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# Anomalous dimension, chiral phase transition and inverse magnetic catalysis in soft-wall AdS/QCD

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ABSTRACT

A modified soft-wall AdS/QCD model with a $z$-dependent bulk scalar mass is proposed. We argue for the necessity of a modified bulk scalar mass from the quark mass anomalous dimension and carefully constrain the form of bulk mass by the corresponding UV and IR asymptotics. After fixing the form of bulk scalar mass, we calculate the mass spectra of (axial-)vector and pseudoscalar mesons, which have a good agreement with the experimental data. The behavior of chiral phase transition is also investigated, and the results are consistent with the standard scenario and lattice simulations. Finally, the issue of chiral magnetic effects is addressed. We find that the inverse magnetic catalysis emerges naturally from the modified soft-wall model, which is consistent with the recent lattice simulations.



## 1. Introduction

Quantum chromodynamics (QCD) in the low energy regime is well known to be very tough to solve for its nonperturbative nature. To tackle the problems of strong interaction, nonperturbative approaches must be used, such as the chiral perturbation theory [1] and the lattice gauge theory [2]. In recent years, another nonperturbative approach motivated from string theory has been developed and used in many fields which are relevant to the strong coupling problems, i.e., the Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence [3–5], which establishes the duality between the weakly coupled supergravity in $AdS_5$ and the $\mathcal{N}=4$ super Yang–Mills gauge theory in the boundary. Particularly, this approach has been used to probe the low energy nonperturbative regime of QCD, and it is usually called holographic QCD or AdS/QCD. There are roughly two approaches in this direction, i.e., the top-down approach using certain brane construction from string theory to characterize low energy hadron physics [6–8] and the bottom-up approach seizing the fundamental features of low energy QCD, such as the chiral symmetry breaking and confining property, to build and constrain the bulk theory. In this work, we mainly focus on the bottom-up approach by which many models have been constructed in recent years, such as the well-known hard-wall and soft-wall models or the light-front AdS/QCD model [9–12]. The hard-wall model [9,10] has a finite cut-off in the extra dimension which mimics the $\Lambda_{QCD}$ energy scale. The chiral symmetry breaking is well reproduced in this model, however, it cannot attain the linear Regge behavior of hadron spectrum which is a typical characterization of QCD confinement. To amend this defect, the soft-wall model [11] was constructed by introducing an infrared (IR) suppressed dilaton term. The soft-wall model has a correct linear Regge behavior, yet it cannot realize the chiral symmetry breaking consistently. A number of studies have been done to improve the soft-wall model, such as [13–17], and also applied to calculate various low energy properties of QCD.

Study of QCD phase transitions under the influence of magnetic field has attracted much attention in recent years [18–30], partly because magnetic fields with strength around $\sqrt{eB} \sim 0.1$–$1.0$ GeV can now be generated in the noncentral heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) [31–33] and the Large Hadron Collider (LHC), and partly because it is such an issue that lattice simulations can approach to study the essential properties of strongly interacting matter, which are related to the vacuum

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and phase structure of QCD. Another motivation is that strong interaction and high magnetic fields coexist during the strong and electroweak phase transitions in the early universe [34,35], and the same happens in certain neutron stars like magnetars [36]. In the research of deconfining and chiral phase transitions under external magnetic field, many works including lattice simulations [37–42] revealed the property of magnetic catalysis (MC), i.e., the increase of chiral condensate or phase transition temperature $T_c$ with magnetic field $B$. However, recent lattice results [43,44] indicated that the transition temperature $T_c$ decreases with increasing $B$, i.e., an inverse magnetic catalysis (IMC), which has spurred the investigation of this specific field and has motivated the study in the holographic framework [45–53]. Most holographic QCD models have the property of MC, i.e., $T_c(B)$ increases with $B$. For exceptions one can refer to [51–53]. Authors of [51] studied the chiral magnetic effects within the Sakai–Sugimoto model, and showed that the IMC appears only at finite chemical potential. The author of [52] found that the IMC manifests in the deconfining phase transition within the hard-wall model and the holographic duals of flavored and unflavored $\mathcal{N}=4$ super Yang–Mills theories on $\mathbb{R}^3 \times S^1$. Ref. [53] showed the same property exists in the soft-wall model, however, the authors also showed that the chiral phase transition has the behavior of MC, contradicting the deconfining transition behavior.

In this work, we put emphasis on the chiral magnetic effects in a soft-wall AdS/QCD model with the modified conformal mass of bulk scalar field. We argue that a $z$-dependent bulk mass is necessary for the mass anomalous dimension in QCD theoretically and for the mass split of chiral parters phenomenally, and is also indispensable for the realization of chiral phase transition. To characterize QCD as real as possible, we carefully constrain the form of the IR-modified conformal mass and the parameters in the model by the meson spectrum and the chiral transition temperature $T_c$. Under an AdS black hole background with constant magnetic field, the behavior of chiral phase transition will be studied following [54,55]. We find that in our model the chiral transition temperature $T_c$ decreases with the increasing magnetic field $B$, which is consistent with the IMC revealed by the recent lattice simulations. It should be noted that the magnetized AdS black hole solution we used was obtained by solving the Einstein–Maxwell system in [56,57], while a full background solution should have been attained by solving the Einstein–Maxwell–Dilaton system when taking the dilaton effect into consideration. As a preliminary try, we just take the simplified version without the back-reaction of dilaton, and only consider the perturbative background solution up to order $B^2$, following [52], so all the results related to the magnetic field $B$ are only valid at small $B$. However, that is enough for us to get the convincing qualitative behavior of the chiral transition under the influence of magnetic field in our model and compare with the other ones. What's more, the quantitative results of IMC consistent with lattice simulations can even be extracted.

The paper is organized as follows. In Sec. 2, we set up the thermal background and give the outline of the modified soft-wall AdS/QCD model. In Sec. 3, we argue and specify the form of the modified bulk scalar mass, then we calculate the mass spectra of (axial-)vector and pseudoscalar mesons and the temperature dependent chiral condensate, which will be found that have a good agreement with the experimental data or lattice results. The parameters of the model will be fixed in the process. In Sec. 4, we turn to the main theme of the paper, i.e., the chiral magnetic effects in the modified soft-wall model. The magnetized AdS black hole solution is reviewed firstly [57], then the chiral thermal transition under the constant magnetic field will be studied. The magnetic dependent behavior of chiral condensate at fixed temperatures is also investigated. In Sec. 5, we give a short summary and conclusion for the study.

## 2. The modified soft-wall model with $z$-dependent bulk mass

As we mainly concern the finite temperature effects, let us consider directly the finite temperature case by introducing an AdS–Schwarzchild black hole with the metric ansatz:

$$ds^2 = e^{2A(z)}\left(-f(z)dt^2 + dx_i dx^i + \frac{1}{f(z)}dz^2\right) \tag{1}$$

with

$$A(z) = -\log\frac{z}{L}; \quad f(z) = 1 - \frac{z^4}{z_h^4}, \tag{2}$$

where $z_h$ is the black hole horizon defined by $f(z_h)=0$, and $L$ is the AdS curvature radius. Below we will set $L=1$ in the calculation. The temperature of the system is then defined by the Hawking formula:

$$T = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z\to z_h} = \frac{1}{\pi z_h}, \tag{3}$$

from which we see that the case with zero temperature corresponds to $z_h \to \infty$ or $f(z)=1$.

The action of the modified soft-wall model can be written as follows:

$$S = -\frac{1}{\kappa}\int d^5x\,\sqrt{-g}\,e^{-\Phi(z)} \times \mathrm{Tr}\left[|DX|^2 + m_5^2|X|^2 + \lambda|X|^4 + \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right], \tag{4}$$

where $D^M X = \partial^M X - iA_L^M X + iXA_R^M$, $g$ the determinant of the metric, and $\Phi(z) = \mu_g^2 z^2$ the dilaton profile. By comparing the two-point correlation function of scalar operator and vector current with the QCD results, the parameters $\kappa$ and $g_5$ can be determined as $\frac{\kappa}{L} = \frac{16\pi^2}{N_c}$ with $N_c$ the color number and $g_5^2 = \frac{3}{4}$ [10,13]. The action also includes a bulk scalar field $X$ and the chiral gauge fields $A_{L,R}^M = A_{L,R}^{M\,a} t^a$ with $t^a$ the generators of $SU(2)_F$ satisfying $\mathrm{Tr}[t^a t^b] = \delta^{ab}/2$. These bulk fields are dual to relevant QCD operators at the boundary $z=0$. The mass of bulk scalar is determined as $m_5^2 = -3$ by the AdS/CFT dictionary $m_5^2 = \Delta(\Delta-4)$ and $\Delta = 3$, which is the dimension of dual operator $\bar{q}_R q_L$ [10]. However, as QCD is not a conformal field theory, we indeed need a $z$-dependent bulk scalar mass which will deviate from $-3$ when $z \neq 0$, so we take $m_5^2(z) = -3 + m(z)$. We will argue for the necessity to consider a $z$-dependent bulk scalar mass and specify the form of $m_5^2(z)$. In general, the bulk scalar $X$ could be decomposed into the scalar meson field $S(x,z)$ and pseudoscalar meson field $\pi(x,z) = \pi^a(x,z)t^a$ in the form of $X = (\frac{\chi}{2} + S)e^{2i\pi}$, where $\chi(z)$ is the vacuum expectation value (VEV) of $X$ which is dual to the chiral condensate $\langle\bar{q}q\rangle$. The bulk gauge field strength has the form as follows:

$$F_{L,R}^{MN} = \partial^M A_{L,R}^N - \partial^N A_{L,R}^M - i[A_{L,R}^M, A_{L,R}^N], \tag{5}$$

where the gauge fields $A_{L,R}^M$ can be recombined into the vector field $V^M = \frac{1}{2}(A_L^M + A_R^M)$ and the axial-vector field $A^M = \frac{1}{2}(A_L^M - A_R^M)$, then the action (4) can be rewritten as

$$S = -\frac{1}{\kappa}\int d^5x\,\sqrt{-g}\,e^{-\Phi(z)} \times \mathrm{Tr}\left[|DX|^2 + m_5^2|X|^2 + \lambda|X|^4 + \frac{1}{2g_5^2}(F_V^2 + F_A^2)\right] \tag{6}$$

with

$$F_V^{MN} = \partial^M V^N - \partial^N V^M - i[V^M, V^N] - i[A^M, A^N], \tag{7}$$

$$F_A^{MN} = \partial^M A^N - \partial^N A^M - i[V^M, A^N] - i[A^M, V^N], \tag{8}$$

and the covariant derivative of $X$ becomes $D^N X = \partial^N X - i[V^N, X] - i\{A^N, X\}$.

We note that the only difference of action (4) from the original soft-wall model [11] is the $z$-dependent bulk scalar mass $m_5(z)$ and a quartic term of $X$ which are necessary for a consistent description of meson spectrum [14–17] and for the realization of chiral phase transition [54,55].

## 3. Bulk scalar mass, meson spectrum and chiral thermal transition

In this section, we will specify the form of the bulk scalar mass $m_5(z)$ which, as noted above, is critical for a reasonable description of the meson spectrum and the chiral thermal transition. In the meantime, the mass spectra of the (axial-)vector and pseudoscalar mesons will be calculated, and the behavior of chiral thermal transition will be investigated. The parameters of the model will be fixed by comparing relevant quantities with the experimental or lattice results.

As has been said, the VEV of bulk scalar field $\chi(z)$ is dual to the quark condensate $\langle\bar{q}q\rangle$, and $m_5$ is related to the dimension of $\langle\bar{q}q\rangle$ by the standard AdS/CFT dictionary $m_5^2 = \Delta(\Delta-4)$, so the quark mass anomalous dimension might induce a modified bulk mass $m_5(z)$ running with $z$. From the respect of QCD, the trace of the stress tensor of quark sector can be written as $T_\mu^\mu = \sum m_q\langle\bar{q}q\rangle$ with $\sum$ summing over different flavors. For arbitrary energy scale $\mu$, we have $d\langle\bar{q}q\rangle/d\log\mu = (\Delta-3)\langle\bar{q}q\rangle$ with $\Delta$ the actual scaling dimension (canonical scaling dimension plus anomalous dimension) of $\langle\bar{q}q\rangle$. As $T_\mu^\mu$ is renormalization-group-invariant, it scales classically and we have $dT_\mu^\mu/d\log\mu = 0$. Taking the derivative of $T_\mu^\mu = \sum m_q\langle\bar{q}q\rangle$ with respect to $\log\mu$, we then obtain

$$\begin{aligned} 0 &= \sum m_q\,\gamma(\alpha)\,\langle\bar{q}q\rangle + \sum m_q\,(\Delta-3)\,\langle\bar{q}q\rangle \\ &= (\gamma(\alpha)+\Delta-3)\,T_\mu^\mu \end{aligned} \tag{9}$$

with the quark mass anomalous dimension defined as $\gamma(\alpha) \equiv \frac{1}{m_q}\frac{d\,m_q}{d\log\mu}$, from which we get $\Delta = 3-\gamma(\alpha)$. As a result, a $z$-dependent bulk scalar mass $m_5(z)$ might be necessary by including the quark mass anomalous dimension in the boundary. We note that a similar argument for the anomalous dimension of the gluon condensate $\langle\mathrm{Tr}\,F_{\mu\nu}^2\rangle$ was given in [62]. However, to fix the specific form of $m_5(z)$ in the soft-wall framework is another issue as one needs to pay attention to the low energy phenomena of QCD, such as the meson spectrum or the chiral and deconfining thermal transitions, etc. Below we will turn to the calculation of the mass spectra of vector, axial-vector and pseudoscalar mesons, and the temperature dependence of chiral condensate will also be addressed. The form of $m_5(z)$ and the parameters of the model will be fixed. After fixing all the ones, we will see that this model can give a very good description of both meson spectrum and chiral phase transition, which are consistent with lattice simulations and experiments.

There have been many studies on the calculation of meson spectrum in a variety of soft-wall models. For some of them one can refer to [14–17]. Study of chiral phase transition in the framework of soft-wall models has also attracted considerable attentions, e.g. [54,55,58]. We only give the corresponding equation of motion (EOM) concerned and then present the numerical calculations which will be compared with the experimental or lattice results. In certain places, the conditions to specify the bulk scalar mass $m_5(z)$ and the model parameters will be noted.

### 3.1. UV condition of the bulk scalar mass $m_5(z)$

Let us first give the EOM of the VEV of bulk scalar field $\chi(z)$ which is closely related to the chiral condensate $\langle\bar{q}q\rangle$. Inserting $X = (\frac{\chi}{2}+S)e^{2i\pi}$ in the action (4), one can derive the EOM of $\chi(z)$ as

$$\begin{aligned} &\chi''(z) + \left(3A'(z) - \Phi'(z) + \frac{f'(z)}{f(z)}\right)\chi'(z) \\ &\quad - \frac{e^{2A(z)}}{f(z)}\partial_\chi V(\chi(z)) = 0 \end{aligned} \tag{10}$$

with the potential $V(\chi) = \frac{1}{2}m_5^2\chi^2 + \frac{1}{8}\lambda\chi^4$.

According to the AdS/CFT dictionary [10], the UV expansion of $\chi(z)$ has the form as follows:

$$\chi(z) = m_q z + \sigma z^3 + \cdots \tag{11}$$

with $m_q$ the current quark mass and $\sigma$ related to the chiral condensate. Now from Eq. (10), the UV asymptotic expression of $m_5(z)$ can be derived as

$$m_5^2(z\sim 0) = -3 - (2\mu_g^2 + \lambda m_q^2/2)z^2 + \cdots. \tag{12}$$

It should be remarked that in this model we do not consider the effects of the quark mass anomalous dimension on the near-boundary behavior of $m_5(z)$ or $\chi(z)$. This is because the UV region of QCD has been identified as the boundary of the bulk theory with conformal property, as dictated by the AdS/CFT dictionary. It is just this identification that validates the bottom-up approach in which the UV physics of QCD can be matched with the boundary ones. However, in the top-down approach with a bias of theoretical considerations, it should be given more careful treatments as the supergravity approximation usually makes the boundary (the UV region of QCD) inaccessible from the bulk. A way to circumvent this problem is to fix the anomalous dimension at certain scale using perturbative QCD, as has been done in [62]. However, in the model addressed here, the UV behavior of $m_5(z)$ or $\chi(z)$ has only little effects on the low-energy phenomena we concerned, and the most important effect comes from the IR region ($z\to\infty$) of the bulk theory, which is identified as the low-energy QCD, so even we might neglect the QCD anomalous dimension on the boundary, this will not affect the results obtained in this work. The IR condition of $m_5(z)$ will be given below by constraint from the mass split of chiral partners, i.e., the vector and axial-vector mesons. Now we turn to the calculation of meson spectrum.

### 3.2. Vector meson

In the axial gauge $V_z = 0$, the EOM of vector meson can be derived from the action (6) as

$$-v_n''(z) + V_v(z)v_n(z) - m_n^2 v_n(z) = 0, \tag{13}$$

where the Schrödinger-like potential has the form:

$$V_v(z) = \frac{1}{2}(A''(z) - \Phi''(z)) + \frac{1}{4}(A'(z) - \Phi'(z))^2. \tag{14}$$

It should be noted that the meson spectrum will be calculated at zero temperature with $f(z)=1$. We see that the EOM of vector meson only includes the parameter $\mu_g$, the inverse of which could be related to the $\Lambda_{\mathrm{QCD}}$ energy scale. This is the same as the original soft-wall model [11], and analytical solution indeed exists in this case. To simplify the discussion, we only present the numerical results, which are shown in Table 1, and $\mu_g = 430$ MeV has been fixed by fitting the experimental data.

**Table 1**
Vector meson spectrum from experiment and our model calculation. The experimental data is taken from [59].

| $\rho$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Exp. (MeV) | 775.5 | 1465 | 1570 | 1720 | 1909 | 2150 | – |
| Theory (MeV) | 860 | 1216 | 1489 | 1720 | 1923 | 2107 | 2275 |

### 3.3. Axial-vector and pseudoscalar mesons

To get the EOM of axial-vector meson, we need to decompose the bulk gauge field $A_\mu^a$ into $A_\mu^a = A_{\mu\perp}^a + \partial_\mu \phi^a$ in the $A_z = 0$ gauge. The radial component $\partial_\mu \phi^a$ has a mixing with the pseudoscalar meson, as we will see later. The EOM of axial-vector meson is then derived from the action (6) as

$$-a_n''(z) + V_a(z) a_n(z) - m_n^2 a_n(z) = 0 \tag{15}$$

with the Schrödinger-like potential

$$V_a(z) = \frac{1}{2}(A''(z) - \Phi''(z)) + \frac{1}{4}(A'(z) - \Phi'(z))^2 + g_5^2 e^{2A(z)} \chi^2(z). \tag{16}$$

We note that the only difference of the EOM of axial-vector meson from that of vector meson is the last term of the potential $V_a(z)$ which induces the mass split of chiral partners phenomenally, and in turn constrains the VEV of bulk scalar field $\chi(z)$ to be linear at large $z$, i.e., $\chi(z \to \infty) = a\, z$. Inserting this asymptotic expression in Eq. (10) and setting $f(z) = 1$, we obtain the IR expression of $m_5(z)$:

$$m_5^2(z \to \infty) = (-2\,\mu_g^2 - \lambda\, a^2/2)\, z^2 - 3 + \cdots. \tag{17}$$

In consideration of Eqs. (12) and (17), the simplest form of $m_5^2(z)$ which satisfies the given UV and IR asymptotics will be used, i.e., $m_5^2(z) = -3 - \mu_m^2 z^2$. Now all the ingredients of the modified soft-wall model are known. The EOM of pseudoscalar meson can also be obtained from the action (6) following the same procedure of derivation of the (axial-)vector meson:

$$\partial_z \left( e^{A-\Phi} \partial_z \phi_n^a \right) + g_5^2\, \chi^2 e^{3A-\Phi} (\pi_n^a - \phi_n^a) = 0, \tag{18}$$

$$m_n^2\, \partial_z \phi_n^a - g_5^2\, e^{2A} \chi^2 \partial_z \pi_n^a = 0. \tag{19}$$

Before calculating the mass spectra of axial-vector and pseudoscalar mesons, we need to specify the other parameters appearing in the model, i.e., $m_q$, $\mu_m$ and $\lambda$, in addition to the parameter $\mu_g$ which has been fixed by the vector meson spectrum to be $\mu_g = 430$ MeV. We found that $m_q$ is most relevant to the mass of the ground state of pseudoscalar meson, which is reasonable as the $\pi$ meson is usually considered as a pseudo Goldstone boson which attains mass by the explicit chiral symmetry breaking originating from the nonzero current quark mass. The chiral transition temperature $T_c$ is most sensitive to the value of $\mu_m$, which indicates a close relation between the modified bulk scalar mass and chiral phase transition. $\lambda$ is associated tightly with the value of $\sigma$ and so is the VEV of bulk scalar, which determines the mass difference of vector and axial-vector mesons. Physically this is also reasonable as we know that it is just chiral symmetry breaking induces the mass split of chiral partners. It should be noted that a satisfying way to specify the real chiral condensate indeed needs holographic renormalization, which will also be referred in the study of chiral thermal transition later. Now we give the best fit of the parameters to the experimental or lattice data: $m_q = 6.3$ MeV, $\mu_m = 1.46$ GeV and $\lambda = 1.76$. The numerical results of the spectra of axial-vector and pseudoscalar mesons are presented in Table 2 and 3.

**Table 2**
Axial-vector meson spectrum from experiment and our model calculation. The data is taken from [59].

| $a_1$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Exp. (MeV) | 1230 | 1647 | 1930 | 2096 | 2270 | – |
| Theory (MeV) | 1283 | 1630 | 1860 | 2057 | 2235 | 2398 |

**Table 3**
Pseudoscalar meson spectrum from experiment and our model calculation. The data is taken from [59].

| $\pi$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Exp. (MeV) | 139.6 | 1300 | 1812 | – |
| Theory (MeV) | 138.6 | 1473 | 1776 | 1995 |

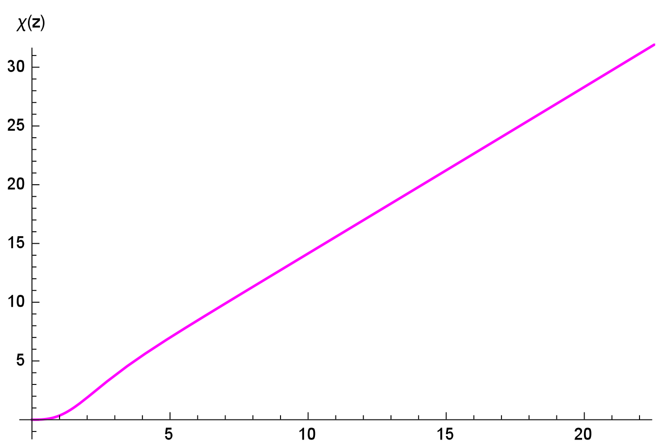


**Fig. 1.** The profile of the VEV of bulk scalar field $\chi(z)$ at $T = 0$.

In calculating the axial-vector meson spectrum, we need the VEV of bulk scalar $\chi(z)$ which can be obtained by solving Eq. (10) at zero temperature ($f(z) = 1$). Details for solving the EOM of $\chi(z)$ at finite temperature can be found in [55,58] and will be outlined later when considering the chiral thermal transition. The profile of $\chi(z)$ is plotted in Fig. 1, where the linear asymptotics at large $z$ can be seen clearly. The value of $\sigma$ can be extracted from the UV expansion of $\chi(z)$ and in our case $\sigma \simeq 0.4$ GeV$^3$. The chiral condensate is related to $\sigma$ by a scaling factor proportional to $N_c$ [53,58].

### 3.4. Chiral thermal transition

Now we consider the behavior of chiral thermal transition. The starting point is Eq. (10), i.e., the EOM of bulk scalar VEV $\chi(z)$, which is dual to the chiral condensate $\langle \bar{q}q \rangle$ by the AdS/CFT dictionary. As our model involves two light flavors, we only discuss chiral phase transition in the two-flavor case. As is well known, in the standard scenario, the chiral phase transition in the chiral limit is second order, while it becomes crossover when the quark mass is finite [60,61]. The chiral transition temperature $T_c$ with physical quark mass is around 170 MeV in the two-flavor case. We remark that another soft-wall model with modified dilaton term has been proposed to address the issue of chiral phase transition in [54,55], and there the correct behaviors of chiral phase transition with both zero and finite quark masses have been realized. In our model, with modified bulk scalar mass, the behaviors of chiral phase transition are the same as those in [54,55].

The temperature dependence of chiral condensate can be derived from Eq. (10) with a regular condition at horizon $z_h$ imposed to get the complete solution, from which the value of $\sigma$ at each temperature can be extracted [55,58]. Using the above fixed pa-

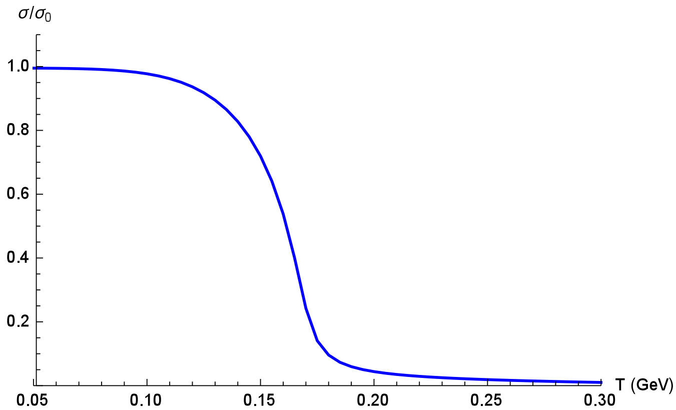


**Fig. 2.** The temperature dependent behavior of the rescaled chiral condensate $\sigma/\sigma_0$.

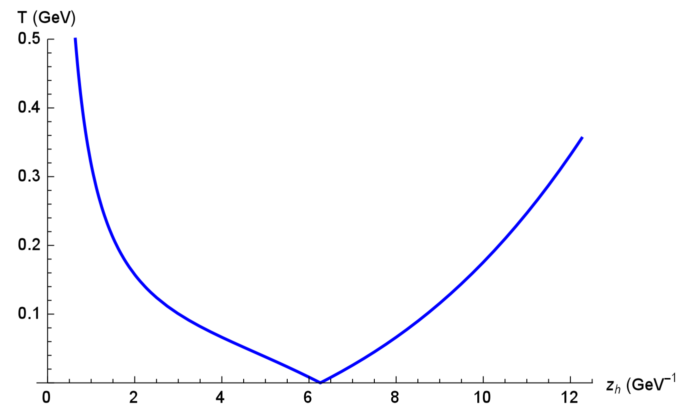


**Fig. 3.** The temperature $T$ as a function of $z_h$ for $\mathcal{B} = 0.1$ GeV$^2$.

rameters, the temperature dependent chiral condensate $\sigma(T)$ is obtained and the numerical result is shown in Fig. 2, where we have rescaled $\sigma$ by the value $\sigma_0 = \sigma(0)$.

From Fig. 2 we can see obviously the crossover behavior of chiral thermal transition, which is completely consistent with the lattice simulations. As noted above, we have tuned the parameter $\mu_m$ to obtain the chiral transition temperature $T_c \simeq 170$ MeV. The zero-temperature value $\sigma(0)$ is approximated by $\sigma(T)$ at $T = 0.0001$ GeV in the real calculation, as we can see that $\sigma(T)$ is almost unchanging when $T < 0.07$ GeV. It should be noted that the solution of $\sigma(T)$ shown in Fig. 2 is not the unique one from solving Eq. (10), which is just as the case in [54,55], e.g., there is also the negative $\sigma(T)$ at low temperatures with the same structure as that in [54,55]. To select the physical solution (as shown in Fig. 2), we need to choose the most stable one thermodynamically by comparing the free energy, which can be derived from the on-shell action of the bulk scalar VEV as

$$\mathcal{F} \equiv \frac{F}{V_3} = -\frac{1}{8}\int dz e^{5A-\Phi}\lambda\chi^4 - \frac{1}{2}(\chi e^{3A-\Phi} f\chi')|_\epsilon. \tag{20}$$

We would like to remark here that both in [54,55] and in this work we need a quartic interaction term of the bulk scalar with positive coupling to realize the spontaneous chiral symmetry breaking and the correct behavior of chiral phase transition. According to the study, if we tune the coupling of the quartic term smaller and smaller, the value of $\sigma$ at near-zero temperature will increase faster and faster until the solution of the bulk scalar VEV $\chi(z)$ cannot be found. As for the modified dilaton profile or the bulk scalar mass, we only need one of these two conditions, as shown in [54,55] and this paper.

It should be emphasized that the actual chiral condensate is not $\sigma$, but related to $\sigma$ by a scaling factor proportional to $N_c$ [53, 58]. However, this will not affect our result as we only consider the rescaled quantity $\sigma/\sigma_0$. Another issue that needs to be clarified is that there indeed exists some indeterminacy in the calculation of $\sigma$, which can be seen from the UV expansion of $\chi(z)$:

$$\begin{aligned}\chi(z) &= m_q z + \sigma z^3 + \frac{1}{4}(4m_q\mu_g^2 + m_q^3\lambda - 2m_q\mu_m^2)z^3\log(cz) + \cdots \\ &= m_q z + \left[\sigma + \frac{1}{4}(4m_q\mu_g^2 + m_q^3\lambda - 2m_q\mu_m^2)\log c\right]z^3 \\ &\quad + \frac{1}{4}(4m_q\mu_g^2 + m_q^3\lambda - 2m_q\mu_m^2)z^3\log z + \cdots\end{aligned} \tag{21}$$

with $c$ an arbitrary constant. However, as the $\log(cz)$ term is independent of temperature, the quantity $\sigma(T) - \sigma(T_0)$ with fixed temperature $T_0$ is definite without ambiguity, and the final form of chiral condensate can be determined by the fact that $\sigma(T)$ approaches to zero at large $T$, which is just the case shown in Fig. 2.

## 4. Chiral magnetic effects and inverse magnetic catalysis

In this section, we investigate the behavior of chiral thermal transition under a background magnetic field, which is the primary motivation of the paper. The magnetic field in the boundary can be modeled by the flavor-diagonal bulk gauge field which is naturally incorporated in the action (4) of the modified soft-wall model. The electric charge of light quarks supply as the source of the background magnetic field defined by the VEV of vector gauge field in the action (6). As a preliminary study, we just take the perturbative magnetized AdS black hole solution of the Einstein–Maxwell system with a constant magnetic field in the $x_3$ direction up to order $B^2$, which has the form as follows [56,57]

$$ds^2 = e^{2A(z)}(-f(z)dt^2 + e^{P(z)}(dx_1^2 + dx_2^2) + e^{Q(z)}dx_3^2 + \frac{1}{f(z)}dz^2) \tag{22}$$

with

$$f(z) = 1 - \frac{z^4}{z_h^4} + \frac{2B^2}{3}z^4\log\frac{z}{z_h}, \tag{23}$$

$$P(z) = -\frac{4}{3}B^2\int_0^z dx\frac{x^3\log(x/z_h)}{1-(x/z_h)^4}, \tag{24}$$

$$Q(z) = \frac{8}{3}B^2\int_0^z dx\frac{x^3\log(x/z_h)}{1-(x/z_h)^4}, \tag{25}$$

where we can see obviously that $f(z_h) = 0$ with $z_h$ the horizon of the black hole. It should be remarked that the real magnetic field $\mathcal{B}$ in the boundary is related to the bulk magnetic field $B$ by a relation $e\mathcal{B} = 1.6\,B/L$ with $e$ the elementary electric charge and $L$ the AdS curvature radius [53]. For convenience, we will set $e$ and $L$ to one below.

The Hawking temperature can then be obtained as

$$T = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z\to z_h} = \frac{1}{4\pi}\left|\frac{4}{z_h} - \frac{2}{3}B^2 z_h^3\right| = \frac{1}{4\pi}\left|\frac{4}{z_h} - \frac{25}{96}\mathcal{B}^2 z_h^3\right|. \tag{26}$$

Generally, there are two possible values of $z_h$ for a given $\mathcal{B}$ and $T$, as shown in Fig. 3, where it is obviously seen that the two branches of the curve $T(z_h)$ join at a point with $T(z_{h0}) = 0$. Physically, one can only choose the descending part with $z_h \leqslant z_{h0}$ from analysis of the scaling behavior of free energy with $T$ at high temperatures [53].

Given the magnetized thermal background, we are ready to investigate the property of chiral thermal transition in the magnetic field. The EOM of the VEV of bulk scalar field $\chi(z)$ can be derived from the action (4) as

$$\chi''(z)+\left(3A'(z)-\Phi'(z)+\frac{f'(z)}{f(z)}\right)\chi'(z) - \frac{e^{2A(z)}}{f(z)}\left(m_5\,\chi(z)+\frac{\lambda}{2}\chi^3(z)\right)=0, \tag{27}$$

which has the same form as Eq. (10), but with a different $f(z)$ containing a magnetic term. The Eq. (27) can be tackled similarly as Eq. (10), and the value of $\sigma$ as a function of $\mathcal{B}$ and $T$ can be extracted from the UV expansion of $\chi(z)$. For fixed magnetic field $\mathcal{B}$, the numerical results of $\sigma(T,\mathcal{B})$ are presented in Fig. 4, where we can already see an IMC effect, i.e., the transition temperature decreases with the increasing magnetic field. It should be noted that in Fig. 4 we also ignore other solutions of $\sigma(T)$ which is unfavorable thermodynamically, as in Sec. 3.4. To characterize this effect quantitatively, we need to extract the chiral transition temperature $T_c$ which can be defined by the extremum of $\partial\sigma/\partial T$. The results are shown in Fig. 5, where one can see clearly that $T_c$ decreases monotonously with $\mathcal{B}$, which is consistent with the recent lattice indications for the IMC phenomenon [43,44].

We shall remark that the conclusions for the case of large enough magnetic fields are indefinite from the sense that the perturbative magnetized background solution we used is only valid for small $B$, and at low temperatures far from $T_c$, even the results with small magnetic field will be unconvincing as the large $z_h$ would lead the high order terms of $B$ to be unnegligible. Nevertheless, with this in mind, we give the numerical calculations of $(\sigma(T,\mathcal{B})-\sigma(T,0))/\sigma(0,0)$ at fixed temperatures in Fig. 6.

We see from Fig. 6 that $\sigma$ increases monotonously with $\mathcal{B}$ at low temperatures, and falls off gradually with $\mathcal{B}$ at higher temperatures. As $T$ is high enough, the effect of magnetic field on $\sigma$ becomes small. This nonmonotonous behavior of the magnetic dependence of chiral condensate at different temperatures is consistent with the lattice simulation [44], at least qualitatively. Though the analysis for the cases of low temperatures and large magnetic fields has gone beyond the apply condition of the background solution, it deserves for us to show the results which might give some guidance to the possible behavior had we taken into consideration the full magnetized background solution.

## 5. Summary and conclusions

In this paper, we have proposed a modified soft-wall AdS/QCD model with a $z$-dependent bulk scalar mass which has been proved to be crucial for a consistent description of both meson spectrum and chiral phase transition in the soft-wall framework. We argued from the respect of quark mass anomalous dimension for the necessity of a modified bulk scalar mass which is constrained by the UV and IR asymptotics. The mass spectra of (axial-)vector and pseudoscalar mesons have been calculated, and

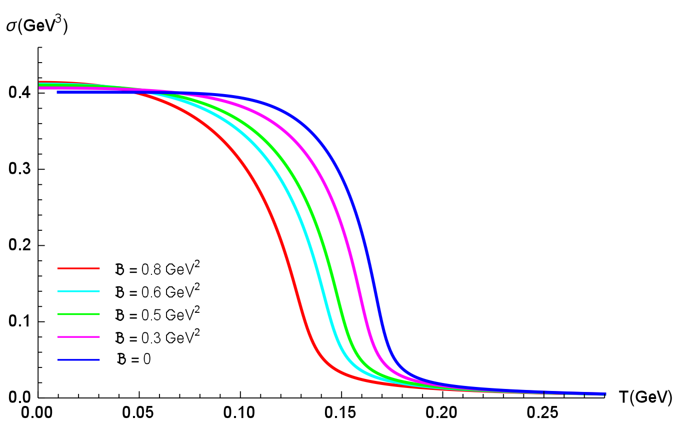


**Fig. 4.** The temperature dependent behavior of $\sigma$ for different magnetic fields $\mathcal{B}$.

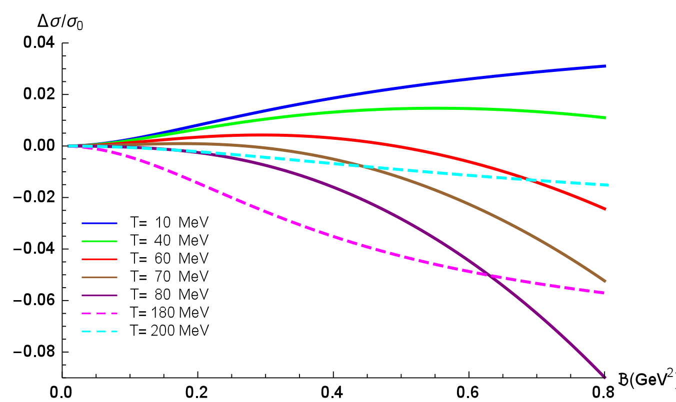


**Fig. 6.** The magnetic dependence of $(\sigma(T,\mathcal{B})-\sigma(T,0))/\sigma(0,0)$ at different temperatures.

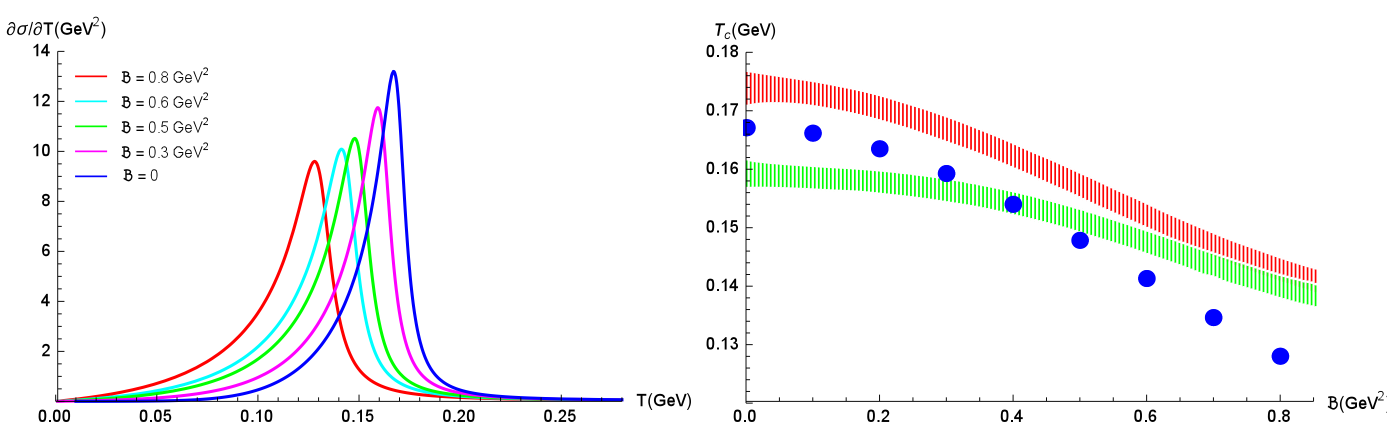


**Fig. 5.** The derivative $\partial\sigma/\partial T$ as a function of $T$ for different magnetic fields $\mathcal{B}$ (left) and the relation of chiral transition temperature $T_c$ with magnetic field $\mathcal{B}$ (right). In the right panel, the green and red bands are the continuum of lattice results determined from the renormalized chiral condensate $\bar{u}u^r+\bar{d}d^r$ and the strange quark number susceptibility, respectively [43], and the blue points denote the numerical results extracted from the extremum of $\partial\sigma/\partial T$ in our model. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the results have a good agreement with the experimental data. The behavior of chiral thermal transition has also been investigated. We remark that in our model the behavior of chiral phase transition consistent with the standard scenario can be realized both in the chiral limit and in the case of finite quark mass, as has been said.

As the main motivation of the paper, we have studied the chiral magnetic effects under a magnetized AdS black hole background, which has been solved from the Einstein–Maxwell system in [57] and has been used to address the similar issues in the original soft-wall AdS/QCD model [53]. It showed that there is no IMC in the original soft-wall model, which can be expected as this model cannot reproduce the chiral symmetry breaking consistently, and furthermore it can neither give a consistent description of meson spectrum nor a correct realization of chiral phase transition. In our model, a modified bulk scalar mass $m_5(z)$ has been introduced to improve the soft-wall model. It is remarkable that only a very simple version of $m_5(z)$ can lead to satisfying results of all the aforementioned low energy physics. We found that the IMC effect emerges naturally from the modified soft-wall model, and the numerical calculations are consistent with the lattice simulations [43,44].

We would like to remark that the full magnetic effects are just incorporated in the expression of $f(z)$ as we consider the chiral magnetic property. It means that the external magnetic field affects the chiral transition behavior only by an indirect interaction with the gauge background in our setup. This is, nevertheless, reasonable physically when we consider effects with light quark masses and around transition temperature for the enhanced influence of the gluonic fluctuations on the quark sector in these cases. However, in the confining phase with low temperatures, the influence of gauge fluctuations on the chiral dynamics will be suppressed, and taken over by the hadronic effects. In this case, one might need to consider the direct interaction of magnetic field with quark matters. This is another reason for that our analyses in the low temperature case are inconclusive, which might also explain the rigidness of the chiral condensate under the influence of magnetic field at low temperatures (see Fig. 4), which is inconsistent with the lattice simulations [44].

We have reached the conclusion that the IMC effect can be realized in the soft-wall model with a simply modified bulk scalar mass, by which a consistent characterization of meson spectrum and chiral phase transition can also be attained. However, to tackle the issue of chiral magnetic effects consistently in the soft-wall framework, we need the full magnetized AdS black hole solution solved from the Einstein–Maxwell–Dilaton system which would be considered in the future.

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