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ORIGINAL ARTICLE

Analysis of simultaneous unbalanced short circuit and open conductor faults in power systems with untransposed lines and six-phase sections

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Transient stability

Abstract The paper introduces a generalized method for analysis of multiple, simultaneous short circuit, open circuit, and open circuit falling conductor faults in mixed three-phase and six-phase power systems with untransposed lines. The method is systematic and suitable for all types of faults, any number of simultaneous faults, and any number of phases. Calculation of all network unbalanced voltages and currents during faults is done in one straightforward step. Coupling among sequence networks in untransposed transmission lines is accounted for. Coupling between the three-sequence networks of the three-phase part and the six-sequence networks of six-phase part is also derived. The method is applied also for transient stability study of mixed three-phase and six-phase power systems during any type of faults. Detailed derivation of the governing equations in each part is presented. Simulation results on the IEEE 300-bus system and the IEEE 30-bus system are given to validate the proposed method.

1. Introduction

Analyzing abnormal conditions in power systems, such as short circuit, and open conductor faults, is important for protection system design and transient stability assessment. Fault analysis is well studied in the literature but only few publications have discussed the analysis of complex simultaneous faults such as multiple faults at different busses, open circuit at multiple branches, complex and unusual phase combination involved in the fault (more than one type of fault at the same bus), cross-country faults, faults in mixed three-phase/higher-order-phase networks, effect of transformers phase shift, and untransposed transmission networks. Multiple simultaneous faults can occur in natural catastrophes, stormy weather and intended attacks.

In [1–5], fault analysis is done in phase coordinates using the three-phase bus admittance matrix. Individual Faults in unbalanced distribution systems are analyzed using three-phase bus impedance matrix in [6,7]. When forming the bus impedance matrix Z_{bus} in phase coordinates for unbalanced multi-phase power systems, each single-phase is considered a bus. So, in three-phase case for example, the size of Z_{bus} will be $3N{*}3N$; where N is the number of three-phase buses. The size will also increase for higher order phases. Accommodating multiple faults at multiple locations in the network in bus

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impedance matrix constitutes a large computational burden. In [8], fault points can be added in modular form but this requires inversion of large augmented matrix containing full admittance matrix is required. Also, this method is suitable for shunt faults only.

Reduced bus impedance matrix in the form of thevenin impedances and thevenin voltages to represent the other parts of the network requires the full Z_{bus} prefault full matrix and the reduction itself is dependent on fault locations.

On the other hand, in symmetrical component reference frame, sequence bus admittance matrices are constants and only the connection between them is altered in the sequence networks according to the fault type. But handling multiple faults is a complex task and in some cases the sequence admittance matrices become coupled. It is well known that untransposed transmission lines can only be decoupled into sequence networks using shunt compensating current sources at the beginning and end of the lines [9,10]. These injected currents introduces coupling between sequence networks and this complicates to a great extent the short circuit analysis in symmetrical components coordinates. Most of commercial short circuit packages have neglected this effect. In [11,12], simultaneous short circuit faults at two different buses have been analyzed using two-port network theory.

The use of six-phase or higher-order-phase transmission lines aims to transmit more power in the same right-of-way and without increasing the voltage level. Only few publications have concerned with fault analyses in six-phase networks such as [13–16] where only basic types of six-phase single faults have been discussed using circuit combinations of the 6-sequence networks.

In this paper, a proposed method of fault analysis in symmetrical components coordinates is presented for networks with mixed three-phase and six-phase parts. The method can accommodate any number of short circuit and open conductor faults at the same time at any number of buses and branches with any phase combination. The addition of fault points/lines is done in modular form in a systematic way and only selected part of admittance matrix is used in the augmented matrix. Untransposed line sections and transformer phase shift are inherently accounted for in the proposed method. Coupling between sequence networks of three-phase and six-phase parts is derived. The use of the proposed method in transient stability analysis is demonstrated.

The paper is organized as follows. Section 2 gives a detailed formulation of the proposed method. Short circuit faults are discussed in Section 3 and open conductor faults are discussed in Section 4. Treatment of untransposed lines is presented in Section 5 while the coupling between three-phase and six-phase sequence networks is investigated in Section 6. The overall network solution is given in Section 7. The use of the proposed method in transient stability analysis is demonstrated in Section 8. Simulation results are given in Section 9, and finally, conclusions are extracted at Section 10.

2. Network modeling

In the proposed method, each branch is represented as shown in Fig. 1. Each branch between bus i and bus j is represented by series impedance $Z_k = R_k + jX_k$, where R_k is branch resistance and X_k is branch reactance, a voltage source E_k in the same direction of current to model the generators, and a voltage source V_k with polarity that opposes current direction to model the abnormal or special condition in which the branch is involved if exists such as short circuit, open circuit, and coupling with other branches. A transformer with complex turns ratio t_k is introduced in Fig. 1 to model the 30° phase shift in Δ/Y transformers in positive and negative sequence networks. For the positive sequence network:

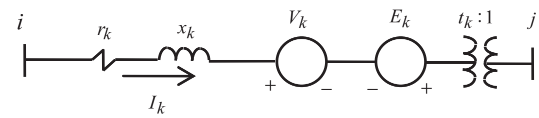

Figure 1 The network branch model.

$$t_k = \begin{cases} 1 & \text{If branch } k \text{ is a line} \\ 1\angle+30^\circ & \text{If branch } k \text{ is a } \Delta/\text{Y} \\ & \quad \text{transformer with Vector Group DY1} \\ 1\angle-30^\circ & \text{If branch } k \text{ is a } \Delta/\text{Y} \\ & \quad \text{transformer with Vector Group DY11} \end{cases} \tag{1}$$

The angle sign is reversed in negative sequence network and the turns-ratio is one in zero sequence network. The branch current is given by

$$I_k = \frac{1}{Z_k}(V_i - t_k V_j + E_k - V_k) \tag{2}$$

The E_k voltage sources exist only in generators' branches which are connected between generators' buses and ground. Line shunt admittances and load admittances are included in added branches between corresponding buses and ground.

For a network with n_{br} branches and n_{bus} buses, Eq. (2) can be written in the form

$$I_{br} = Z_{br}^{-1}(AV_{bus} + E_{br} - V_{br}) \tag{3}$$

where

$$I_{br} = [\,I_1 \quad \cdots \quad I_{nbr}\,]^T, \quad E_{br} = [\,E_1 \quad \cdots \quad E_{nbr}\,]^T,$$

$$V_{br} = [\,V_1 \quad \cdots \quad V_{nbr}\,]^T, \quad V_{bus} = [\,V_1 \quad \cdots \quad V_{nbus}\,]^T,$$

Z_{br} is a diagonal matrix with $Z_{br-kk} = Z_k$, and A is a modified branch to node incidence matrix with

$$A_{ki} = \begin{cases} 1 & \text{If branch } k \text{ starts at node } i \\ -t_k & \text{If branch } k \text{ ends at node } i \\ 0 & \text{Otherwise} \end{cases} \tag{4}$$

Multiplying both sides of Eq. (3) byA^{*T}, the conjugate transpose of matrix A, gives:

$$A^{*T}I_{br} = A^{*T}Z_{br}^{-1}AV_{bus} + A^{*T}Z_{br}^{-1}E_{br} - A^{*T}Z_{br}^{-1}V_{br} \tag{5}$$

The bus admittance matrix is given by [5]:

$$Y_{bus} = A^{*T}Z_{br}^{-1}A \tag{6}$$

And from KCL,

$$J_{bus} = A^{*T}I_{br} \tag{7}$$

where J_{bus} is a vector of nodal external current sources.

So, Eq. (5) becomes

$$J_{bus} = Y_{bus}V_{bus} + A^{*T}Z_{br}^{-1}E_{br} - A^{*T}Z_{br}^{-1}V_{br} \tag{8}$$

The bus impedance matrix is given by

$$Z_{bus} = Y_{bus}^{-1} \tag{9}$$

From (8), the bus voltages can be calculated as follows

$$V_{bus} = Z_{bus}(J_{bus} - A^{*T}Z_{br}^{-1}E_{br} + A^{*T}Z_{br}^{-1}V_{br}) \tag{10}$$

Branch admittance matrix is defined as

$$Y_{br} = Z_{br}^{-1} \tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (3) gives

$$I_{br} = (Y_{br} - Y_{br}AZ_{bus}A^{*T}Y_{br})(E_{br} - V_{br}) + Y_{br}AZ_{bus}J_{bus} \tag{12}$$

It is assumed that no external sources other than the E.M.F voltage sources exist. So that

$$J_{bus} = 0 \tag{13}$$

Defining

$$Y = (Y_{br} - Y_{br}AZ_{bus}A^{*T}Y_{br}) \tag{14}$$

Eq. (12) becomes

$$I_{br} + YV_{br} = YE_{br} \tag{15}$$

As only short circuit branches, open circuit branches, and coupling branches will have internal voltages V_k, Eq. (15) will be written only for those special branches as follows

$$I_{br-red} + Y_{red}V_{br} = Y_{red}E_{br} \tag{16}$$

where Y_{red} is the Y matrix with only rows corresponding to the special branches present and other rows omitted. It should be noted here that Eq. (15) still can be used to calculate the current at the other branches after solution of special branches internal voltages.

Eq. (16) is written for each sequence network s as follows:

$$I_{br-red-s} + Y_{red-s}V_{br-s} = Y_{red-s}E_{br-s} \tag{17}$$

where $s = 0, \ldots, n_s - 1$ and n_s is the number of sequence networks which is three for three-phase and six for six-phase networks.

The number of special branches in three-phase network, m_3 and in six-phase network, m_6 are given by

$$m_3 = n_{3sc} + n_{3oc} + 2n_{3ut} + n_{3cp} \tag{18}$$

$$m_6 = n_{6sc} + n_{6oc} + 2n_{6ut} + n_{6cp} \tag{19}$$

where n_{3sc}, n_{6sc} are number of short circuit branches in three-phase and six-phase networks, respectively, n_{3oc}, n_{6oc} are number of open circuit branches in three-phase and six-phase networks, respectively, n_{3ut}, n_{6ut} are number of untransposed line branches in three-phase and six-phase networks, respectively, and n_{3cp}, n_{6cp} are number of coupling branches between three-phase and six-phase sequence networks in the three-phase side and six-phase side, respectively.

If Eq. (17) is written for the 3-sequence networks of the three-phase part of the network and for the 6-sequence networks of the six-phase part of the network, we have then $3m_3 + 6m_6$ complex equations in $6m_3 + 12m_6$ complex unknowns which are the special branches currents and internal voltages. The other equations will be given according to the type of each special branch as shown in the following sections.

It should be emphasized here that the advantage of this formulation is that it gives direct relations between network branch currents and branch internal voltages which is used in conjunction with specific fault or abnormal condition to solve for network currents and voltages. It should be also noted that there is no need, in the proposed method, to calculate pre-fault voltages or currents or Thévenin equivalents at faulted buses and branches. The only need is to calculate the generators' internal E.MFs.

3. Short circuit branches

Short circuit at a certain bus is simulated by adding a special branch between the faulted bus and ground in all sequence networks. The equations of any fault branch in phase coordinates can be put in the form

$$B_3V_{3p} - C_3I_{3p} - F_3V_{3p} = O_{3\times1} \tag{20}$$

where V_{3p}, I_{3p} are vectors of three-phase phase-to-ground voltages and currents, respectively, $O_{3\times1}$ is a zero column vector with size 3×1, and the matrices B_3, C_3, F_3 depend on the phase combination of the fault. Similarly, in six-phase networks

$$B_6V_{6p} - C_6I_{6p} - F_6V_{6p} = O_{6\times1} \tag{21}$$

As an example, for a six-phase fault at a bus as shown in Fig. 2 where phase a is shorted to ground through impedance Z_a, phases b, d are shorted together through impedance Z_{bd}, and phases c, e are shorted together to ground through impedances Z_{ce-g}, Z_{cx}, and Z_{ex} while phase f is unfaulted, the following equations can be given

$$\begin{aligned} &V_a = Z_aI_a, \quad V_b = Z_{bd}I_b + V_d, \quad I_b + I_d = 0 \\ &V_c = Z_{cx}I_c + Z_{ce-g}(I_c + I_e), \\ &V_e = Z_{ce-g}(I_c + I_e) + Z_{ex}I_e, \quad I_f = 0 \end{aligned} \tag{22}$$

$$\begin{aligned} &\begin{bmatrix} 1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&0&0&0\\0&0&0&0&1&0\\0&0&0&0&0&0 \end{bmatrix}\begin{bmatrix}V_a\\V_b\\V_c\\V_d\\V_e\\V_f\end{bmatrix} \\ &- \begin{bmatrix} Z_a&0&0&0&0&0\\0&Z_{bd}&0&0&0&0\\0&0&Z_{ce-g}+Z_{cx}&0&Z_{ce-g}&0\\0&1&0&1&0&0\\0&0&Z_{ce-g}&0&Z_{ce-g}+Z_{ex}&0\\0&0&0&0&0&1 \end{bmatrix}\begin{bmatrix}I_a\\I_b\\I_c\\I_d\\I_e\\I_f\end{bmatrix} \\ &- \begin{bmatrix} 0&0&0&0&0&0\\0&0&0&1&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0 \end{bmatrix}\begin{bmatrix}V_a\\V_b\\V_c\\V_d\\V_e\\V_f\end{bmatrix} = O_{6x1} \end{aligned} \tag{23}$$

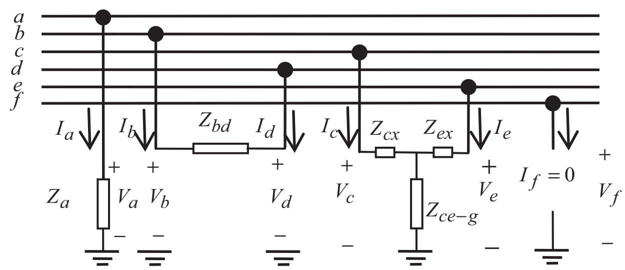

Figure 2 Phase combination of the six-phase fault.

Eq. (22) can be put in the form (21) as shown in (23) at the bottom of the next page. Transforming phase voltages and currents in Eq. (20) to sequence coordinates gives

$$(B_3 - F_3)T_3V_{3s} - C_3T_3I_{3s} = O_{3\times 1} \tag{24}$$

where V_{3s}, I_{3s} are the vectors of sequence internal voltages and currents, respectively, and T_3 is the three-phase transformation matrix between phase and sequence coordinates and it is given by

$$T_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{25}$$

where $a = 1\angle 120^\circ, I_{3p} = T_3I_{3s}, I_{3p} = [I_a \quad I_b \quad I_c]^T$, $I_{3s} = [I_0 \quad I_1 \quad I_2]^T$. I_0, I_1, and I_2 are zero, positive, and negative sequence currents, respectively. Similarly, for six-phase network

$$(B_6 - F_6)T_6V_{6s} - C_6T_6I_{6s} = O_{6\times 1} \tag{26}$$

where T_6 is the six-phase transformation matrix between phase and sequence coordinates [13] and is given by

$$T_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & b^5 & b^4 & b^3 & b^2 & b \\ 1 & b^4 & b^2 & 1 & b^4 & b^2 \\ 1 & b^3 & 1 & b^3 & 1 & b^3 \\ 1 & b^2 & b^4 & 1 & b^2 & b^4 \\ 1 & b & b^2 & b^3 & b^4 & b^5 \end{bmatrix} \tag{27}$$

where $b = -a^2$. Eq. (26) gives the relation between the six-phase sequence voltages and currents of the short circuit branches shown in Fig. 3.

For cross-country faults, i.e. faults including phases of different buses, the size of matrices V_{6p}, I_{6p} will increase as they will include six-phase currents and six-phase voltages for each involved bus in the fault. The size of the matrices $B_6, F_6, C_6, O_{6\times 1}$ will also increase accordingly.

4. Open conductor faults

The equations of any open branch in phase variables can be formulated as follows

$$B_3V_{3p} - C_3I_{3p} = O_{3\times 1} \tag{28}$$

where the matrices B_3, C_3 depend on the phase combination of the open fault. Similarly, in six-phase networks

$$B_6V_{6p} - C_6I_{6p} = O_{6\times 1} \tag{29}$$

As an example, for a six-phase open conductor fault at a branch where phase a is not open, phase b, c, e, f conductors are opened through impedances Z_b, Z_c, Z_e, Z_f, respectively, while phase d is opened through infinite impedance, the following equations can be given

$$V_a = 0, \quad V_b = Z_bI_b, \quad V_c = Z_cI_c, \quad I_d = 0,$$
$$V_e = Z_eI_e \quad V_f = Z_fI_f \tag{30}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_b & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_c & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_e & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_f \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \\ I_e \\ I_f \end{bmatrix} = O_{6\times 1} \tag{31}$$

Transforming Eqs. (28) and (29) to sequence coordinates gives

$$B_3T_3V_{3s} - C_3T_3I_{3s} = O_{3\times 1} \tag{32}$$

$$B_6T_6V_{6s} - C_6T_6I_{6s} = O_{6\times 1} \tag{33}$$

which gives the required relations between sequence internal voltages and currents in both three-phase and six-phase networks.

For open branch falling conductor faults, the original branch should be divided into two branches with an added internal bus between them as shown in Fig. 4 for the three-phase case. One of the two branches has almost zero impedance and the other will have the whole branch impedance. The open branch falling conductor fault is then simulated by simultaneous open conductor fault at the zero impedance branch and short circuit at the fictitious internal added bus.

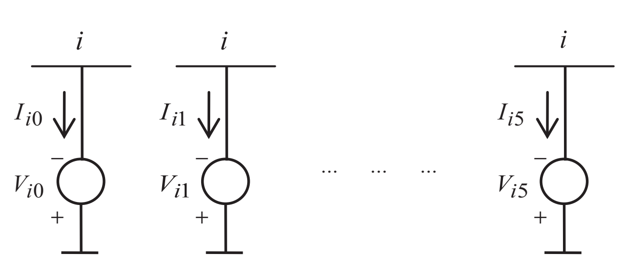

Figure 3 Sequence fault currents and voltages.

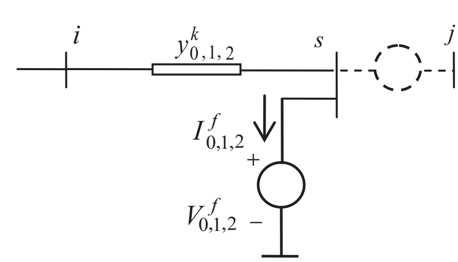

Figure 4 Treatment of open branch falling conductor fault.

5. Untransposed transmission lines

When transmission lines are not transposed, the mutual impedances between phases are unequal, this leads to a sequence impedance matrix which is not diagonal and so, the sequence networks are coupled. The sequence series admittance matrix of a transmission line branch k and the sequence half shunt admittance matrix at the beginning and end of the line are in the form of Eq. (34).

$$Y^{kz}_{012} = \begin{bmatrix} y^{kz}_{00} & y^{kz}_{01} & y^{kz}_{02} \\ y^{kz}_{10} & y^{kz}_{11} & y^{kz}_{12} \\ y^{kz}_{20} & y^{kz}_{21} & y^{kz}_{22} \end{bmatrix}, \quad \frac{Y^{ks}_{012}}{2} = \begin{bmatrix} y^{ks}_{00} & y^{ks}_{01} & y^{ks}_{02} \\ y^{ks}_{10} & y^{ks}_{11} & y^{ks}_{12} \\ y^{ks}_{20} & y^{ks}_{21} & y^{ks}_{22} \end{bmatrix} \tag{34}$$

In order to use only diagonal impedance elements in Y^{kz}_{012}, and $Y^{ks}_{012}/2$ matrices, i.e. decoupled sequence networks, compensating currents must be injected at both ends of the line in all sequence networks [9]. The three-phase case is shown in Fig. 5 where the relation between injected sequence currents and sequence node voltages is given by

$$\begin{bmatrix} I^k_{i0} \\ I^k_{j0} \\ I^k_{i1} \\ I^k_{j1} \\ I^k_{i2} \\ I^k_{j2} \end{bmatrix} + \begin{bmatrix} 0 & 0 & y^{kz}_{01}+y^{ks}_{01} & -y^{kz}_{01} & y^{kz}_{02}+y^{ks}_{02} & -y^{kz}_{02} \\ 0 & 0 & -y^{kz}_{01} & y^{kz}_{01}+y^{ks}_{01} & -y^{kz}_{01} & y^{kz}_{02}+y^{ks}_{02} \\ y^{kz}_{10}+y^{ks}_{10} & -y^{kz}_{10} & 0 & 0 & y^{kz}_{12}+y^{ks}_{12} & -y^{kz}_{12} \\ -y^{kz}_{10} & y^{kz}_{10}+y^{ks}_{10} & 0 & 0 & -y^{kz}_{12} & y^{kz}_{12}+y^{ks}_{12} \\ y^{kz}_{20}+y^{ks}_{20} & -y^{kz}_{20} & y^{kz}_{21}+y^{ks}_{21} & -y^{kz}_{21} & 0 & 0 \\ -y^{kz}_{20} & y^{kz}_{20}+y^{ks}_{20} & -y^{kz}_{21} & y^{kz}_{21}+y^{ks}_{21} & 0 & 0 \end{bmatrix} \times \begin{bmatrix} V^k_{i0} \\ V^k_{j0} \\ V^k_{i1} \\ V^k_{j1} \\ V^k_{i2} \\ V^k_{j2} \end{bmatrix} = O_{6\times 1} \tag{35}$$

For six-phase and any number of phases, the elements of the admittance matrix of the untransposed line k; Y^k are

$$Y^k_{iu,jv} = \begin{cases} 0, & u = v \\ -y^{kz}_{uv}, & u \neq v \end{cases}$$
$$Y^k_{iu,iv} = Y^k_{ju,jv} = \begin{cases} 0, & u = v \\ y^{kz}_{uv} + y^{ks}_{uv}, & u \neq v \end{cases} \tag{36}$$
$$u, v = 0, 1, \ldots, n_s - 1$$

6. Coupling between three-phase and six-phase sequence networks

Coupling between three-phase and six-phase sequence network will be derived based on transformer connections between three-phase and six-phase parts of the network. One connection is given in the following as an example for illustration in Fig. 6 where a Δ connection is used in the three-phase side and a center-tap connection is used in the six-phase side and the center-tap points are connected together to form the six-phase neutral point. From figure, it can be shown that

$$V_{6p} = nG_1G_2V_{3p} \tag{37}$$

where V_{6p}, V_{3p} are six-phase and three-phase per unit phase-to-ground voltages, respectively. G_1 gives the transformer transfer matrix, and G_2 gives the transformation between primary per unit phase-to-ground voltages and per unit phase-to-phase voltages; namely

$$\begin{bmatrix} V_{6a} \\ V_{6b} \\ V_{6c} \\ V_{6d} \\ V_{6e} \\ V_{6f} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_{3a} \\ V_{3b} \\ V_{3c} \end{bmatrix} \tag{38}$$

Transferring (38) into sequence coordinates gives

$$T_6V_{6s} - \frac{1}{\sqrt{3}}G_1G_2T_3V_{3s} = O_{6\times 1} \tag{39}$$

The current relation is

$$I_{3p} = -\frac{1}{2\sqrt{3}}G_2^TG_1^TI_{6p} \tag{40}$$

where I_{6p}, I_{3p} are six-phase and three-phase per unit line currents, respectively. Transforming (40) to sequence coordinates gives

$$T_3I_{3s} + \frac{1}{2\sqrt{3}}G_1^TG_2^TT_6I_{6s} = O_{3\times 1} \tag{41}$$

Eqs. (39) and (41) give the relation between the currents and voltages of three special coupling branches in the 3-sequence networks at the three-phase side and six special coupling branches in the 6-sequence networks at the six-phase side as shown in Fig. 7. This connection makes the following relations between base phase-to-ground voltage and base current of three-phase and six-phase networks

$$V_{base-pg-6ph} = \frac{\sqrt{3}}{n}V_{base-pg-3ph} \tag{42}$$

$$I_{base-6ph} = \frac{n}{2\sqrt{3}}I_{base-3ph} \tag{43}$$

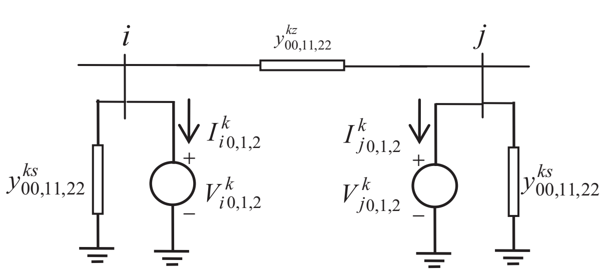

Figure 5 Sequence networks of three-phase untransposed line.

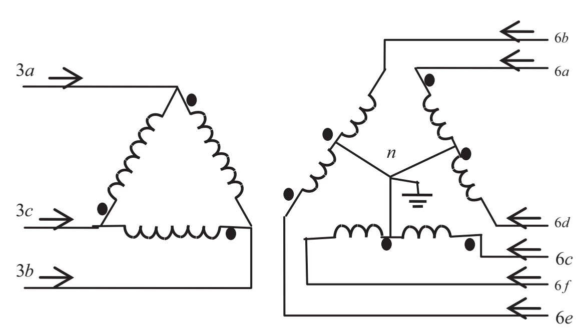

Figure 6 Type 1 three/six phase connection.

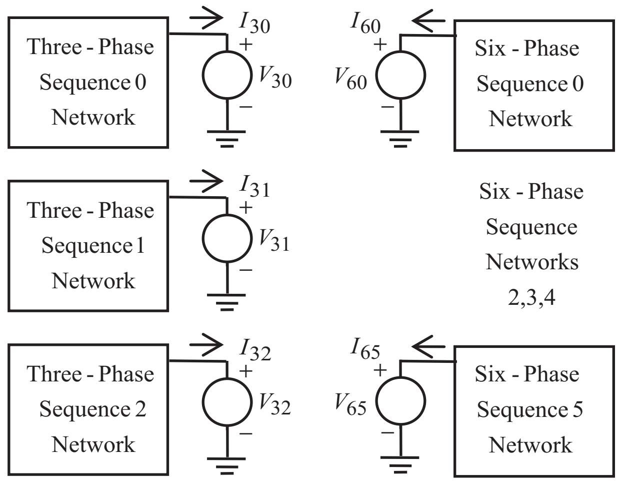

Figure 7 Coupling between three-phase and six-phase sequence networks.

where n is the ratio between primary turns and half the secondary turns. Inclusion of coupling transformer leakage impedance is also straightforward but is not given here due to lack of space.

7. The overall network solution

For both three-phase and six-phase parts, Eq. (17), for special branches currents for all sequences, (24) and (26), for each short circuit branch, (32) and (33), for each open fault branch, (35), for each untransposed transmission line, (39) and (41) for each pair of coupling branches between three-phase and six-phase parts, are solved together for currents and voltages of special branches. The total number of complex equations is $6m_3 + 12m_6$ which is equal to number of complex unknowns. After that, the current of any branch can be evaluated by Eq. (15).

8. Transient stability analysis

Using Eq. (15) the currents at the generators' branches can be calculated as

$$I_{gbr} + Y_{redg} V_{br} = Y_{redg} E_{br} \tag{44}$$

where I_{gbr} is the vector of currents at the generators' branches and Y_{redg} is the matrix Y in (14) with only rows corresponding to generators' branches present and other rows omitted. The active output power of generator i is given by

$$P_{gi} = \mathrm{Real}(E_{gbr-i} I^{*}_{gbr-i}) \tag{45}$$

And the system dynamic equations are given by

$$\frac{h_{gi}}{\pi f_0} \frac{d\Delta\omega_{gi}}{dt} = P_{mgi} - r_{gi}\Delta\omega_{gi} - P_{gi} - d_{gi}\Delta\omega_{gi} \tag{46}$$

$$\frac{d\delta_{gi}}{dt} = \Delta\omega_{gi} \tag{47}$$

$$\omega_{gi} = \omega_0 + \Delta\omega_{gi} \tag{48}$$

where, for generator i, h_{gi} is the per unit generator inertia constant, and f_0 is the system nominal frequency, P_{mgi} is the per unit mechanical input power, $\Delta\omega_{gi}$ is the generator angular speed deviation from nominal speed, d_{gi} is the generator damping coefficient, ω_0 is the synchronous frame angular speed and δ_{gi} is the angle of the generator internal E.M.F with respect to the voltage of the slack bus of the power flow solution. Generator internal voltages are calculated after pre-fault power flow solution and loads are converted to constant impedance and are modeled as added branches between load buses and ground.

9. Applications and results

The proposed method is applied for analysis of the IEEE 300-bus system [17] during simultaneous unbalanced open circuit, and short circuit faults. The system contains 69 generators and three subsystems. The following modifications are applied to the system.

1- All generator transformers in subsystem 3 are assumed Δ/Grounded Y transformers with vector group DY1. All other transformers are assumed star-earthed at both terminals.
2- Generators' positive, negative, and zero sequence reactances are: $X_{g1} = X_{g2} = 0.1$ pu, and $X_{g0} = 0.05$ pu. All generators have solidly-earthed star connections.
3- For transmission lines, negative sequence reactances are assumed equal to positive sequence ones, while zero sequence values are twice the values of positive sequence ones.
4- Five three-phase transmission lines are assumed to be untransposed and they are given in Table 1. The sequence impedance matrices of these lines will have off-diagonal elements which are assumed to be half the positive sequence impedances.
5- Transformer reactances are assumed equal in all sequences.
6- A six-phase sub-network containing two transmission lines and 3 buses, shown in Fig. 8 is assumed to be connected between bus 81, and bus 188, connecting sub-networks 2, and 3.Without loss of generality, the lines are assumed well-transposed and their sequence reactances are given in Table 2. Each of the 3 buses is assumed to have a load of $50 + j30$ MVA. The two transformers connecting three-Phase and six-phase networks are assumed to be of the connection shown in Fig. 6.

Table 1 Un-transposed branches.

(195–219)	(194–219)	(212–215)
(134–135)	(155–161)	

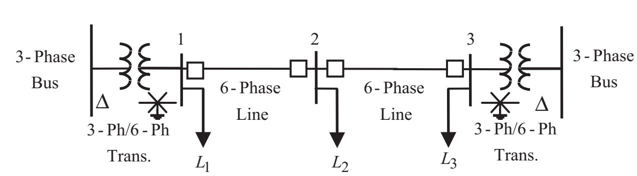

Figure 8 The six-phase sub-network.

Table 2 Six-phase lines parameters.

$Z_s = 0.1$ pu $Z_m = 0.05$ pu
$Z_0 = Z_s + 5Z_m$
$Z_i = Z_s - Z_m,\ i = 1, \ldots, 5$

Pre-fault generators' E.M.F are obtained after power flow solution of the system taking into account the phase shift introduced by Δ/Y transformers. For power flow solution, the six-phase sub-network is represented by three-phase equivalent of six-phase lines and loads [5].

17 simultaneous abnormal conditions are simulated in the system. These include 8 open circuit faults in three-phase sub-network with 2 of them falling to ground, 8 short circuit faults in three-phase sub-network, and one short circuit fault in the six-phase sub-network. Details of these faults are described in Tables 3–5; respectively. All results are obtained using MATLAB.

Results of the open branch faults are given in Table 6 including phase currents and voltages. Similar results for the three-phase short circuit faults are given in Table 7. Currents and voltages for the two falling conductors are given in Table 8.

Results of the Six-Phase Short circuit are presented in Table 9 giving the six-phase currents and voltages.

Transient stability of the IEEE 300-bus system is simulated using the proposed method for a short circuit fault in the six-phase sub-network. Fault specifications are as given in Table 5 but the fault is bolted and at bus 1 in the six-phase sub-network. The per unit inertia constants, damping constants, and regulator constants of the generators are given in Table 10. Governor action is disabled and input mechanical power is assumed constant. The magnitudes of internal EMFs are assumed also constants. The fault is applied at $t = 0.1$ s and is cleared after 2 cycles (40 ms). The speed of the first 5 generators as well as the difference between their angle and the angle of the slack generator, (generator No. 56), is given in Fig. 9 where the system is shown to retain its stability with a very little change in the generators' angles.

Another example is given for the 6 generators, IEEE 30-bus system [17]. A six-phase sub-network as shown in Fig. 8 is connected between buses 3, and 14 with a load of 10 + $j6$ MVA at each six-phase bus. A six-phase bolted fault with the same specifications as Table 5 is simulated at bus 1 of the six-phase sub-network at $t = 0.1$ and is cleared after 2 cycles. The response of the speed of generators 2–6 and the difference between their angles and the angle of the slack generator

Table 3 Three-phase open branch faults.

Branch buses	Open phases
(9006–9007)	c, b through $Z_b = 0.01$ pu
(9023–9026)	b, c, a through $Z_a = 0.02$ pu
(2–6)	a, b, c Three phases Fall near bus 6
(45–74)	a, b
(129–133)	a, c, b through $Z_b = 0.03$ pu
(217–220)	a, b, c through $Z_a = 0.01$ pu $Z_b = 0.02$ pu $Z_c = 0.03$ pu
(63–64)	a, b, c
(7130–130)	c, a through $Z_a = 0.02$ pu Phase c falls near bus 130

Table 4 Three-phase short circuit faults.

Faulted buses	Faulted phases
6	a, b to ground bolted fault
20	c to ground through $Z_{fc} = 0.02$ pu
	a to b through $Z_{fab} = 0.01$ pu
23	a, b, c bolted fault
121	b to c to ground through $Z_{fbc-g} = 0.01$ pu
171	a, c to ground bolted fault
221	a to c bolted fault
7011	a, b, c to ground through $Z_{fa} = 0.01$ pu
	$Z_{fb} = 0.02$ pu $Z_{fc} = 0.03$ pu
9533	c to ground bolted fault

Table 5 Six-phase short circuit faults.

Faulted buses	Faulted phases
2	a to ground bolted fault
	b to d through $Z_{fbd} = 0.01$ pu
	c to e to ground through $Z_{fce-g} = 0.02$ pu

Table 6 Three-phase open conductor currents and voltages.

Branch (9006–9007)	
$I_a = 0.6739 \angle -24.27°$ kA	$V_a = 0.0$ kV
$I_b = 0.6811 \angle -166.24°$ kA	$V_b = 0.0030 \angle -76.24°$ kV
$I_c = 0.0$ kA	$V_c = 0.2816 \angle 158.51°$ kV
Branch (9023–9026)	
$I_a = 0.0438$ *angle* $-44.38°$ kA	$V_a = 0.0004 \angle 45.62°$ kV
$I_b = 0.0$ kA	$V_b = 3.7224 \angle 144.15°$ kV
$I_c = 0.0$ kA	$V_c = 3.6712 \angle 96.25°$ kV
Branch (2–6)	
$I_a = 0.0$ kA	$V_a = 0.0006 \angle -68.63°$ kV
$I_b = 0.0$ kA	$V_b = 0.0006 \angle 176.97°$ kV
$I_c = 0.0$ kA	$V_c = 38.8210 \angle -63.41°$ kV
Branch (45–74)	
$I_a = 0.0$ kA	$V_a = 29.1189 \angle 33.32°$ kV
$I_b = 0.0$ kA	$V_b = 31.7059 \angle -52.91°$ kV
$I_c = 0.4168\angle 87.42°$ kA	$V_c = 0.0$ kV
Branch (129–133)	
$I_a = 0.0$ kA	$V_a = 44.6902 \angle 0.49°$ kV
$I_b = 0.2994 \angle -81.82°$ kA	$V_b = 4.7521 \angle 8.18°$ kV
$I_c = 0.0$ kA	$V_c = 32.0045 \angle 100.86°$ kV
Branch (217–220)	
$I_a = 0.8182 \angle -152.08°$ kA	$V_a = 1.5582 \angle -62.08°$ kV
$I_b = 0.0969 \angle 30.24°$ kA	$V_b = 0.3692 \angle 120.24°$ kV
$I_c = 0.6923 \angle 33.41°$ kA	$V_c = 3.9552 \angle 123.41°$ kV
Branch (63–64)	
$I_a = 0.0$ kA	$V_a = 35.4824 \angle -106.81°$ kV
$I_b = 0.0$ kA	$V_b = 34.5977 \angle 130.55°$ kV
$I_c = 0.0$ kA	$V_c = 33.5402 \angle 13.46°$ kV
Branch (7130–130)	
$I_a = 69.9229 \angle -34.76°$ kA	$V_a = 2.6632 \angle 55.24°$ kV
$I_b = 64.7511 \angle -176.48°$ kA	$V_b = 0.0$ kV
$I_c = 0.0$ kA	$V_c = 4.4431 \angle -67.16°$ kV

Table 7 Three-phase short circuit currents and voltages.

Bus 6	
$I_a = 6.9051 \angle -83.23°$ kA	$V_a = 0.0$ kV
$I_b = 7.7100 \angle 153.95°$ kA	$V_b = 0.0$ kV
$I_c = 0.0$ kA	$V_c = 38.8213 \angle 116.59°$ kV
Bus 20	
$I_a = 11.2791 \angle -59.62°$ kA	$V_a = 17.8144 \angle -50.22°$ kV
$I_b = 11.2791 \angle 120.38°$ kA	$V_b = 21.2846 \angle -93.96°$ kV
$I_c = 7.5410 \angle 22.28°$ kA	$V_c = 19.9459 \angle 112.28°$ kV
Bus 23	
$I_a = 8.1249 \angle -87.82°$ kA	$V_a = 0.0$ kV
$I_b = 8.6860 \angle 164.99°$ kA	$V_b = 0.0$ kV
$I_c = 9.0962 \angle 37.80°$ kA	$V_c = 0.0$ kV
Bus 121	
$I_a = 0.0$ kA	$V_a = 64.1109 \angle -19.40°$ kV
$I_b = 9.1017 \angle 161.83°$ kA	$V_b = 8.1302 \angle -176.89°$ kV
$I_c = 8.9460 \angle 21.65°$ kA	$V_c = 8.1302 \angle -176.89°$ kV
Bus 171	
$I_a = 23.7652 \angle -96.28°$ kA	$V_a = 0.0$ kV
$I_b = 0.0$ kA	$V_b = 56.9005 \angle -140.36°$ kV
$I_c = 22.7064 \angle 20.49°$ kA	$V_c = 0.0$ kV
Bus 221	
$I_a = 17.8942 \angle -131.41°$ kA	$V_a = 38.6312 \angle 35.55°$ kV
$I_b = 0.0$ kA	$V_b = 77.8255 \angle -145.36°$ kV
$I_c = 17.8942 \angle 48.59°$ kA	$V_c = 38.6312 \angle 35.55°$ kV
Bus 7011	
$I_a = 67.5753 \angle -89.51°$ kA	$V_a = 1.2869 \angle 0.49°$ kV
$I_b = 57.0013 \angle 154.97°$ kA	$V_b = 2.1711 \angle -115.03°$ kV
$I_c = 47.9656 \angle 30.50°$ kA	$V_c = 2.7404 < 120.50°$ kV
Bus 9533	
$I_a = 0.0$ kA	$V_a = 1.5040 \angle -32.69°$ kV
$I_b = 0.0$ kA	$V_b = 1.5204 \angle -133.32°$ kV
$I_c = 23.6764 \angle 7.71°$ kA	$V_c = 0.0$ kV

Table 8 Falling conductor currents and voltages.

Branch (2–6) near bus 6	
$I_a = 11.5667 \angle -77.28°$ kA	$V_a = 0.0$ kV
$I_b = 12.2210 \angle 162.69°$ kA	$V_b = 0.0$ kV
$I_c = 12.0337 \angle 38.21°$ kA	$V_c = 0.0$ kV
Branch (7130–130) near bus 130	
$I_a = 0.0$ kA	$V_a = 129.1042 \angle 23.15°$ kV
$I_b = 0.0$ kA	$V_b = 110.1065 \angle -120.74°$ kV
$I_c = 5.2154 \angle 68.43°$ kA	$V_c = 0.0$ kV

Table 9 Six-phase short circuit faults voltages and currents.

Bus 2	
$I_a = 1.1774 \angle -67.31°$ kA	$V_a = 0.0$ kV
$I_b = 0.9530 \angle -96.44°$ kA	$V_b = 162.1079 \angle -114.80°$ kV
$I_c = 1.2780 \angle 170.93°$ kA	$V_c = 118.4896 \angle -158.56°$ kV
$I_d = 0.9530 \angle 83.56°$ kA	$V_d = 194.5797 \angle -134.19°$ kV
$I_e = 1.1157 \angle 30.77°$ kA	$V_e = 118.4896 \angle -158.56°$ kV
$I_f = 0.0$ kA	$V_f = 229.4339 \angle 80.20°$ kV

(generator No. 1) are shown in Fig. 10 where it is obvious that the system retains its stability depending only on the inherent damping although the governors' control is disabled.

Table 10 Generators' dynamic constants.

$h_{gi} = 10$ second, $r_{gi} = 0$ for slack generator
$h_{gi} = 3$ second, $r_{gi} = 0$ for other generators
$d_{gi} = 0.1$ for all generators $f_0 = 50$ Hz

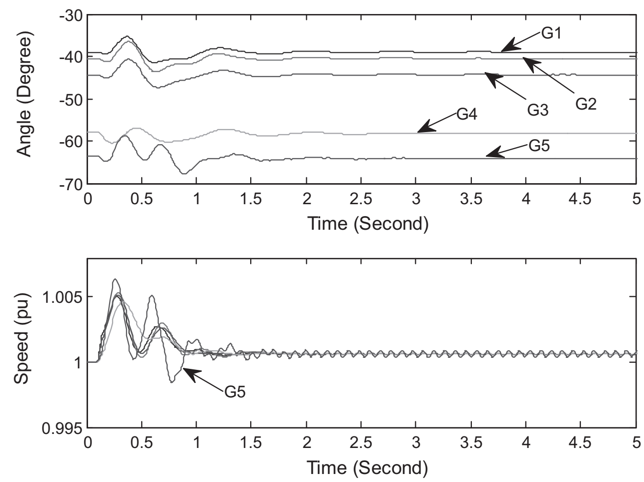

Figure 9 Response of the first 5 generators in the IEEE 300-bus system.

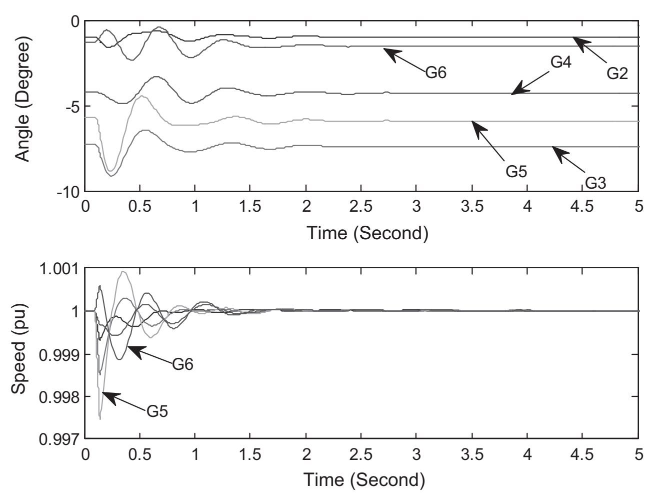

Figure 10 Response of generators 2 to 6 in the IEEE 30-bus system.

It should be emphasized here that stabilization is not the purpose of this paper but the way transient stability is simulated for simultaneous complicated abnormal conditions. All voltages and currents of the system can be easily computed. Dynamics of exciters, voltage regulators and power system stabilizers (PSS) can also be added easily to the method.

10. Conclusion

A new method for analyzing simultaneous open circuit and short circuit faults for mixed three-phase and six-phase power systems is proposed in this paper. The method accounts for transformer phase shifts and un-transposed transmission lines. Generalized treatment of all types of faults is given for three-phase and six-phase systems. Coupling between three-phase and six-phase sub-network is handled in a systematic way. The algorithm is general and can be applied to any number of phases. All network currents and voltages can be easily computed at one step. The use of the algorithm for simulating system transient stability after any complicated abnormal condition is demonstrated. Results are given for the IEEE 300-bus and IEEE 30-bus systems after modifications and inclusion of six-phase sub-network.

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