International Conference on Computational Heat and Mass Transfer-2015

Influence of Thermal Radiation and Chemical Reaction on MHD Flow, Heat and Mass Transfer over a Stretching Surface

Manjula Jonnadula\textsuperscript{a*}, Padma Polarapu \textsuperscript{b}, Ganeswara Reddy M\textsuperscript{c}. Venakateswarlu. M\textsuperscript{d}

\textsuperscript{a}Department of Mathematics, SBIT, Khammam, Telangana State, India, Pin: 507 111
\textsuperscript{b}Department of Mathematics, S. R. Govt. Arts & Science College, Kothagudem, Telangana state, Pin: 507 101
\textsuperscript{c}Department of Mathematics, A. N. University Campus, Ongole, Andhra Pradesh, India, Pin: 523 001
\textsuperscript{d}Department of Mathematics, V. R. S. Engineering College, Vijayawada, Andhra Pradesh, India, Pin: 520 007

Abstract

In this present work, we study the flow, heat and mass transfer characteristics of an electrically conducting, incompressible and viscous fluid past a stretching porous medium. The governing partial differential equations of the flow were transformed to a system of non-linear ordinary differential equations by using similarity transformation. The system of non-linear equations is solved by using an efficient numerical technique known as Keller Box method. The effects of radiation parameter, chemical reaction parameter, thermophoresis parameter and Brownian motion parameter on longitudinal velocity, dimensionless transverse velocity, temperature, concentration, skin friction coefficient, Nusselt number and Sherwood number in the flow regime are presented graphically and discussed in detail. Comparison is made with previously published works and found to be in good agreement.

\textsuperscript{©} 2015 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

Keywords: Thermal radiation; Chemical reaction; MHD; Heat and mass transfer; stretching surface.

1. Introduction

Heat and mass transfer problems on mixed convection boundary layer flow due to a stretching surface in a saturated porous channel have received considerable interest because of its various applications in geophysics, polymer processing of a chemical engineering plant, paper production, liquid films in condensation process and energy related engineering problems that include both polymer and metal sheets. For example, it occurs in the fiber and granular insulation materials, astrophysical flows, solar power technology, aerodynamic extrusion of polymer sheets, high performance insulation buildings, the manufacturing of ceramics or glassware, food processing, packed bed chemical reactors, missiles, satellites, transpiration cooling and continuous filament extrusion from a dye. In

\textsuperscript{*} Corresponding author.

E-mail address: manjulajonnadulap@gmail.com

1877-7058 © 2015 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

Peer-review under responsibility of the organizing committee of ICCHMT – 2015
doi:10.1016/j.proeng.2015.11.489
several practical applications in the presence of two types of chemical effects namely, homogeneous and heterogeneous reactions mass transfer takes place by diffusive operations which involve the molecular diffusion of species.

A homogeneous reaction is one which is analogous to internal source of heat generation that occurs uniformly throughout a given phase, whereas a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. In the first order chemical reaction the rate of reaction is directly proportional to the concentration. The diffusive species can be absorbed or generated due to different types of chemical reaction with the ambient fluid which can greatly influenced by the properties and quality of finished products.

Anjali Devi and Ganga [1] investigated the effects of viscous and joules dissipation on MHD flow, heat and mass transfer past a stretching porous surface embedded in a porous medium. Awad et al. [2] demonstrated the thermophoresis effects on magneto-nanofluid over a stretching sheet. Aziz et al. [3] studied the effects of variable reactive index and heat generation on MHD flow over an inclined radiating plate with the temperature dependent thermal conductivity. Chakrabarti and Gupta [3] examined the hydromagnetic flow and heat transfer over a stretching sheet. Ghaly and Seddeek [4] have studied the effect of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate with temperature dependent viscosity. The effects of chemical reaction and thermal stratification on MHD flow, heat and mass transfer over a vertical stretching surface in the presence of heat source was proposed by Kandasamy et al. [5-6]. The influence of chemical reaction on heat and mass transfer by natural convection vertical surfaces in porous media was investigated by Postelnicu [7] by considering Soret and Dufour effects. Exothermic chemical reactions heat up while endothermic reactions cool down their surroundings. Reddy [8] presented the influence of thermal radiation, viscous dissipation and hall current on MHD convection flow over a stretching flat plate. Reddy et al. [9] demonstrated the effects of viscous dissipation and heat source on unsteady MHD flow over a stretching sheet. Thermal radiation and chemical reaction effects on MHD mixed convective boundary layer slip flow in a porous medium with heat source and ohmic heating is analyzed by Reddy [10]. Tak and Lodha [11] examined the effects of transverse magnetic field and viscous dissipation on flow and heat transfer over a permeable stretching surface. Yohannes and Shankar [12] proposed the influence of viscous dissipation and chemical reaction on MHD flow, heat and mass transfer of nanofluids through a porous media due to a stretching sheet. Ziami et al. [13] studied the boundary layer flow and heat transfer over a nonlinearly permeable stretching/shrinking sheet in a nonofluid.

2. Mathematical Analysis

We consider the steady, incompressible, laminar, two-dimensional boundary layer flow with heat and mass transfer of a viscous and electrically conducting fluid over a stretching porous plate embedded in a porous medium in the presence of a thermal radiation including chemical reaction. Consider a polymer sheet emerging out of a slit at $x = 0, y = 0$ and subsequently being stretched, as in a polymer extrusion process. Let us assume that the speed at a point in the plate is proportional to the power of its distance from the slit and the boundary layer approximations are applicable. The wall is kept at a constant temperature $T_w$ and concentration $C_w$, higher than the ambient temperature $T_0$ and concentration $C_0$, respectively. Also, it is assumed that there exists a homogeneous chemical reaction of first order with rate constant $K_r$ between the diffusing species and the fluid. The concentration of the diffusing species is very small in comparison to other chemical species. It is assumed that the porous medium is homogeneous and present everywhere in local thermodynamic equilibrium. The chemical reactions are taking place in the flow. The fluid has constant kinematic viscosity and thermal diffusivity and that the Boussinesq approximation may be adapted for steady laminar flow. The flow is subjected to a uniform transverse magnetic field of strength $B = B_0$ which is applied in the positive y-direction, normal to the surface in the presence of porous medium. The rest of the properties of the fluid and the porous medium are assumed to be constant. The magnetic field Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison to the applied magnetic field. The fluid is assumed to be in thermal equilibrium and no slip occurs. The Hall Effect is also neglected. The pressure gradient and external forces are neglected. The physical flow model and coordinate system is shown in Figure 1. Under the assumptions mentioned above, governing equations for the conservation of mass, momentum, and thermal energy equations by using usual boundary-layer approximations in the presence of thermal radiation, chemical reaction can be written in Cartesian coordinates $x$ and $y$. 

Mathematical Analysis

We consider the steady, incompressible, laminar, two-dimensional boundary layer flow with heat and mass transfer of a viscous and electrically conducting fluid over a stretching porous plate embedded in a porous medium in the presence of a thermal radiation including chemical reaction. Consider a polymer sheet emerging out of a slit at $x = 0, y = 0$ and subsequently being stretched, as in a polymer extrusion process. Let us assume that the speed at a point in the plate is proportional to the power of its distance from the slit and the boundary layer approximations are applicable. The wall is kept at a constant temperature $T_w$ and concentration $C_w$, higher than the ambient temperature $T_0$ and concentration $C_0$, respectively. Also, it is assumed that there exists a homogeneous chemical reaction of first order with rate constant $K_r$ between the diffusing species and the fluid. The concentration of the diffusing species is very small in comparison to other chemical species. It is assumed that the porous medium is homogeneous and present everywhere in local thermodynamic equilibrium. The chemical reactions are taking place in the flow. The fluid has constant kinematic viscosity and thermal diffusivity and that the Boussinesq approximation may be adapted for steady laminar flow. The flow is subjected to a uniform transverse magnetic field of strength $B = B_0$ which is applied in the positive $y$-direction, normal to the surface in the presence of porous medium. The rest of the properties of the fluid and the porous medium are assumed to be constant. The magnetic field Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison to the applied magnetic field. The fluid is assumed to be in thermal equilibrium and no slip occurs. The Hall Effect is also neglected. The pressure gradient and external forces are neglected. The physical flow model and coordinate system is shown in Figure 1. Under the assumptions mentioned above, governing equations for the conservation of mass, momentum, and thermal energy equations by using usual boundary-layer approximations in the presence of thermal radiation, chemical reaction can be written in Cartesian coordinates $x$ and $y$.
The corresponding boundary conditions for the velocity, temperature, and concentration fields are given as follows:

\[ \left. \begin{align*}
\mathbf{u} & = \mathbf{0}, \quad T = T_\infty, \quad C = C_\infty \\
\mathbf{u} & = \mathbf{v}, \quad T = T_w, \quad C = C_w
\end{align*} \right|_{x = 0} 
\]

Here, \( \mathbf{u} \) and \( \mathbf{v} \) are the components of the velocity field in the \( x \) and \( y \) directions, respectively, \( T \) is the temperature, and \( C \) is the concentration of the fluid.

By using the Rosseland approximation (Brewster [14]), the radiative heat flux \( q_r \) is given by

\[ q_r = -\frac{\sigma T^4}{\kappa} \]

where \( \sigma \) is the Stephan-Boltzmann constant and \( \kappa \) is the adsorption coefficient, respectively. It should be noted that by using the Rosseland approximation and the present analysis is limited to optically thick fluids. We assume that the temperature variation within the flow is small such that \( T' \) may be expressed as a linear combination of temperature. Expanding \( T' \) about \( T_\infty \) in Taylor series and neglecting higher order terms yields:

\[ T' = T_\infty - \frac{1}{4} T'' \]
In view of equations (6) and (7), equation (3) reduces to
\[ \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha [1 + R] \frac{\partial^2 T}{\partial y^2} + \frac{\gamma}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\sigma B_0^2}{\rho c_p} \right) u^2 \]  
where \( \alpha = K / \rho c_p \) is the thermal diffusivity and \( R = (16 \sigma T_e^4) / (3 \kappa K) \) is the radiation parameter.

The first, second and third terms on the RHS of Eq. (8) denote the thermal radiation, viscous dissipation and magnetic heating terms, respectively.

We introduce the following dimensionless variables [15]:

\[
\psi(x, y) = \left[ \frac{2\nu x U(x)}{1 + m} \right]^{\frac{1}{2}} F(\eta), \quad \eta = \left[ \frac{(1 + m)U(x)}{2\nu x} \right]^{\frac{1}{2}} y = \left[ \frac{(1 + m)ax^{m-1}}{2\nu x} \right]^{\frac{1}{2}} y,
\]

\[
\nu_u(x) = -\lambda \sqrt{\frac{ax(m+1)}{2}} x^{m-1}, \quad n = 2m, \phi = \frac{C - C_m}{C_w - C_m}, \theta(\eta) = \frac{T - T_0}{T_w - T_0},
\]

\[
\psi(x, y) = \left[ \frac{2\nu x U(x)}{1 + m} \right]^{\frac{1}{2}} F(\eta), \quad \eta = \left[ \frac{(1 + m)U(x)}{2\nu x} \right]^{\frac{1}{2}} y = \left[ \frac{(1 + m)ax^{m-1}}{2\nu x} \right]^{\frac{1}{2}} y,
\]

\[
\nu_u(x) = -\lambda \sqrt{\frac{ax(m+1)}{2}} x^{m-1}, \quad n = 2m, \phi = \frac{C - C_m}{C_w - C_m}, \theta(\eta) = \frac{T - T_0}{T_w - T_0},
\]

where \( \lambda > 0 \) for suction the stretching plate and \( \psi \) is the stream function.

The velocity components are given by \( u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \)  \( (10) \)

It is obvious that the equation of continuity (1) is satisfied.

From equations (2), (4) and (8), we obtain the coupled, nonlinear, dimensionless partial differential equations for momentum, energy and species conservation for the regime:

\[
F^{(1)} + FF^{(1)} - \beta F^{(1)} - (R^{(1)} + M) F^{(1)} = 0\]  
\[
(1 + R) \theta^{(1)} + Pr F \theta^{(1)} - 2\beta Pr F \theta^{(1)} = -Ec Pr \left[ F^{(1)} + M F^{(1)} \right] \]

\[
\phi^{(1)} + ScF \phi^{(1)} - 2\beta Scm \phi^{(1)} + \frac{Nt}{Nb} \phi^{(1)} - \gamma Sc \phi = 0 \quad (13)
\]

The corresponding boundary conditions can be written as

\[ F(0) = \lambda, \quad F^{(1)}(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \]

\[ F^{(1)}(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0, \quad (14) \]

where stretching parameter \( \beta = \frac{2m}{m+1} \), Magnetic parameter \( M = \frac{2\sigma B_0^2}{\rho a(m+1)x^{m-1}} \), Permeability parameter \( R_i = \frac{K_p a(m+1)}{2\nu} x^{m-1} \), Prandtl number \( Pr = \frac{\mu c_p}{K} \), Eckert number, \( Ec = \frac{a^2}{c_p T_0} \), Schmidt number \( Sc = \frac{v}{D} \)

Chemical reaction parameter \( \gamma = \frac{2K_r}{(1 + m)ax^{m-1}} \), Thermophoresis parameter \( Nt = \frac{\tau D T}{v T_w} (T_w - T_0) \), Brownian motion parameter \( Nb = \frac{\tau D}{v (C_w - C_m)} \).

Since equations (11) – (13) are highly non-linear, it is difficult to find the closed form solutions for general value of parameter \( \beta \). However, a numerical solution of these equations can be obtained when \( \beta = 1 \) (i.e. \( m = 1 \)).

It may be noted that, when \( \beta = 1 \) \( m = 1 \) or the velocity of the stretching plate is \( ax \), i.e. the plate stretches with a velocity varying linearly with distance. In this case, the equations (11), (12) and (13) are reduced to
\[ F^{11} + FF^{11} - F^{11} = 0 \]  
\[ (1 + R) \theta^{11} + Pr F \theta' - 2 Pr F \theta'' = -Ec Pr \left[ F^{11} + MF' \right] \]  
\[ \phi^{11} + ScF \phi' - 2Sc \phi F' + \frac{Nt}{Nb} \theta^{11} - \gamma \phi F' = 0 \]

with boundary conditions given in (14).

Now, we are interested to study the quantities of practical interest, the skin friction coefficient \( C_f \) and the local Nusselt number \( Nu \) and Sherwood number \( Sh \). The parameters respectively, characterize the surface drag, wall heat transfer rate and mass transfer rate. These quantities are defined by:

\[ C_f = \frac{2 \tau_w}{\rho U^2}, \quad Nu = \frac{q_w x}{k(T_w - T_\infty)}, \quad Sh = \frac{q_w x}{k(C_w - C_\infty)} \]  
(18)

where \( \tau_w = \mu \frac{\partial u}{\partial y} \), \( q_w = (1 + R) K \frac{\partial T}{\partial y} \), \( q_w = D_w \frac{\partial C}{\partial y} \) at \( y = 0 \).

Using (18) and (19), the dimensionless skin friction coefficient, wall heat and mass transfer rates become:

\[ C_f = \sqrt{\frac{2(m+1)}{Re_x}} F^{11}(0), \quad Nu = -(1 + R) \sqrt{\frac{(m+1)Re_x}{2}} \theta'(0), \quad Sh = \sqrt{\frac{(m+1)Re_x}{2}} \phi'(0) \]  
(20)

where \( Re_x = \frac{\nu_x(x)}{v} \) is the local Reynolds number.

The associated expression of dimensionless skin friction coefficient \( C_f \), reduced Nusselt number \( Nu \), and reduced Sherwood number \( Sh \) are defined as:

\[ C_f = C_f \sqrt{\frac{Re_x}{2(m+1)}}, \quad Nu = \frac{Nu}{(1 + R) \sqrt{\frac{2}{(m+1)Re_x}}}, \quad Sh = Sh \sqrt{\frac{2}{(m+1)Re_x}} \]  
(21)

3. Numerical Solution

Since equations (11) – (13) are highly non-linear, it is difficult to find the closed form solutions. Thus, the solutions of these equations with the boundary conditions (14) are solved numerically using the Keller-box method [10]. The convergence of the method depends on the choice of the initial guesses. The following initial guesses are chosen: \( f_o(\eta) = 1 - e^{-\eta}, \quad \theta_o(\eta) = e^{-\eta}, \quad \phi_o(\eta) = e^{-\eta} \)  
(22)

The choices of the initial guesses depend on the convergence criteria and the wall shear stress \( f^{11}(0) \) is commonly used as a convergence criterion because in the boundary layer flow calculations the greatest error appears in the wall shear stress parameter as it is explained in Cebeci and Bradshaw [16]. Thus, we used this convergence criterion in the present study. A uniform grid of size 0.01 is chosen to satisfy the convergence criterion of \( 10^{-7} \) in our study, which gives about six decimal places accurate to most of the prescribed quantities. From the process of numerical computation, the skin friction coefficients, reduced Nusselt number and reduced Sherwood number are presented in graphs.

4. Results And Discussion

In the solutions, the effects of thermal radiation and chemical reaction on heat and mass transfer characteristics through a stretching porous surface on MHD flow are considered. The transformed non-linear ordinary differential equations (15)–(17) subject to the boundary conditions (14) were solved numerically using Keller box method as described in Cebeci and Bradshaw [17]. Velocity, temperature and concentration profiles were obtained and we applied the results to compute the skin friction coefficient, the Nusselt number and local Sherwood number in equation (21). The results were discussed for the different values of the parameters graphically. The effects of various governing parameters on the skin friction coefficient \( F^{11}(0) \), Nusselt number \( -\theta'(0) \), Sherwood number \( -\phi'(0) \) are shown in the graphs. It is observed that increasing the values of \( M \) results in a decrease in the skin friction coefficient. The effect of suction is to decrease the skin friction coefficient. The rate of heat transfer \( \theta'(0) \) enhances for development in both magnetic parameter and Eckert number. The rate of heat transfer \( \theta'(0) \) of
the fluid decreases with rise of suction parameter. It is clear that increase in Pr decrease the rate of heat transfer \( \theta^i (0) \). In the investigation following default parameter values are adopted for computations: \( M = 2, \lambda = 1, R = 0.5, \Pr = 0.71, Nt = Nb = Sc = \gamma = 0.1, R_i = 100, Ec = 0.2 \). All graphs therefore correspond to these values unless specially indicated on the appropriate graph.

Figure 2 reveals the influence of magnetic parameter \( M \) on dimensionless transverse velocity. It is seen that the velocity increases steadily and then converges closely for transverse velocity. But it is noticed that an increase in magnetic parameter \( M \) decreases the dimensionless transverse velocity. This is because the application of transverse magnetic field in an electrically conducting fluid produces a retarding Lorentz force. The force slows down the motion of the fluid in the boundary layer and hence reduces velocity at the expense of increasing its temperature. The radiation parameter \( R \) is responsible to the thickening of the thermal boundary. This enables the fluid to release the heat energy from the flow region and causes the structure to cool. This is true because the Rosseland approximation results in an increase in temperature. It is observed that the temperature increases as thermal radiation parameter increases, which clear from Figure 3. Figures 4 – 6 depict effects of Chemical reaction parameter, thermophoresis parameter and Brownian motion parameter on the concentration of the flow field. The Chemical reaction parameter and Brownian motion parameter are found to decrease the concentration of the flow field at all points. Whereas a reverse trend is observed with increasing the values of thermophoresis parameter \( Nt \) which is clear from Figure 6. The effects of the magnetic parameter \( M \) on skin friction coefficient are shown in Figure 7. This Figure indicates a decrease in the values of skin friction coefficient with the increase magnetic parameter. The rate of heat transfer \( \theta^i (0) \) for various values of pertinent parameters is illustrated in Figure 8. It is observed from this Figure that the effect of increasing the suction parameter is to decrease rate of heat transfer. Figure 9 depicts the influence of magnetic parameter on the rate of heat transfer \( \theta^i (0) \) and the rate of mass transfer \( \phi^i (0) \). It is noticed that the increase of \( M \) increases the rate of heat transfer \( \theta^i (0) \) whereas an opposite trend is observed for the rate of mass transfer. The rate of heat transfer \( \theta^i (0) \) and the rate of mass transfer \( \phi^i (0) \) for different values of Prandtl number \( Pr \) are shown in Figure 10. It is found that the rate of heat transfer decreases for increase of Prandtl number, whereas an opposite trend is seen for the rate of mass transfer. Figures 11 – 16 elucidate the impact of various physical parameters on the rate of mass transfer \( \phi^i (0) \). It is found that an increase in the values of Thermophoresis parameter \( Nt \) Prandtl number and suction parameter decreases the rate of mass transfer \( \phi^i (0) \) as depicted in Figures 11- 14, whereas an opposite trend is observed with increasing the values of Brownian motion parameter and Thermophoresis parameter \( Nt \) which is clear from Figures 15 & 16. The effect of increasing \( Nb \) is to increases the rate of mass transfer \( \phi^i (0) \) which is clear from Figure 16.

![Figure 2 Non-dimensional transverse velocity profiles for different values of M](image2.png)

![Figure 3 Effect of on temperature profiles.](image3.png)

![Figure 4 Dimensionless concentration distribution for various values of γ](image4.png)
Figure 5 Effect of $Nb$ on concentration profiles.

Figure 6 Effect of $Nt$ on concentration profiles.

Figure 7 Influence of $M$ on Skin Friction Coefficient.

Figure 8 Influence of $\lambda$ on rate of heat transfer.

Figure 9 Influence of $M$ over rate of heat transfer and rate of mass transfer.

Figure 10 Influence of $Pr$ over rate of heat transfer and rate of mass transfer.

Figure 11 Influence of $Nt$ over rate of mass transfer.

Figure 12 Influence of $Pr$ on rate of mass transfer.

Figure 13 Influence of $\lambda$ over rate of rate of mass transfer.

Figure 14 Influence of $Pr$ over rate of mass transfer

Figure 15 Influence of $Nt$ on rate of mass transfer.

Figure 16 Influence of $Nb$ over rate of mass transfer.
5. CONCLUSION

In the present paper a boundary layer analysis to study the combined effects of thermal radiation and chemical reaction on MHD flow past a stretching porous surface embedded in a porous medium is presented. The governing equations associated to the boundary conditions were transformed to a two point non – linear ordinary differential equations with the help of similarity transformation equations. The solutions of the problem were numerically solved with the help of Keller box method. The expressions for the Skin fiction heat transfer and mass transfer rates are also derived. The following results were summarized as follows:

- The velocity profile of the dimensionless transverse velocity decreases with an increase in magnetic parameter.
- The temperature increases with increase in the values of radiation parameter
- The concentration profile increases with increase in the values of values of thermophoresis parameter where as an opposite trend is observed with increase in the values of chemical reaction parameter, Brownian motion parameter
- The Skin Friction coefficient decreases with increasing suction parameter.
- The rate of heat transfer $\theta'(0)$ increases with increasing $M$ where as an opposite trend is seen with increase in the values of $\alpha, Pr$.
- The rate of mass transfer $\phi'(0)$ increases with increasing the values of $Nt, Nb$ and decreases with increase in the values of $M, Pr, \lambda$.

References