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Multicriteria analysis with fuzzy pairwise comparison

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Abstract

The analytic hierarchy process (AHP) is a popular method for solving multicriteria analysis (MA) problems involving qualitative data. However, this method is often criticized due to its use of an unbalanced scale of judgements and its inability to adequately handle the inherent uncertainty and imprecision of the pairwise comparison process. This paper presents a fuzzy approach for tackling qualitative MA problems in a simple and straightforward manner. As a result, effective decisions can be made based on adequate modeling of the uncertainty and imprecision in human behavior. An empirical study of a tender selection problem at Monash municipal government of Victoria in Australia is conducted. The result shows that the approach developed is simple and comprehensible in concept, efficient in computation, and robust in modeling human evaluation processes which make it of general use for solving practical qualitative MA problems.

Keywords: Multicriteria analysis; Fuzzy numbers; Pairwise comparison

1. Introduction

Multicriteria analysis (MA) often requires the decision maker (DM) to provide qualitative assessments for determining (a) the performance of each alternative with respect to each criterion and (b) the relative importance of the

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evaluation criteria with respect to the overall objective of the problem. As a result, uncertain, imprecise and subjective data are usually present which make the decision-making process complex and challenging [4,6,17,20,23].

Attempts to handle this uncertainty, imprecision and subjectiveness are carried out basically by means of probability theory and/or fuzzy set theory. The former focuses on the stochastic nature of the decision-making process while the latter concerns the subjectiveness and imprecision of human behavior. As suggested by Chen and Hwang [4], Dubois and Prade [7], Efstathiou [8] and Zimmermann [23,24], stochastic methods such as statistical analysis cannot adequately handle the subjectiveness and imprecision of the human decision-making process.

The analytic hierarchy process (AHP) of Saaty [14,15] is a popular method for tackling MA problems involving qualitative data, and has successfully been applied to many actual decision situations. Pairwise comparison is used in the decision-making process to form a reciprocal decision matrix, thus transforming qualitative data to crisp ratios and making the process simple and easy to handle. An eigenvector method is used to solve the reciprocal matrix for determining the criteria importance and alternative performance. The simple additive weighting (SAW) method [4,9] is used to calculate the utility for each alternative across all criteria. However this method is often criticized because of (a) its use of an unbalanced scale of estimations and (b) its inability to adequately handle the uncertainty and imprecision associated with the mapping of the DM's perception to a crisp number [5,13].

Buckley [2] and Laarhoven and Pedrycz [12] extend Saaty's AHP to deal with the imprecision and subjectiveness in the pairwise comparison process. Triangular or trapezoidal fuzzy numbers are used to express the DM's assessments on alternatives with respect to each criterion. After the criteria are weighted, the overall utilities of alternatives, known as fuzzy utilities (represented by fuzzy numbers), are aggregated by fuzzy arithmetic [11] using the SAW method. To prioritize the alternatives, their fuzzy utilities need to be compared and ranked. However this comparison process can be quite complex and may produce unreliable results due to (a) considerable computations required, (b) inconsistent ranking outcomes with different ranking approaches, and (c) counter-intuitive ranking outcomes under some circumstances [1,4,6,23].

To facilitate the pairwise comparison process and to avoid the complex and unreliable process of comparing fuzzy utilities, this paper presents an MA approach for effectively solving MA problems involving qualitative data. Triangular fuzzy numbers are used in the pairwise comparison process to express the DM's subjective assessments. The concept of fuzzy extent analysis is applied to solve the fuzzy reciprocal matrix for determining the criteria importance and alternative performance. To avoid the complex and unreliable process of comparing fuzzy utilities, the α-cut concept is used to transform the fuzzy performance matrix representing the overall performance of all alternatives with respect to each criterion into an interval performance matrix. Incorporated with the DM's attitude towards risk, an overall performance index is obtained for each alternative across all criteria by applying the concept of the degree of similarity to the ideal solution using the vector matching function.

In what follows, we first present some basic concepts to pave the way for the methodology development. We then present an MA approach for tackling the general MA problem involving qualitative assessments. Finally we present an empirical study of a real tender selection problem in Monash municipal government of Victoria, Australia to illustrate the applicability of the approach developed.

2. Basic concepts

2.1. Triangular fuzzy numbers

A fuzzy number is a convex fuzzy set [21], characterized by a given interval of real numbers, each with a grade of membership between 0 and 1. Its membership function is piecewise continuous, and satisfies the following conditions:

(a) $\mu_A(x) = 0$ for each $x \in (-\infty, a_1] \cup [a_4, +\infty)$,
(b) $\mu_A(x)$ is non-decreasing on $[a_1, a_2]$ and non-increasing on $[a_3, a_4]$,
(c) $\mu_A(x) = 1$, for each $x \in [a_2, a_3]$,
where $a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4$ are real numbers in the real line R.

Triangular fuzzy numbers are a special class of fuzzy number, defined by three real numbers, often expressed as (a_1, a_2, a_3). Their membership functions are usually described as

$$\mu_A(x) = \begin{cases} (x - a_1)/(a_2 - a_1), & a_1 \leqslant x \leqslant a_2, \\ (a_3 - x)/(a_3 - a_2), & a_2 \leqslant x \leqslant a_3, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where a_2 is the most possible value of fuzzy number A, and a_1 and a_3 are the lower and upper bounds, respectively which is often used to illustrate the fuzziness of the data evaluated.

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be two positive triangular fuzzy numbers. The basic fuzzy arithmetic operations on these fuzzy numbers are defined as

(a) *Inverse*:

$$A^{-1} = \left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right).$$

(b) *Addition*:

$$A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3).$$

(c) *Subtraction*:

$$A - B = (a_1 + b_3, a_2 - b_2, a_3 - b_1).$$

(d) *Scalar multiplication*:

$$\forall k > 0,\ k \in R,\ kA = (ka_1, ka_2, ka_3),$$
$$\forall k < 0,\ k \in R,\ kA = (ka_3, ka_2, ka_1).$$

(e) *Multiplication*:

$$AB = (a_1 b_1, a_2 b_2, a_3 b_3).$$

(f) *Division*:

$$\frac{A}{B} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right).$$

Fuzzy numbers are intuitively easy to use in expressing the DM's qualitative assessments [2,10,12,13,17–19]. To facilitate the making of pairwise comparison, triangular fuzzy numbers defined in Table 1 are used. A triangular fuzzy number $\bar{x}$ expresses the meaning of 'about x', where $1 \leqslant x \leqslant 9$, with its membership function defined as in (1). Fuzzy number $\bar{9}$ used by Juang and Lee [10] is revised here to better reflect the decision situation involved.

2.2. Fuzzy synthetic extent analysis

Assume that $X = \{x_1, x_2, \ldots, x_n\}$ is an object set, and $U = \{u_1, u_2, \ldots, u_m\}$ is a goal set. According to the method of fuzzy extent analysis [3], fuzzy extent analysis can be performed with respect to each object for each goal, respectively, resulting in m extent analysis values for each object, given as $\mu_i^1, \mu_i^2, \mu_i^m$, $i = 1, 2, \ldots, n$, where all μ_i^j $(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m)$ are fuzzy numbers representing the performance of the object x_i with regard to each goal u_j.

By using fuzzy synthetic extent analysis, the value of fuzzy synthetic extent with respect to the ith object $x_i\,(i = 1, 2, \ldots, n)$ that represents the overall performance of the object across all goals involved can be determined by

Table 1
Fuzzy numbers used for making qualitative assessments

Fuzzy number	Membership function
$\bar{1}$	$(1, 1, 3)$
$\bar{x}$	$(x - 2, x, x + 2)$ for $x = 3, 5, 7$
$\bar{9}$	$(7, 9, 11)$

$$S_i = \frac{\sum_{j=1}^{m} \mu_i^j}{\sum_{i=1}^{n} \sum_{j=1}^{m} \mu_i^j}, \quad i = 1, 2, \ldots, n. \tag{2}$$

3. The MA approach

The general MA decision problem usually consists of (a) a number of alternatives, denoted as $A_i\,(i = 1, 2, \ldots, n)$, (b) a set of evaluation criteria $C_j\,(j = 1, 2, \ldots, m)$, (c) a qualitative or quantitative assessment $x_{ij}\,(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$ (referred to as *performance ratings*) representing the performance of each alternative A_i with respect to each criterion C_j, leading to the determination of a decision matrix for the alternatives, and (d) a weighting vector $W = (w_1, w_2, \ldots, w_m)$ (referred to as *criteria weights*) representing the relative importance of the evaluation criteria with respect to the overall objective of the problem.

With the problem structure defined above, mainstream fuzzy MA models in the context of multiattribute additive value theory have been developed along the line of the evaluation approach involving three phases [4,6,19,23]: (a) the determination of the criteria importance and alternative performance, (b) the aggregation of the assessments with respect to all criteria for each alternative, and (c) the ranking of the alternatives based on their aggregated overall assessments (fuzzy utilities). The main problem with this approach lies in (a) the inappropriateness of handling the uncertainty and imprecision of the decision-making process and (b) the complex and unreliable process of comparing fuzzy utilities.

To circumvent these drawbacks, this paper presents a fuzzy MA approach based on the synthesis of the following concepts, including (a) fuzzy set theory, (b) AHP, (c) fuzzy extent analysis, (d) α-cut concept, (e) ideal solution, and (f) vector matching function. As a result, the cognitive burden of the DM is greatly reduced, the subjectiveness and imprecision of the evaluation process are adequately handled, and the complex and unreliable process of comparing fuzzy utilities is avoided, resulting in effective decisions being made in solving practical qualitative MA problems.

The ranking procedure starts at the determination of the criteria importance and alternative performance. By using the fuzzy numbers defined in Table 1, a fuzzy reciprocal judgement matrix for criteria importance (W) or alternative performance with respect to a specific criterion (C_j) can be determined as

$$C_j \text{ or } W = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \ldots & \bar{a}_{1k} \\ \bar{a}_{21} & \bar{a}_{22} & \ldots & \bar{a}_{2k} \\ \ldots & \ldots & \ldots & \ldots \\ \bar{a}_{k1} & \bar{a}_{k2} & \ldots & \bar{a}_{kk} \end{bmatrix}, \tag{3}$$

where

$$\bar{a}_{ls} = \begin{cases} \bar{1}, \bar{3}, \bar{5}, \bar{9}, & l < s, \\ 1, & l = s, \ l, s = 1, 2, \ldots, k; \ k = m \text{ or } n, \\ 1/\bar{a}_{sl}, & l > s. \end{cases} \tag{4}$$

With the application of the fuzzy extent analysis on (3) by (1) and (2), the corresponding criteria weights (w_j) or alternative performance ratings (x_{ij}) with respect to a specific criterion C_j can then be determined as

$$x_{ij} \text{ or } w_j = \frac{\sum_{s=1}^{k} \bar{a}_{ls}}{\sum_{l=1}^{k} \sum_{s=1}^{k} \bar{a}_{ls}}, \tag{5}$$

where $i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$ and $k = m$ or n depending on whether the reciprocal judgement matrix is for assessing the performance ratings of alternatives or the weights of the criteria involved.

As a result, the decision matrix (X) and the weight vector (W) for the MA decision problem can be respectively determined as

$$X = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1m} \\ x_{21} & x_{22} & \ldots & x_{2m} \\ \ldots & \ldots & \ldots & \ldots \\ x_{n1} & x_{n2} & \ldots & x_{nm} \end{bmatrix}, \tag{6}$$

$$W = (w_1, w_2, \ldots, w_m), \tag{7}$$

where x_{ij} represents the resultant fuzzy performance assessment of alternative $A_i\,(i = 1, 2, \ldots, n)$ with respect to criterion C_j and w_j is the resultant fuzzy weight of the criterion $C_j\,(j = 1, 2, \ldots, m)$ with respect to the overall objective of the problem.

A fuzzy performance matrix (8) representing the overall performance of all alternatives with respect to each criterion can therefore be obtained by multiplying the weighting vector by the decision matrix. The arithmetic operations on these fuzzy numbers are based on interval arithmetic [11].

$$Z = \begin{bmatrix} w_1 x_{11} & w_2 x_{12} & \ldots & w_m x_{1m} \\ w_1 x_{21} & w_2 x_{22} & \ldots & w_m x_{2m} \\ \ldots & \ldots & \ldots & \ldots \\ w_1 x_{n1} & w_2 x_{n2} & \ldots & w_m x_{nm} \end{bmatrix}. \tag{8}$$

By using an α-cut on the performance matrix (8), an interval performance matrix can be derived as in (9), where $0 \leqslant \alpha \leqslant 1$. The value of α represents the DM's degree of confidence in his/her fuzzy assessments regarding alternative ratings and criteria weights. A larger α value indicates a more confident DM, meaning that the DM's assessments are closer to the most possible value a_2 of the triangular fuzzy numbers (a_1, a_2, a_3).

$$Z_\alpha = \begin{bmatrix} [z^\alpha_{11l}, z^\alpha_{11r}] & [z^\alpha_{12l}, z^\alpha_{12r}] & \cdots & [z^\alpha_{1ml}, z^\alpha_{1mr}] \\ [z^\alpha_{21l}, z^\alpha_{21r}] & [z^\alpha_{22l}, z^\alpha_{22r}] & \cdots & [z^\alpha_{2ml}, z^\alpha_{2mr}] \\ \cdots & \cdots & \cdots & \cdots \\ [z^\alpha_{n1l}, z^\alpha_{n1r}] & [z^\alpha_{n2l}, z^\alpha_{n2r}] & \cdots & [z^\alpha_{nml}, z^\alpha_{nmr}] \end{bmatrix}. \tag{9}$$

Incorporated with the DM's attitude towards risk using an optimism index λ, an overall crisp performance matrix is calculated as in (10), where $z^{\lambda'}_{ij\alpha} = \lambda z^\alpha_{ijr} + (1-\lambda) z^\alpha_{ijl}$, $\lambda \in [0, 1]$.

$$Z^{\lambda'}_\alpha = \begin{bmatrix} z^{\lambda'}_{11\alpha} & z^{\lambda'}_{12\alpha} & \cdots & z^{\lambda'}_{1m\alpha} \\ z^{\lambda'}_{21\alpha} & z^{\lambda'}_{22\alpha} & \cdots & z^{\lambda'}_{2m\alpha} \\ \cdots & \cdots & \cdots & \cdots \\ z^{\lambda'}_{n1\alpha} & z^{\lambda'}_{n2\alpha} & \cdots & z^{\lambda'}_{nm\alpha} \end{bmatrix} \tag{10}$$

In practical applications, $\lambda = 1, \lambda = 0.5$, and $\lambda = 0$ are used to indicate that the DM involved has an optimistic, moderate, or pessimistic view, respectively. An optimistic DM is apt to prefer higher values of his/her fuzzy assessments, while a pessimistic DM tends to favor lower values.

To facilitate the vector matching process, a normalization process in regard to each criterion is applied to (10) by using (11), resulting in a normalized performance matrix expressed as in (12):

$$z^\lambda_{ij\alpha} = \frac{z^{\lambda'}_{ij\alpha}}{\sqrt{\sum_{i=1}^{n} (z^{\lambda'}_{ij\alpha})^2}}, \tag{11}$$

$$Z^\lambda_\alpha = \begin{bmatrix} z^\lambda_{11\alpha} & z^\lambda_{12\alpha} & \cdots & z^\lambda_{1m\alpha} \\ z^\lambda_{21\alpha} & z^\lambda_{22\alpha} & \cdots & z^\alpha_{2m\alpha} \\ \cdots & \cdots & \cdots & \cdots \\ z^\lambda_{n1\alpha} & z^\lambda_{n2\alpha} & \cdots & z^\lambda_{nm\alpha} \end{bmatrix}. \tag{12}$$

Zeleny [22] first introduced the concept of the ideal solution in decision analysis as the best or desired decision outcome for given decision situation. Hwang and Yoon [9] further extended this concept to include the negative ideal solution to avoid the worst decision outcome. This concept has since been widely used in developing various methodologies for solving practical decision problems [16,19,20]. This is due to (a) its simplicity and comprehensibility in concept, (b) its computation efficiency, and (c) its ability to measure the relative performance of the decision alternatives in a simple mathematical form.

In line with this concept, the positive ideal solution $A^{\lambda+}_\alpha$ and the negative ideal solution $A^{\lambda-}_\alpha$ can be determined by selecting the maximum value and the minimum value across all alternatives with respect to each criterion, given as in

(13). They respectively represent the best possible and the worst possible results among the alternatives across all criteria.

$$\begin{aligned} A_{\alpha}^{\lambda+} &= (z_{1\alpha}^{\lambda+}, z_{2\alpha}^{\lambda+}, \ldots, z_{m\alpha}^{\lambda+}), \\ A_{\alpha}^{\lambda-} &= (z_{1\alpha}^{\lambda-}, z_{2\alpha}^{\lambda-}, \ldots, z_{m\alpha}^{\lambda-}), \end{aligned} \tag{13}$$

where

$$\begin{aligned} z_{j\alpha}^{\lambda+} &= \max\,(z_{1j\alpha}^{\lambda}, z_{2j\alpha}^{\lambda}, \ldots, z_{nj\alpha}^{\lambda}), \\ z_{j\alpha}^{\lambda-} &= \min\,(z_{1j\alpha}^{\lambda}, z_{2j\alpha}^{\lambda}, \ldots, z_{nj\alpha}^{\lambda}). \end{aligned} \tag{14}$$

By applying the vector matching function, the degree of similarity between each alternative and the positive ideal solution and the negative ideal solution can be calculated, respectively by

$$S_{i\alpha}^{\lambda+} = \frac{A_{i\alpha}^{\lambda} A_{\alpha}^{\lambda+}}{\max\,(A_{i\alpha}^{\lambda} A_{i\alpha}^{\lambda}, A_{\alpha}^{\lambda+} A_{\alpha}^{\lambda+})}, \tag{15}$$

$$S_{i\alpha}^{\lambda-} = \frac{A_{i\alpha}^{\lambda} A_{\alpha}^{\lambda-}}{\max\,(A_{i\alpha}^{\lambda} A_{i\alpha}^{\lambda}, A_{\alpha}^{\lambda-} A_{\alpha}^{\lambda-})}, \tag{16}$$

where $A_{i\alpha}^{\lambda} = (z_{i1\alpha}^{\lambda}, z_{i2\alpha}^{\lambda}, \ldots, z_{im\alpha}^{\lambda})$ is the ith row of the overall performance matrix in (12), representing the corresponding performance of alternative $A_i\,(i \in \{1, 2, \ldots, n\})$ in regard to each criterion $C_j\,(j = 1, 2, \ldots, m)$. The larger the value of $S_{i\alpha}^{\lambda+}$and $S_{i\alpha}^{\lambda-}$, the higher the degree of similarity between each alternative and the positive ideal solution and the negative ideal solution, respectively [19].

A preferred alternative should have a higher degree of similarity to the positive ideal solution, and at the same time a lower degree of similarity to the negative ideal solution [9,16,19,20,22]. Therefore, an overall performance index for each alternative with the DM's α level of confidence in his/her fuzzy assessments and λ degree of optimism towards risk can be determined by (17) as

$$P_{\alpha i}^{\lambda} = \frac{S_{i\alpha}^{\lambda+}}{S_{i\alpha}^{\lambda+} + S_{i\alpha}^{\lambda-}}, \quad i = 1, 2, \ldots, n. \tag{17}$$

The larger the index value, the more preferred the alternative.

In summarizing the discussion above, we present the steps required for the approach developed as follows:

1. Formulate the decision problem as an MA problem and identify the hierarchical structure of the problem.
2. Determine the decision matrix as expressed in (6) by (3)–(5) using the AHP method based on the fuzzy numbers defined in Table 1.
3. Obtain the weighting vector (7) for the criteria by (3)–(5) using the AHP method based on the fuzzy numbers defined in Table 1.

4. Determine the fuzzy performance matrix (8) by multiplying the decision matrix obtained at Step 2 by the weighting vector determined at Step 3.
5. Obtain the interval performance matrix (9) by using an α-cut on the performance matrix determined at Step 4.
6. Determine the crisp performance matrix (10) by incorporating the DM's attitude towards risk represented by an optimism index λ.
7. Calculate the normalized performance matrix (12) by (11).
8. Determine the positive ideal solution and the negative ideal solution by (13) and (14).
9. Calculate the degree of similarity between each alternative and the positive ideal solution and the negative ideal solution by (15) and (16).
10. Determine the overall performance index for each alternative by (17).
11. Rank the alternatives in the descending order of their corresponding performance index values.

4. An empirical study

In this section we present an empirical study of a tender selection problem faced by a local government in Victoria, Australia to illustrate the applicability of the approach developed. The complexity of the tender selection process is highlighted, and the need for a structured approach to make fair and consistent tender selection decisions is demonstrated.

Monash municipal government is one of the largest local governments in Victoria, Australia. Every year it has a large number of public projects to be contracted out to various private companies in order to serve the local community effectively. As the demand for accountability and transparency of the decision-making process in the local government department increases, a formal procedure of evaluating and selecting the most qualified tender (the contracting company) for a specific project on a competitive basis becomes an important issue in its daily operations.

Selecting the best-qualified tender from available tenders for a specific project is a complex decision-making process in which the overall performance of the available tenders needs to be evaluated with respect to multiple selection criteria. Subjective (qualitative) assessments are often involved with regard to the criteria importance and tender performance, resulting in fuzzy and imprecise data being used which requires the use of a fuzzy approach for effectively tackling this kind of decision problem.

Various selection criteria may be involved depending on the type of project present. In the current discussion, a specific project (Public Building Maintenance at Monash Municipal Government) worthwhile up to two million Australian dollars in a two-year period is considered. Nine tender submissions have been received. After the initial screening process with respect to a few rigorous cutoff criteria such as the conformity to the tending requirements and the maximum project cost allowed, etc., three tenders are left for further evaluation. Based on the comprehensive discussion of the relevant government department, four selection criteria including the Tending Cost Attractiveness (C_1), Technical Capability (C_2), References (C_3), and Services (C_4) have been identified. Fig. 1 describes the hierarchical structure of the tender performance evaluation process. We discuss these selection criteria individually below.

(a) *Tending Cost Attractiveness* (C_1). Tending cost attractiveness of a tender submission concerns about the proposed cost of the project with respect to the allocated budget and other tender submissions. From the government point of view, it is preferable to spend less money on the project as long as the project can be completed satisfactorily. Based on the available information, a fuzzy reciprocal judgement matrix (C_1) based on the pairwise comparison process using the fuzzy numbers defined in Table 1 was obtained as follows

$$C_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} \bar{1} & \bar{3} & \bar{9} \\ \bar{3}^{-1} & \bar{1} & \bar{5} \\ \bar{9}^{-1} & \bar{5}^{-1} & \bar{1} \end{bmatrix} \end{array}.$$

(b) *Technical Capability* (C_2). The technical capability of a tender refers to the tender's ability to carry out the project technically in a satisfactory manner. This is often determined subjectively depending on the information provided by the tender together with specific technical requirements of the project. Along with the AHP method based on the fuzzy numbers defined in Table 1, a fuzzy reciprocal judgement matrix (C_2) for determining the performance of three tenders with respect to this criterion was given as follows

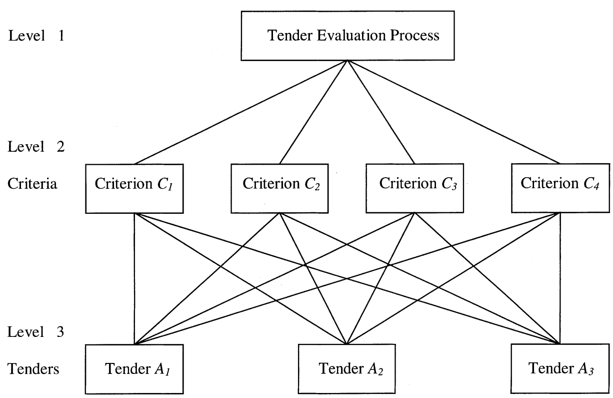

Fig. 1. The hierarchical structure of the tender performance evaluation process.

$$C_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} \bar{1} & \bar{3} & \bar{5}^{-1} \\ \bar{3}^{-1} & \bar{1} & \bar{9} \\ \bar{5} & \bar{9}^{-1} & \bar{1} \end{bmatrix} \end{array}.$$

(c) *References* (C_3). The reference of the tenders is related to the past performance of the tenders involving the same or similar types of project. This is often determined subjectively by asking the involved companies listed in their tender submissions. Similarly, a reciprocal fuzzy judgement matrix (C_3) was determined based on the available information as

$$C_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} \bar{1} & \bar{9}^{-1} & \bar{7} \\ \bar{9} & \bar{1} & \bar{3}^{-1} \\ \bar{7}^{-1} & \bar{3} & \bar{1} \end{bmatrix} \end{array}.$$

(d) *Services* (C_4). The Services criterion concerns about the services provided by the tender after the completion of the project. Factors such as after-sale training, maintenance availability, reliability, etc. usually need to be considered. For this specific project, the fuzzy reciprocal judgement matrix (C_4) with respect to the tender performance in regard to this criterion was obtained as

$$C_4 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} \bar{1} & \bar{7}^{-1} & \bar{3} \\ \bar{7} & \bar{1} & \bar{9} \\ \bar{3}^{-1} & \bar{9}^{-1} & \bar{1} \end{bmatrix} \end{array}.$$

By applying (3)–(5) using the fuzzy extent analysis on these reciprocal judgement matrices, the performance ratings (x_{ij}) of alternative $A_i\,(i = 1, 2, 3)$ with respect to each criterion $C_j\,(j = 1, 2, 3, 4)$ were calculated as

$$X_1 = \left[\frac{\bar{1} + \bar{3} + \bar{9}}{\bar{1} + \bar{3} + \bar{9} + \bar{3}^{-1} + \bar{1} + \bar{5} + \bar{9}^{-1} + \bar{5}^{-1} + \bar{1}}, \right.$$
$$\frac{\bar{3}^{-1} + \bar{1} + \bar{5}}{\bar{1} + \bar{3} + \bar{9} + \bar{3}^{-1} + \bar{1} + \bar{5} + \bar{9}^{-1} + \bar{5}^{-1} + \bar{1}},$$
$$\left. \frac{\bar{9}^{-1} + \bar{5}^{-1} + \bar{1}}{\bar{1} + \bar{3} + \bar{9} + \bar{3}^{-1} + \bar{1} + \bar{5} + \bar{9}^{-1} + \bar{5}^{-1} + \bar{1}} \right],$$

$$X_2 = \left[\frac{\bar{1} + \bar{3} + \bar{5}^{-1}}{\bar{1} + \bar{3} + \bar{5}^{-1} + \bar{3}^{-1} + \bar{1} + \bar{9} + \bar{5} + \bar{9}^{-1} + \bar{1}}, \frac{\bar{3}^{-1} + \bar{1} + \bar{9}}{\bar{1} + \bar{3} + \bar{5}^{-1} + \bar{3}^{-1} + \bar{1} + \bar{9} + \bar{5} + \bar{9}^{-1} + \bar{1}}, \frac{\bar{5} + \bar{9}^{-1} + \bar{1}}{\bar{1} + \bar{3} + \bar{5}^{-1} + \bar{3}^{-1} + \bar{1} + \bar{9} + \bar{5} + \bar{9}^{-1} + \bar{1}} \right],$$

$$X_3 = \left[\frac{\bar{1} + \bar{9}^{-1} + \bar{7}}{\bar{1} + \bar{9}^{-1} + \bar{7} + \bar{9} + \bar{1} + \bar{3}^{-1} + \bar{7}^{-1} + \bar{3} + \bar{1}}, \frac{\bar{9} + \bar{1} + \bar{3}^{-1}}{\bar{1} + \bar{9}^{-1} + \bar{7} + \bar{9} + \bar{1} + \bar{3}^{-1} + \bar{7}^{-1} + \bar{3} + \bar{1}}, \frac{\bar{7}^{-1} + \bar{3} + \bar{1}}{\bar{1} + \bar{9}^{-1} + \bar{7} + \bar{9} + \bar{1} + \bar{3}^{-1} + \bar{7}^{-1} + \bar{3} + \bar{1}} \right],$$

$$X_4 = \left[\frac{\bar{1} + \bar{7}^{-1} + \bar{3}}{\bar{1} + \bar{7}^{-1} + \bar{3} + \bar{7} + \bar{1} + \bar{9} + \bar{3}^{-1} + \bar{9}^{-1} + \bar{1}}, \frac{\bar{7} + \bar{1} + \bar{9}}{\bar{1} + \bar{7}^{-1} + \bar{3} + \bar{7} + \bar{1} + \bar{9} + \bar{3}^{-1} + \bar{9}^{-1} + \bar{1}}, \frac{\bar{3}^{-1} + \bar{9}^{-1} + \bar{1}}{\bar{1} + \bar{7}^{-1} + \bar{3} + \bar{7} + \bar{1} + \bar{9} + \bar{3}^{-1} + \bar{9}^{-1} + \bar{1}} \right],$$

where

$$X_1 = (x_{11}, x_{21}, x_{31}), \quad X_2 = (x_{12}, x_{22}, x_{32}),$$

$$X_3 = (x_{13}, x_{23}, x_{33}), \quad X_4 = (x_{14}, x_{24}, x_{34}).$$

As a result, the decision matrix for the tender selection problem was determined using fuzzy arithmetic [11] as

$$X = \begin{bmatrix} (0.27, 0.63, 1.32) & (0.06, 0.12, 0.58) & (0.17, 0.36, 0.74) & (0.06, 0.18, 0.50) \\ (0.13, 0.31, 0.76) & (0.24, 0.50, 1.04) & (0.23, 0.46, 0.91) & (0.37, 0.75, 1.40) \\ (0.04, 0.06, 0.24) & (0.12, 0.30, 0.70) & (0.06, 0.18, 0.50) & (0.04, 0.06, 0.25) \end{bmatrix}.$$

To determine the relative importance of the selection criteria, fuzzy pairwise comparison process is conducted, resulting in a fuzzy reciprocal judgement matrix (W) as

$$W = \begin{matrix} & C_1 & C_2 & C_3 & C_4 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{matrix} \begin{bmatrix} \bar{1} & \bar{3} & \bar{7} & \bar{5} \\ \bar{3}^{-1} & \bar{1} & \bar{9} & \bar{3} \\ \bar{7}^{-1} & \bar{9}^{-1} & \bar{1} & \bar{3}^{-1} \\ \bar{5}^{-1} & \bar{3}^{-1} & \bar{3} & \bar{1} \end{bmatrix}$$

Similarly, the weighting vector was determined by (3)–(5) using fuzzy extent analysis as

$$w_1 = \frac{\bar{1}+\bar{3}+\bar{7}+\bar{5}}{\bar{1}+\bar{3}+\bar{7}+\bar{5}+\bar{3}^{-1}+\bar{1}+\bar{9}+\bar{3}+\bar{7}^{-1}+\bar{9}^{-1}+\bar{1}+\bar{3}^{-1}+\bar{5}^{-1}+\bar{3}^{-1}+\bar{3}+\bar{1}},$$

$$w_2 = \frac{\bar{3}^{-1}+\bar{1}+\bar{9}+\bar{3}}{\bar{1}+\bar{3}+\bar{7}+\bar{5}+\bar{3}^{-1}+\bar{1}+\bar{9}+\bar{3}+\bar{7}^{-1}+\bar{9}^{-1}+\bar{1}+\bar{3}^{-1}+\bar{5}^{-1}+\bar{3}^{-1}+\bar{3}+\bar{1}},$$

$$w_3 = \frac{\bar{7}^{-1}+\bar{9}^{-1}+\bar{1}+\bar{3}^{-1}}{\bar{1}+\bar{3}+\bar{7}+\bar{5}+\bar{3}^{-1}+\bar{1}+\bar{9}+\bar{3}+\bar{7}^{-1}+\bar{9}^{-1}+\bar{1}+\bar{3}^{-1}+\bar{5}^{-1}+\bar{3}^{-1}+\bar{3}+\bar{1}},$$

$$w_4 = \frac{\bar{5}^{-1}+\bar{3}^{-1}+\bar{3}+\bar{1}}{\bar{1}+\bar{3}+\bar{7}+\bar{5}+\bar{3}^{-1}+\bar{1}+\bar{9}+\bar{3}+\bar{7}^{-1}+\bar{9}^{-1}+\bar{1}+\bar{3}^{-1}+\bar{5}^{-1}+\bar{3}^{-1}+\bar{3}+\bar{1}}.$$

Based on the fuzzy arithmetic [11], the criteria weights were calculated as

$$w_1 = (0.17, 0.45, 1.05), \quad w_2 = (0.16, 0.38, 0.87),$$

$$w_3 = (0.02, 0.04, 0.19), \quad w_4 = (0.04, 0.13, 0.41).$$

A fuzzy performance matrix for the problem was therefore obtained by (8) as

$$Z = \begin{bmatrix} (0.046, 0.284, 1.386) & (0.010, 0.046, 0.505) & (0.003, 0.014, 0.141) & (0.002, 0.023, 0.205) \\ (0.022, 0.140, 0.798) & (0.038, 0.190, 0.905) & (0.005, 0.018, 0.173) & (0.015, 0.098, 0.574) \\ (0.007, 0.027, 0.252) & (0.019, 0.114, 0.609) & (0.001, 0.007, 0.095) & (0.002, 0.008, 0.103) \end{bmatrix}.$$

Let $\alpha = 0.5$, $\lambda = 0.5$ (for a moderate DM). The performance index for each tender and its corresponding ranking was determined by applying Eqs. (6)–(17). Table 2 shows the result. Obviously tender A_1 is the best choice in this situation.

Similarily, letting $\alpha = 0.0, 0.1, 0.3, 0.7, \ldots, 0.9, 1.0$, and $\lambda = 0.0$ (for a pessimistic DM), $\lambda = 1.0$ (for an optimistic DM), we can calculate the overall performance index for each tender and determine its corresponding ranking. Figs. 2–4 show the results, respectively.

Table 2
Performance index and ranking of the tenders

Tenders	Performance index	Ranking
A_1	0.77	1
A_2	0.67	2
A_3	0.28	3

The results in Figs. 2–4 show that tender A_1 is clearly the best choice under almost any degree of confidence of the DM with various attitudes towards risk. It is also clear that this method can adequately reflect the uncertainty and imprecision associated with the DM's subjective judgement in human thinking. It provides the DM an appropriate tool to better understand the decision problem and his/her decision behavior. Effective and consistent decisions can therefore be made.

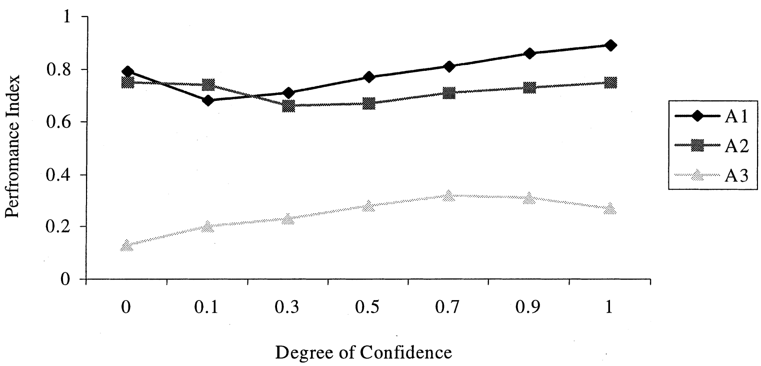

Fig. 2. Performance index and ranking of the tenders for a moderate DM.

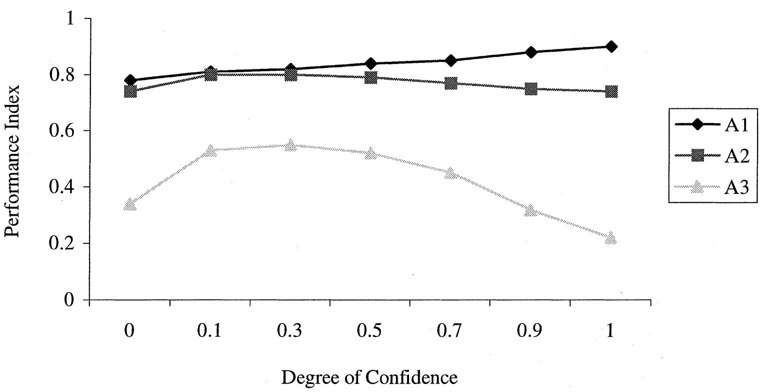

Fig. 3. Performance index and ranking of the tenders for a pessimistic DM.

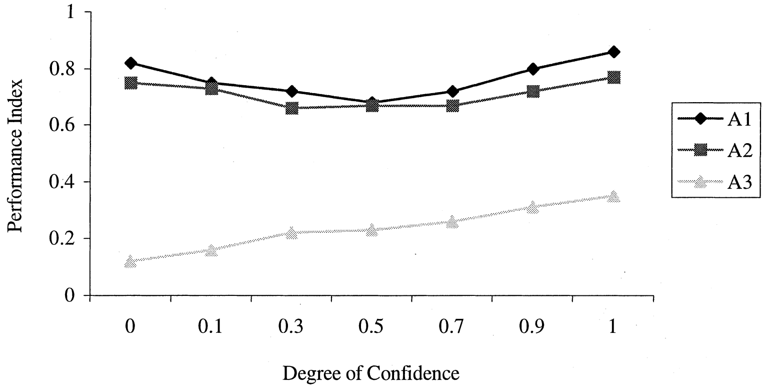

Fig. 4. Performance index and ranking of the tenders for an optimistic DM.

For the sake of comparison, we have used Saaty's traditional AHP method to treat the same problem. The resulting pairwise comparison matrices for tender performance with respect to each criterion and criteria importance were obtained individually as follows:

$$C_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} 1 & 3 & 9 \\ 1/3 & 1 & 5 \\ 1/9 & 1/5 & 1 \end{bmatrix} \end{array}, \quad C_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} 1 & 3 & 1/5 \\ 1/3 & 1 & 9 \\ 5 & 1/9 & 1 \end{bmatrix} \end{array},$$

$$C_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} 1 & 1/9 & 7 \\ 9 & 1 & 1/3 \\ 1/7 & 35 & 1 \end{bmatrix} \end{array}, \quad C_4 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} 1 & 1/7 & 3 \\ 7 & 1 & 9 \\ 1/3 & 1/9 & 1 \end{bmatrix} \end{array},$$

$$W = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 1 & 3 & 7 & 5 \\ 1/3 & 1 & 9 & 3 \\ 1/7 & 1/9 & 1 & 1/3 \\ 1/5 & 1/3 & 3 & 1 \end{bmatrix} \end{array}.$$

The procedure of the AHP for solving these reciprocal matrices is well established [14,15]. Here we only present the final overall performance index for each tender and its corresponding ranking, as shown in Table 3. It is clear that tender A_1 is the best choice.

Same results were obtained with the traditional AHP method and the approach developed. This would give the DM reasonable assurance in making

Table 3
Performance index and ranking of the tenders with the traditional AHP

Tenders	Performance index	Ranking
A_1	0.51	1
A_2	0.37	2
A_3	0.12	3

his/her decisions in the tender selection process. However in comparison with the traditional AHP method, the approach developed clearly has its advantages. These advantages include (a) better modeling of the uncertainty and imprecision associated with the pairwise comparison process, (b) cognitively less demanding on the DM, and (c) adequate reflection of the DM's attitude toward risk and their degrees of confidence in their subjective assessments. Real experience in applying the approach developed in selecting the most appropriate tender at the Monash municipal government has reinforced these findings.

5. Conclusion

The AHP method is widely used for tackling MA decision problems in real situations. Despite its simplicity in concept and efficiency in computation, it suffers from a few shortcomings. To improve the AHP method, this paper presents an MA approach using fuzzy pairwise comparison for effectively solving the general MA decision problem involving qualitative data.

An empirical study of a real selection situation faced by Monash municipal government in Victoria, Australia is conducted using the approach developed. It shows that the approach developed is favorable for solving practical MA problems involving qualitative data. The underlying concept of the approach developed is simple and comprehensible, and the computation involved is efficient. In particular, the approach developed can adequately handle the inherent uncertainty and imprecision of the human decision-making process and provide the flexibility and robustness needed for the DM to better understand the decision problem and their decision behaviors. These merits of the approach developed facilitate its use in real situations for making effective decisions.

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