordinate  $\xi = a > 0$  corresponding to a plate uniformly accelerated normal to itself with the proper acceleration  $a^{-1}$ . For a minimally coupled scalar field,  $\varepsilon$  corresponds to a cosmological constant induced on the plate. In the conformally coupled case

$$\varepsilon = - \left( 1 - \frac{3}{2A} \right) p. \quad (14)$$

The vacuum stress induced on the brane is as following [20]

$$p = p_p^{(R)} + p_f^{(R)}, \quad (15)$$

where for the pole and finite contributions one has

$$p_p^{(R)} = \frac{B_4}{4sa^4} A F_{R,-1}^{(as)}, \quad A = aA_s, \quad (16)$$

$$p_f^{(R)} = \frac{B_4}{4a^4} A [F_{R,0}^{(as)} + F_R^{(1)}(0)], \quad (17)$$

and the coefficients are defined by following expressions

$$B_4 = \frac{1}{(4\pi)^{3/2} \Gamma(3/2)}, \quad (18)$$

$$F_R^{(1)}(s) = -\frac{1}{\pi} \cos \frac{\pi s}{2} \int_0^\infty dx x^2 \int_\rho^\infty dz z^{-s} \left[ \frac{K_z(x)}{\bar{K}_z(x)} + \frac{1}{r} \sum_{l=0}^N \frac{(-1)^l U_l(\cos \theta)}{(1+r^2)^{l/2}} \right], \quad (19)$$

$$F_{R,-1}^{(as)} = -\frac{2}{\pi} \Gamma\left(\frac{3}{2}\right) \sum_{j=0}^1 \frac{(-1)^j}{\Gamma(j+1)\Gamma(\frac{3}{2}-j)} \sum_{m=0}^{3-2j} U_{3-2j,m} B\left(m + \frac{1}{2}, \frac{3}{2}\right), \quad (20)$$

$$\begin{aligned} F_{R,0}^{(as)} = & -\frac{1}{\pi} \Gamma\left(\frac{3}{2}\right) \sum_{j=0}^1 \frac{(-1)^j}{\Gamma(j+1)\Gamma(\frac{3}{2}-j)} \sum_{m=0}^{3-2j} U_{3-2j,m} B\left(m + \frac{1}{2}, \frac{3}{2}\right) \\ & \times \left[ \psi(m+2) + \psi(j+1) - \psi\left(m + \frac{1}{2}\right) - \psi\left(\frac{3}{2}\right) \right] \\ & + \frac{1}{\pi} \left( \sum_{l=1,4-l=\text{even}}^3 + \sum_{l=4}^N \right) (-1)^l B\left(\frac{l-3}{2}, \frac{3}{2}\right) \sum_{m=0}^l U_{lm} B\left(m + \frac{1}{2}, \frac{3}{2}\right), \end{aligned} \quad (21)$$

where  $\psi(x) = d \ln \Gamma(x) / dx$  is the digamma function and  $B(x, y)$  is the beta function.

The surface energy per unit surface of the plate can be found integrating the energy density from Eq. (11),

$$E^{(R, \text{surf})} = \int d^4x \sqrt{|\det g_{ik}|} \langle 0 | T_0^{(\text{surf})0} | 0 \rangle = a \langle 0 | \tau_0^0 | 0 \rangle = a\varepsilon. \quad (22)$$

### 3. Vacuum energy–momentum tensor in dS bulk

To find the VEV's induced by the surface (8) in dS spacetime, first we will consider the corresponding quantities in the coordinates  $(\tau, \xi, \mathbf{x}')$  with metric (6). These quantities can be found from the corresponding results in the Rindler spacetime by using the standard transformation formula for the conformally related problems [22]:

$$\langle T_i^k[g'_{lm}, \varphi] \rangle = \Omega^{-5} \langle T_i^k[g_{lm}^R, \varphi_R] \rangle. \quad (23)$$

If the classical energy–momentum tensor is traceless then the classical action is invariant under the conformal transformation. It must be noted that trace anomalies in stress tensor i.e., the nonvanishing  $T_i^i$  for a conformally invariant field after renormalization originate from some quantum behavior [26]. The trace anomaly is related to the divergent part of effective action, in the absence of boundaries in odd spacetime dimensions the conformal anomaly is absent [22] (see also Appendix A of the present Letter). Under the conformal transformation  $g'_{ik} = \Omega^2 g_{ik}^R$ , the  $\varphi_R$  field will change by the rule

$$\varphi(x') = \Omega^{-3/2} \varphi_R(x'), \quad (24)$$

where the conformal factor is given by expression (5). The scalar field  $\varphi_R(x')$ , satisfy following mixed boundary condition

$$(A_R + B_R \hat{n}_R^l \nabla'_l) \varphi_R = 0, \quad \xi = a, \quad \hat{n}_R^l \nabla'_l = \delta_1^l. \quad (25)$$

Now by comparing boundary conditions (3) and (25) and taking into account Eq. (24), one obtains the relation between the coefficients in the boundary conditions:

$$A = \frac{1}{\Omega} \left( A_R + \frac{3}{2} B_R n^l \nabla_l \Omega \right), \quad B = B_R, \quad x \in S. \quad (26)$$

To evaluate the expression  $n^l \nabla_l \Omega$  we need the components of the normal to  $S$  in coordinates  $x^i$ . They can be found by transforming the components  $\hat{n}^l = \delta_1^l / \Omega$  in coordinates  $x'^i$ :

$$n^l = \left( 0, \frac{a}{\alpha} (\cos \theta - r/\alpha), -\frac{a}{\alpha r} \sin \theta, 0, 0 \right). \quad (27)$$

Now it can be easily seen that  $n^l \nabla_l \Omega = -\sqrt{\alpha^2 - r^2} / \alpha^2$  and, hence, the relation between the Robin coefficients in the Rindler and dS problems takes the form

$$A = \frac{a A_R}{\sqrt{\alpha^2 - r^2}} - \frac{3 a B_R}{2 \alpha^2}, \quad B = B_R. \quad (28)$$

As for the energy–momentum tensor the spatial part is anisotropic, the corresponding part in coordinates  $x^i$  is more complicated:

$$\langle T_i^k[g_{lm}, \varphi] \rangle = \Omega^{-5} \langle T_i^k[g_{lm}^R, \varphi_R] \rangle, \quad i, k = 0, 3, 4, \quad (29)$$

$$\langle T_1^1[g_{lm}, \varphi] \rangle = \frac{(\cos \theta - r/\alpha)^2}{\Omega^7} \langle T_1^1[g_{lm}^R, \varphi_R] \rangle + \frac{1 - r^2/\alpha^2}{\Omega^7} \sin^2 \theta \langle T_2^2[g_{lm}^R, \varphi_R] \rangle, \quad (30)$$

$$\langle T_1^2[g_{lm}, \varphi] \rangle = \frac{(r/\alpha - \cos \theta) \sin \theta}{r \Omega^7} \{ \langle T_1^1[g_{lm}^R, \varphi_R] \rangle - \langle T_2^2[g_{lm}^R, \varphi_R] \rangle \}, \quad (31)$$

$$\langle T_2^2[g_{lm}, \varphi] \rangle = \frac{1 - r^2/\alpha^2}{\Omega^7} \sin^2 \theta \langle T_1^1[g_{lm}^R, \varphi_R] \rangle + \frac{(r/\alpha - \cos \theta)^2}{\Omega^7} \langle T_2^2[g_{lm}^R, \varphi_R] \rangle. \quad (32)$$

#### 4. Conclusion

In the present Letter we have investigated the surface Casimir densities in dS spacetime for a conformally coupled massless scalar field which satisfies the Robin boundary condition (3) on a hypersurface described by Eq. (8). The coefficients in the boundary condition are given by relations (28) with constants  $A_R$  and  $B_R$  and, in general, depend on the point of the hypersurface. The latter is the conformal image of the flat boundary moving by uniform proper acceleration in the Minkowski spacetime. We have assumed that the field in dS spacetime is in the state conformally related to the Fulling–Rindler vacuum. The energy–momentum tensor in dS spacetime is generated from the corresponding results in the Rindler spacetime by using the standard formula for the energy–momentum tensors in conformally related problems in combination with the appropriate coordinate transformation. The Rindler energy–momentum tensor is taken from Ref. [21], where the general case of the curvature coupling parameter is considered. For a minimally coupled scalar field, the surface energy–momentum tensor induced by quantum vacuum effects corresponds to a source of a cosmological constant type located on the plate and with the cosmological constant determined by the proper acceleration of the plate. By using the conformal relation between the Rindler and dS spacetimes and the results from [27], in Ref. [17] the vacuum expectation value of the bulk energy–momentum tensor is evaluated for a conformally coupled scalar field which satisfies the Robin boundary condition on a curved brane in dS spacetime. By making use the same technique and the conformal properties of the surface energy–momentum tensor, from the results of the [20] we have obtained the surface vacuum energy–momentum tensor induced on the brane in dS spacetime, which is a conformal image of a uniformly accelerated plate in the Minkowski spacetime. As it has been shown recently in [28] (see also [17,29]), the surface densities induced by quantum fluctuations of bulk fields can serve as a natural mechanism for the generation of cosmological constant in braneworld models of the Randall–Sundrum type with the value in good agreement with recent cosmological observations.

#### Appendix A

As we have seen in previous sections the vacuum expectation values of the surface energy–momentum tensor contain pole and finite contributions. The remaining pole term is a characteristic feature for the zeta function regularization method and has been found for many other case of boundary geometries. In the conformally coupled case, fluctuations of the stress tensor trace is as

$$\langle T_i^i(x) \rangle = cK(x), \quad (\text{A.1})$$

where  $c$  is a constant, and

$$K(x) = \zeta(s|A)(x)|_{s=0}, \quad (\text{A.2})$$

here  $\zeta(s|A)$  is the zeta function related to an elliptic operator  $A$  [30]. One can represent the zeta function as following

$$\zeta(s|A) = \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-1} K(t), \quad (\text{A.3})$$

where

$$K(t) = \left( \frac{1}{4\pi t} \right)^{3/2} \sum_k \exp(-\lambda_k t), \quad (\text{A.4})$$

is the heat kernel in four dimension, the  $\lambda_k$ 's are the one-particle energies with the quantum number  $k$ . Now the ultraviolet divergences of the vacuum energy are determined from the behavior of the integrand in Eq. (A.1) at the lower integration limit and, hence, from the asymptotic expansion of the heat kernel for  $t \rightarrow 0$

$$K(t) \sim \left(\frac{1}{4\pi t}\right)^{3/2} \sum_{k=0,1/2,1,\dots} B_k t^k. \quad (\text{A.5})$$

This expansion is known for a very general manifold, if the underlying manifold is without boundary, only coefficients with integer numbers enter, otherwise half integer power of  $t$  are present. The  $B_k$  are given by

$$B_k = \int_M dv a_k(x) + \int_{\partial M} ds c_k(y), \quad (\text{A.6})$$

the Seely–de Witt coefficients  $a_k$  vanish for half-odd integers, these coefficients are independent of the applied boundary condition, but the coefficients do depend on the spin of the field in equation [22,31,32]. The coefficients  $c_k$  are functions of the second fundamental form of the boundary (extrinsic curvature), the induced geometry on the boundary (intrinsic curvature), and the nature of boundary condition imposed. The simplest first of  $a_k$  and  $c_k$  coefficients for a manifold with boundary are given in [22].

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