Design of Digital Robust Controller for a Class-D Amplifier Using A2DOF

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Abstract

In recent years, it is desired that the bandwidth of a class-D amplifier is widened using sampling frequencies as low as possible. For example, it is expected in the application of the power supply of a low frequency immunity test, or an audio power amplifier, or the power amplifier of vibration generator. In this paper, it is shown that the bandwidth of the class-D amplifier can be widened to 20[kHz] by an A2DOF (Approximate 2-Degree-Of-Freedom) digital controller with 500[kHz] sampling frequencies. The controller is implemented by a DSP (digital signal processor). It is shown from experiments that 20[kHz] bandwidth can be maintained even if load changes.

Keywords: class-D amplifier; digital robust control; A2DOF; DSP

1. Introduction

In recent years, various kinds of the application of a class-D amplifier is considered. For example, the applications are the power supply of a low frequency immunity test, or the audio power amplifier, or the power amplifier of vibration generator. In these applications, it is expected that the bandwidth of the class-D amplifier is
widened using sampling frequencies as low as possible from the limit implementing digital controllers in microprocessors. In the reference [1-3], sampling frequencies more than several MHz are used. However, good control algorithms cannot be realized in usual microprocessors with this sampling frequency. The authors have already proposed the method [4] that is suitable for designing a controller of PWM power amplifiers. This method is using an A2DOF (Approximate 2-Degree-Of-Freedom) system and the performance is better than the method [5] but the bandwidth is not wide enough. It is desired to widen the bandwidth for various applications. In this paper, it is shown that the bandwidth of the class-D amplifier can be widened to 20[kHz] by the A2DOF digital controller with 500[kHz] sampling frequencies. This proposed digital controller is actually realized by a DSP (Digital Signal Processor). It is shown from experiments that 20[kHz] bandwidth can be maintained even if load changes.

2. Class-D amplifier

A class-D amplifier of Fig.1 has been made. A full-bridge chopper circuit is used for power amplification, and the DC power-supply voltage $E$ is 30 [V]. The values of LC filter are $L_0 = 44$ [μH] and $C_0 = 0.235$ [μF].

If the frequency of a control input $u$ is smaller than that of a carrier wave, the state equation of the class-D amplifier of Fig. 1 is derived from the state equalizing method as follows:

$$
\begin{align*}
\dot{x}(t) &= A_0 x(t) + B_0 u(t) \\
y &= C x(t)
\end{align*}
$$

$$
A_0 = \begin{bmatrix}
\frac{1}{C_0 R_0} & \frac{1}{L_0} \\
-\frac{1}{C_0} & \frac{1}{L_0}
\end{bmatrix} \\
B_0 = \begin{bmatrix}
0 \\
K_p
\end{bmatrix}
$$

(1)

Here $K_p$ is the steady-state gain, and is 0.002. $R_0$ is the total resistance consisting of coil and ON resistance of FET, etc., and is about 2 [Ω]. The delay time depending on AD conversion time and DSP computing time is considered to be an input dead time existing in the controlled object. Moreover a delay element $1/z$ is added to the input of the controlled object for replacing the current feedback with the input feedback and the output feedback.

Then the state equation of the system considering these two delay elements is expressed as follows:

$$
\begin{align*}
x_d(k + 1) &= A_{dw} x_d(k) + B_{dw} v(k) \\
y(k) &= C_{dw} x_d(k)
\end{align*}
$$

(2)

$$
A_{dw} = \begin{bmatrix} A_d & B_d \\ 0 & 0 \end{bmatrix} \\
B_{dw}(k) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
A_d = \begin{bmatrix} e^{A_d T} & e^{A_d (T-L_d)} \int_0^{L_d} e^{A_d \tau} B_d d\tau \\ 0 & 0 \end{bmatrix}
$$

$$
B_d = \begin{bmatrix}
\int e^{-\xi_1(k)} B_d d\tau \\
\xi_1(k) = u(k) \\
C_d = \begin{bmatrix} C & 0 \end{bmatrix} \\
C_{dw} = \begin{bmatrix} C_d & 0 \end{bmatrix}
\end{bmatrix}
$$

Load changes and DC power supply changes are considered as parameter changes of Eq. (1). The parameter changes are transformed to equivalent disturbance input to eq. (1). Therefore, in order to suppress the influence of...
these parameter changes, it is necessary to constitute the control systems whose pulse transfer functions from equivalent disturbances to the output \( y \) become as small as possible in their amplitudes. In the following section, a simple design method to attain such a performance will be presented.

3. Design of A2DOF digital controller

At first the pulse transfer function between the reference input \( r \) and the output \( y \) is set up as follows:

\[
W_{ry}(z) = \frac{(1 + H_1)(1 + H_2)(1 + H_3)(z + H_4)(z - n_1)(z - n_2)}{(1 - n_1)(1 - n_2)(z + H_1)(z + H_2)(z + H_3)(z + H_4)}
\]  

(3)

Here \( n_1, n_2 \) are the zeros of the controlled object (2). If \( H_1, H_2 \) and \( H_3 \) are set up as \( |H_1| > |\text{Re}(H_2)| \), \( |H_1| > |\text{Re}(H_3)| \), then \( W_{ry}(z) \) will be approximated as follows:

\[
W_{ry}(z) \approx \frac{1 + H_1}{z + H_1} \equiv W_m(z)
\]  

(4)

Eq. (4) is specified to realize the required bandwidth. \( W_{ry}(z) \) of Eq. (3) is determined by the following state feedback and feed forward:

\[
v(t) = -F_x (t) + G H_4 r(t) \quad \xi_1(t) + G r \quad x^*(t) = \begin{bmatrix} x_1(t) & x_2(t) & \xi_1(t) & \xi_2(t) \end{bmatrix}^T
\]  

(5)

When the current feedback from \( x_2 \) is replaced with the input feedback and output feedback, the model matching system of only voltage feedback is obtained. The control system added the system for robustness to the model matching system using only output feedback is shown in Fig. 2. \( K(z) \) of Fig. 2 is as follows:

\[
K(z) = \frac{k_1 z^{-1}}{z - 1 + k_1}
\]  

(6)

\( W_{Ob}(z) \) is transfer function between the the equivalent disturbance \( Q \) and the output \( y \) of the model matching system. The transfer functions of \( r - y \) and \( Q - y \) of Fig. 2 are derived as follows:

\[
y = \frac{(1 + H_1)(z - 1 + k_1) W_s(z)}{(z + H_1)(z - 1 + k_2 W_s(z))} r \quad y = \frac{(z - 1) W_s(z)}{(z - 1 + k_2 W_s(z))} W_{Ob}(z) Q
\]  

(7)

\[
W_s(z) = \frac{(1 + H_1)(1 + H_2)(z - n_1)(z - n_2)}{(1 - n_1)(1 - n_2)(z + H_2)(z + H_3)}
\]  

(8)

Here, if \( W_s(z) \approx 1 \) in the control bandwidth needed, then Eqs. (7) is approximated as follows:

\[
y \approx \frac{1 + H_1}{z + H_1} r \quad y \approx \frac{z - 1}{z - 1 + k_2} W_{Ob}(z) Q
\]  

(9)

From Eq. (9), it prove that the characteristics from \( r \) to \( y \) can be determined by \( H_1 \), and the characteristics from \( Q \) to \( y \) can be determined independently by \( k_2 \). This means that the control system of Fig. 2 is the A2DOF, and as much as \( k_2 \) decrease, the sensitivity against disturbance becomes low. When the controller of Fig. 2 is transformed equivalently, the digital integral type A2DOF control system is gotten as shown in Fig. 3.
4. Experimental studies

The experimental setup system is shown in Fig. 4. The A2DOF controller was designed as the sampling frequency 500 [kHz]. The designed controller was equipped with DSP TMS320LF2801. The experimental results of step responses and the closed-loop gain characteristics using A2DOF controller at resistive load \( R_L = 8 \, \Omega \) and \( R_L = 4 \, \Omega \) are shown in Fig. 5 and Fig. 6, respectively. The step responses are almost the same and the bandwidths are maintained over 20 kHz even if load changes. The experimental results using PI controller are shown in Fig. 7 and Fig. 8. The step responses at \( R_L = 4 \, \Omega \) is changed and the bandwidths are decreased to about 15 kHz. From these result it turned out that the control system using the A2DOF controller is robust.

5. Conclusion

In this paper, it has been shown that the bandwidth of the class-D amplifier can be widened to 20 [kHz] and the good robust performance can be realized using the A2DOF digital controller with 500[kHz] sampling frequency. The proposed digital controller was equipped in DSP, and it is verified by experiments that the desirable performances are attained. As a result, the proposed controller can be used for the power supply of immunity tests, the audio amplifier and the power amplifier of vibration generator.

References