Moment-Curvature Model for Steel Plate-Concrete Composite Beams

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Abstract

The rapid development of composite structures has led to a new design concept of the steel plate-concrete composite beam in which a thin steel plate is attached to the reinforced concrete beam. The shear connection is established by the use of headed studs and the joined parts can act as a unit due to the adequate bond between the steel and concrete. The behavior of this type of structures can be classified as intermediate between composite and reinforced concrete structures.

The experimental studies have shown that the methods described in standard codes allow to determine the ultimate load of the structure, while the structure’s deflection is undetermined. More sophisticated analyses of beams are more accurate but they are used in practical design rather reluctantly. The purpose of this paper is to present a moment-curvature model which can be used in simplified calculation of deflection of steel plate-concrete composite beams. Instead of adopting the complex theoretical analysis, a trilinear model is represented by pre-cracking, post-crack and post-yielding stages. The additional deflection due to shear slip at the interface plane is taken into account. The slip effect always exists both in fully and partially composite beams and experimental studies have shown that it has a significant impact on the response of the structure. Tension stiffening effects are also considered since beam rigidity is reduced when cracking is developing due to an increase of the applied load. For validation purposes a theoretical analysis based on the presented model was performed and compared with the experimental results. The predicted results of the analytical studies were compared with the measurements of the tested beam to confirm the validity of the proposed method. It was confirmed that including the shear slip effect has significantly improved the accuracy of prediction.

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1. Introduction

The steel plate-concrete composite beam is a new design concept which requires comprehensive research before applying such beams in civil engineering designs. In the case of the flexure behavior they can be accurately calculated with the standard procedure specified by codes for reinforced concrete structures [1]. However, including the basic phenomena such as cracking and slip at the interface in the theoretical analysis of such beams makes the analysis complicated, and, in particular, it makes it difficult to correctly determine the deflection of this type of elements.

Steel plate-concrete composite beams are innovative designs therefore there is little literature available. The researchers dealing with similar issues, include Nie [2] who investigated the effects of shear slip on the deformation of steel–concrete composite beams. The methods for calculation the deflection of reinforced concrete beams strengthened with FRP plates can be found in [3]. A similar problem was considered in [4]. Wang [5] describes a method for calculating the maximum deflection of traditional steel-concrete composite beams with partial shear interaction.

This paper presents a trilinear moment-curvature model which allows a description of the behavior of steel plate-composite beams until ultimate strain in concrete is reached and crushing occurs. It may be used successfully in practical applications because of a simplified manner of the analysis of this type of construction.

**Nomenclature**

- $a_L$: distance from the centroid of the reinforcement to the centroid of the steel plate
- $A_L$: area of the steel plate
- $a_{s1}$: distance from the centroid of the reinforcement to bottom edge of the beam
- $A_{s1}$: area of the longitudinal steel reinforcement
- $b$: width
- $d$: effective depth
- $E_c$: modulus of elasticity of concrete
- $E_s$: modulus of elasticity of steel
- $f_c$: compressive strength of concrete
- $f_{ct}$: tensile strength of concrete
- $f_{y,s1}$: yield strength of reinforcement
- $f_{y,L}$: yield strength of steel plate
- $L$: length of the beam
- $\alpha$: static diagram coefficient dependent on boundary conditions and load arrangement
- $\varepsilon_y$: steel yield strain
- $\varepsilon_u$: ultimate compressive strain in concrete
- $\lambda$: factor of effective height of the compression zone
- $\Delta f$: additional deflection due to slip according to specific static diagram
2. Flexure behavior and deflection

The experimental results [6], theoretical studies and numerical analysis confirm that this type of structures have demonstrate a specific response to the applied load. It can be schematically represented by a moment-deflection or moment-curvature curve shown in Fig. 2.

![Fig. 2. Idealized load-deflection (moment-curvature) curve for steel plate-concrete composite structures.](image)

This structure is characterized by the behavior similar to reinforced concrete structures, but the description of the phenomena characteristic of composite structures should not be neglected in particular the slip at the interface. In the presented model it is assumed that the beam's response can be divided into three characteristic stages: pre-cracking, post-cracking and post-yielding. The elastic behavior in the first phase, when the load does not exceed the cracking moment, can be observed. When the cracking moment is reached, the first crack is assumed to occur and there is a significant reduction of the rigidity of the structure. It is mainly caused by the shear slip and it results in an increase of deflection compared with the structure's deflection calculated by the analysis using the theory applicable to the reinforced concrete structure, assuming that the steel plate in this case is a kind of longitudinal reinforcement. When the yielding moment is reached, it causes the plasticization of the steel plate, which greatly reduces global stiffness. The development of the plastic zone is terminated when the ultimate concrete strain is reached, which is identified with the beam failure.

3. Moment-curvature model

The presented model was derived based on the following assumptions: a. plain sections remain plain, b. the appearance of the first cracks is postulated when the stress in the concrete reaches the tensile strength of the concrete, c. the tensile strength of the cracked concrete is ignored, d. the strain in bonded reinforcement is the same as that in the surrounding concrete and e. the interface shear force is proportional to slip.

It is desirable to introduce the following dimensionless relationships:
\[ n = \frac{E_s}{E_c} \]  
(1)

\[ \alpha_{s1} = \frac{a_{s1}}{d} \]  
(2)

\[ \alpha_L = \frac{a_L}{d} \]  
(3)

\[ \rho_{s1} = \frac{n \cdot A_{s1}}{b \cdot d} \]  
(4)

\[ \rho_L = \frac{n \cdot A_L}{b \cdot d} \]  
(5)

\[ \rho_{sl} = \rho_{s1} + \rho_L \]  
(6)

\[ \omega_{s1} = \frac{f_{y,s1}}{f_c} \]  
(7)

\[ \omega_L = \frac{f_{y,L}}{f_c} \]  
(8)

3.1. Uncracked beam

![Diagram showing strains and stresses distribution in the uncracked phase.](image)

Fig. 3. Strains and stresses distribution in the uncracked phase.
The relative height of compression of zone can be expressed as:

\[
\xi_{cr} = \frac{1}{2}(\alpha_{sL} + 1)^2 + \alpha_L \rho_L + \rho_{sL}
\]

\[
1 + \alpha_{sL} + \rho_{sL}
\]

(9)

\[
\xi'_{cr} = 1 + \alpha_{sL} - \xi_{cr}
\]

(10)

The curvature of the beam at the cracking moment can be calculated as:

\[
\phi_{cr} = \frac{\varepsilon_{ct}}{\xi_{cr} d} = \frac{f_{ct}}{E_c \xi_{cr} d}
\]

(11)

The dimensionless cracking moment can be represented by the formula:

\[
m_{cr} = \frac{1}{3}(1 + \alpha_{sL}) \left[ (1 + \alpha_{sL})^2 - 3 \xi_{cr} \xi'_{cr} \right] + \rho_L (1 + \alpha_L - \xi_{cr})^2 + \rho_{sL} (\xi_{cr} - 1)^2
\]

(12)

3.2. Cracked beam

![Diagram showing strains and stresses distribution in the cracked phase.](image)

Fig. 4. Strains and stresses distribution in the cracked phase.

The relative height of compression of zone can be expressed as:

\[
\xi_s = \sqrt{\rho_{sL}^2 + 2(\rho_{sL} + \rho_L (1 + \alpha_L)) - \rho_{sL}}
\]

(13)
\[ \xi' = 1 + \alpha_L - \xi \quad (14) \]

The curvature of the beam at yielding can be calculated as:

\[ \varphi_y = \frac{\varepsilon_y}{\xi' d} \quad (15) \]

The dimensionless yielding moment can be represented by the formula:

\[ m_y = \frac{\xi^3 + 3 \rho L \xi^2 + 3 \rho_{s1} (\xi - 1)^2}{3 \cdot n \cdot \xi'} \quad (16) \]

3.3. Failure of the beam

The relative height of compression of zone can be expressed as:

\[ \xi_u = \frac{\rho_{s1} \varepsilon_{s1} + \rho_L \varepsilon_L}{\lambda \cdot n} \quad (17) \]

The curvature of the beam at failure can be calculated as:

\[ \varphi_u = \frac{\varepsilon_u}{\xi_u d} \quad (18) \]

The dimensionless ultimate moment can be represented by the formula:
4. Additional deflection due to slip

In the case of the design of traditional composite structures calculations are based on the assumption of full composite action due to the used shear connectors which ensure an adequate rigidity. However, in reality the slip does occur and in the case of steel plate-concrete composite beams it has a significant influence on the value of deflection. In the literature [2, 7, 8] this problem is fairly well recognized for traditional composite beams, and solution of finding additional deflection for the basic static diagram such as simply supported beam, cantilever beam or continuous beam is provided.

We can obtain additional curvature due to the slip at the interface for steel plate-concrete composite beam using relationship:

\[
\Delta \varphi = \frac{\Delta f}{\alpha \cdot L^2}
\]

5. Model validation, results and discussion

The model presented in this paper is fully defined when \( M_{cr}, \varphi_{cr}, M_{y}, \varphi_{y}, M_{ucr}, \varphi_{ucr} \) are determined. The model was verified by referring it against the experimental results [6].

The results of the analysis in the form of the moment-deflection curve are presented in Fig. 6 and the moment-curvature curve is shown in Fig. 7. A good correlation of results can be observed despite the simplified approach used for the analysis. The comparison of the analysis results of analysis with and without consideration of the shear slip shows a significant impact of this phenomenon on the global behavior of the beam. For verification purposes of the serviceability limit state of the deflection consideration of this effect is indispensable for adequate modeling of the response of the steel plate-concrete composite beam.

![Fig. 6. Load-deflection curve.](image-url)
6. Conclusions

Design of steel plate-concrete composite beams requires a complex theoretical analysis, which is associated with the determination of the deflection value. The presented model of moment-curvature relationship for the beam allows the determination of the curvature of the section and finally the deflection. Only three values of curvature and corresponding moments need to be determined related to cracking, yielding and failure of the beam, respectively. Moreover, it is necessary to take into account the slip of the plate and cracking of the reinforced concrete beam, because these effects have a significant impact on the deflection value. Despite its simplicity, the model can be successfully used in the design practice, which is confirmed by the presented verification.

References