VISCOUS UNSTEADY GUST AERODYNAMICS OF
A FLAT PLATE AIRFOIL BY A LOCALLY
ANALYTICAL METHOD

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Abstract—A mathematical model is developed to analyze the unsteady viscous transverse gust aero-
dynamics of a flat plate airfoil in an incompressible laminar flow at moderate values of the Reynolds
number. The steady flow is described by the Navier-Stokes equations. The unsteady viscous flow is
assumed to be a small perturbation to this steady viscous flow, with the resulting system of linear partial
differential equations coupled to the steady flow field through the unsteady boundary conditions.
Solutions for both the steady and unsteady viscous flow fields are obtained by developing locally analytical
solutions. For the steady flow, this is accomplished by first locally linearizing the nonlinear convective
terms in the Navier-Stokes equations. The significant effects of Reynolds number and reduced frequency
on the transverse gust generated unsteady aerodynamics on the airfoil are then demonstrated.

NOMENCLATURE

\( C \)—airfoil chord
\( k \)—reduced frequency, \( \omega C/U_c \)
\( P \)—dimensionless unsteady pressure
\( \text{Re} \)—Reynolds number, \( U_c C/v \)
\( U \)—nondimensional steady velocity in x direction
\( \eta \)—nondimensional unsteady velocity in y direction
\( y \)—coordinate in normal flow direction
\( \Delta x \)—step size in x direction
\( \psi \)—nondimensional unsteady stream function
\( \xi \)—nondimensional unsteady vorticity
\( \omega_o \)—gust frequency
\( C_l \)—unsteady lift coefficient
\( L \)—unsteady lift
\( P \)—dimensionless steady pressure
\( u \)—nondimensional unsteady velocity in x direction
\( U_c \)—magnitude of free-stream velocity
\( x \)—coordinate in mean flow direction
\( \Delta C_p \)—unsteady pressure difference, \( P_{lower} - P_{upper} \)
\( \Delta y \)—step size in y direction
\( \Psi \)—nondimensional steady stream function
\( \zeta \)—nondimensional steady vorticity

INTRODUCTION

The interaction of an airfoil with a convected disturbance, a gust, is of significance to a variety
of applications. As a result, considerable progress has been made in predicting the unsteady
aerodynamic response of an airfoil to a gust. These analyses are typically limited to inviscid
potential flows, with the unsteady flow assumed to be a small perturbation to a uniform mean flow
and the Kutta condition imposed. Models have considered convected gusts transported with the
mean velocity of the flow, including both transverse gusts, Sears [1], and linearly combined
transverse and chordwise gusts, Horlock [2].

The effect of viscosity on flat plate airfoil unsteady aerodynamics, thereby removing the need
for the Kutta condition, has also been briefly considered. Yates [3] formulated an incompressible
viscous theory with a zero thickness boundary layer for a flat plate airfoil at zero mean angle of
attack executing harmonic motions or subject to a convected transverse gust. Also, the low
Reynolds number incompressible Oseen flow model has been used to calculate zero mean angle
of attack oscillating flat plate aerodynamics [4, 5]. These analyses utilize classical aerodynamic
solution techniques, resulting in integral equation solutions. Although such classical models and
solution techniques are of value, advances in numerical techniques permit the flow physics modeling
to be extended. In this regard, Schroeder and Fleeter [6] developed a model and locally analytical
solution to predict the unsteady aerodynamics of a flat plate airfoil executing harmonic torsional
oscillations at small mean angle of attack to an incompressible laminar flow at low Reynolds
number values.
In this paper, the effects of Reynolds number and reduced frequency on the unsteady aerodynamics of a flat plate airfoil generated by a convected transverse gust are analyzed. This is accomplished by developing a mathematical model which significantly extends the modeling and locally analytical solution initially proposed in Ref. [6]. In particular, the model developed herein analyzes the steady viscous flow at moderate values of the Reynolds number past a flat plate airfoil and the unsteady viscous aerodynamic interaction of this steady flow field with a convected transverse gust. The unsteady viscous flow is assumed to be a small perturbation to the steady viscous flow. The steady flow field, described by the Navier–Stokes equations, is nonuniform and nonlinear and is also independent of the unsteady flow. The small perturbation unsteady viscous flow field is described by a system of linear partial differential equations that are coupled to the steady flow field, thereby modeling the strong dependence of the unsteady aerodynamics on the steady flow.

Locally analytical solutions for both the steady and the unsteady viscous flow fields are developed. In this method, the discrete algebraic equations which represent the flow field equations are obtained from analytical solutions in individual grid elements. For the steady viscous flow, this is accomplished by first locally linearizing the nonlinear convective terms in the Navier–Stokes equations. General analytical solutions to the flow field equations are then determined. Locally analytical solutions are then developed by applying these solutions to individual grid elements, with the integration and separation constants determined from the boundary conditions in each grid element. The complete flow field solutions are obtained through the application of the global boundary conditions. It should be noted that the nonlinear character of the complete steady flow field is preserved as the flow is only locally linearized, i.e. independently linearized solutions are obtained in individual grid elements.

MATHEMATICAL MODEL

The two-dimensional flow field together with the cartesian coordinate system are schematically depicted in Fig. 1. For harmonic time dependence at a frequency $\omega$, the flow field is described by the nondimensional continuity and Navier–Stokes equations, written in terms of the vorticity, $\zeta$, and the stream function, $\psi$, in equation (1)

\begin{align}
\nabla^2 \zeta &= \zeta_{xx} + \zeta_{yy} = \text{Re}(k\zeta_x + \bar{u}\zeta_x + \bar{v}\zeta_y), \\
\nabla^2 \psi &= -\zeta,
\end{align}

(1a)

(1b)

where $\zeta = \bar{\zeta} - \bar{u}$, and $\bar{u} = \bar{\psi}_{x}$; $\bar{v} = -\bar{\psi}_{y}$.

The flow field is decomposed into steady and harmonic unsteady components, with the unsteady component assumed to be a small perturbation to the steady component

$$
\tilde{\zeta}(x, y, t) = \zeta(x, y) + e^{i\omega t}\zeta(x, y),
$$

(2a)

Fig. 1. Flow field schematic.
Viscous unsteady gust aerodynamics

\( \dot{\psi}(x, y, t) = \Psi(x, y) + e^{ikx}\psi(x, y), \)  
\( \dot{u}(x, y, t) = U(x, y) + e^{ikx}u(x, y), \)  
\( \dot{v}(x, y, t) = V(x, y) + e^{ikx}v(x, y), \)  
\( \dot{p}(x, y, t) = P(x, y) + e^{ikx}p(x, y), \)

where

\( \xi \approx \zeta, \quad \psi \approx \Psi, \quad u \approx U, \quad v \approx V, \quad p \approx P. \)

The equations describing the steady and unsteady viscous flow fields are determined by substituting equations (2) into (1), and grouping together the time independent and the time dependent terms. For the unsteady flow, the second order terms are neglected as small compared to the first order terms. Also, as the linearized unsteady flow is assumed to be harmonic, the \( e^{ikx} \) is dropped, for convenience.

The resulting coupled nonlinear partial differential equations describing the steady flow field, equation (3), are independent of the unsteady flow. The vorticity equation is nonlinear, with the stream function described by a linear Poisson equation which is coupled to the vorticity equation through the vorticity source term. The pressure is also described by a linear Poisson equation with the source terms dependent on the steady flow field.

\( \nabla^2 \xi = \text{Re}(U_{xx} + V_{yy}), \)
\( \nabla^2 \psi = -\zeta, \)
\( \nabla^2 p = -2(U_x V_y - V_x U_y). \)

The resulting coupled linear partial differential equations describing the unsteady harmonic flow field are given in equation (4). The unsteady flow is coupled to the steady flow field. In particular, in both the unsteady vorticity transport and pressure equations, the variable coefficients are dependent on the steady flow field with the unsteady stream function coupled to the solution for the unsteady vorticity.

\( \nabla^2 \zeta = \text{Re}(ki\xi + U_{xx} + V_{yy} + u_{xx} + v_{yy}), \)
\( \nabla^2 \psi = -\zeta, \)
\( \nabla^2 p = -2[(u_x V_y + v_y U_x) - (v_x U_y + U_y V_x)]. \)

Boundary conditions

The steady flow boundary conditions specify no slip between the fluid and the surface and that the velocity normal to the surface is zero

\( \Psi = \text{constant}, \quad \text{on solid surfaces}, \)
\( \zeta = -U_y = -\Psi_{yy}, \quad \text{on solid surfaces}. \)

The unsteady boundary conditions require that the velocity of the fluid is equal to that of the airfoil surfaces. Thus the unsteady normal velocity boundary condition defines the convected transverse gust whereas the unsteady chordwise velocity component is zero because the fluid is viscous.

\( v(x, o) = -v_g \exp[-ikx], \)
\( u(x, o) = 0. \)

The corresponding unsteady stream function and vorticity boundary conditions are specified from their definitions and the above unsteady boundary conditions

\( \psi(x, o) = \frac{-v_g}{ik} \exp[-ikx] \)
\( \zeta(x, o) = ikv_g \exp[-ikx] - u,(x, o). \)
LOCALLY ANALYTICAL SOLUTIONS

Locally analytical solutions are obtained for the unsteady and steady viscous flow fields. In this method, the discrete algebraic equations which represent the aerodynamic equations are obtained from analytical solutions in individual local grid elements. This is accomplished by dividing the flow field into computational grid elements. In each individual element the nonlinear convective terms of the Navier–Stokes equations which describe the steady flow are locally linearized. The nonlinear character of the steady flow field is preserved as the flow is only locally linearized, that is, independently linearized in individual grid elements. Analytical solutions to the linear equations describing both the steady and the unsteady flow fields in each element are then determined. The solution for the complete flow field is obtained through the application of the global boundary conditions and the assembly of the locally analytic solutions in the individual grid elements.

STEADY FLOW FIELD

The steady vorticity transport is described by equation (3) which is nonlinear because of the convective terms $U\zeta_x + V\zeta_y$. These terms are locally linearized by assuming that the velocity components $U$ and $V$, which are the coefficients of the vorticity, are constant in each individual grid element, that is, locally linearized

$$U = \frac{2A}{Re}, \quad V = \frac{2B}{Re},$$

where $A$ and $B$ are constants in an individual grid element, taking on different values in each grid element. The resulting locally linearized vorticity equation is given in equation (9)

$$2A\zeta_x + 2B\zeta_y = \zeta_{xx} + \zeta_{yy}. \quad (9)$$

This locally linearized equation can be solved analytically to determine the steady vorticity, $\zeta$, in a grid element, thereby providing the functional relationships between the vorticity in an individual grid element and the boundary values specified on that grid element. This vorticity transport equation is elliptic. Therefore, to obtain a unique solution for the typical uniform grid element with center $(x_0, y_0)$, Fig. 2, boundary conditions must be specified on all four boundaries. These boundary conditions are expressed in an implicit formulation in terms of the nodal values.

![Fig. 2. Computational grid element.](image-url)
of the vorticity along the boundaries of the element. A second order polynomial is used to approximate the vorticity on each of the boundaries

\[ \zeta(x, y_0 + \Delta y) = a_1^\xi + a_2^\xi x + a_3^\xi x^2, \]
\[ \zeta(x_0 + \Delta x, y) = b_1^\xi + b_2^\xi y + b_3^\xi y^2, \]
\[ \zeta(x, y_0 - \Delta y) = c_1^\xi + c_2^\xi x + c_3^\xi x^2, \]
\[ \zeta(x_0 - \Delta x, y) = d_1^\xi + d_2^\xi y + d_3^\xi y^2, \]

(10)

where \( a_1^\xi, b_1^\xi, c_1^\xi, d_1^\xi \) are constants determined from the three nodal points on each boundary side and the \( x \) and \( y \) distances are all measured from the center of the element \((x_0, y_0)\).

The analytical solution to equation (9) subject to the boundary conditions specified in equation (10) is determined by separation of variables

\[
\zeta(x, y) = \exp(Ax + By) \sum_{n=1}^{\infty} \left\{ [B_1^n \sinh(E_{1n}x) + B_2^n \cosh(E_{1n}x)] \sin(\lambda_{1n}(y + \Delta y)) + [B_1^n \sinh(E_{2n}y) + B_2^n \cosh(E_{2n}y)] \sin(\lambda_{2n}(x + \Delta x)) \right\}. \]

(11)

The locally analytical solution for the stream function is obtained by a procedure analogous to that used for the vorticity. First, the flow region is subdivided into computational grid elements.

The stream function is described by a linear Poisson equation which is coupled to the vorticity, equation (3b). This stream function Poisson equation also is elliptic. Therefore, to obtain a unique analytical solution for the typical grid element, continuous conditions must be specified on all four boundaries. As for the vorticity transport equation, continuous boundary conditions are represented in an implicit formulation in terms of the nodal values of the stream function by second order polynomials in \( x \) or \( y \) as measured from the element center \((x_0, y_0)\)

\[ \Psi(x, y_0 + \Delta y) = a_1^\eta + a_2^\eta x + a_3^\eta x^2, \]
\[ \Psi(x_0 + \Delta x, y) = b_1^\eta + b_2^\eta y + b_3^\eta y^2, \]
\[ \Psi(x, y_0 - \Delta y) = c_1^\eta + c_2^\eta x + c_3^\eta x^2, \]
\[ \Psi(x_0 - \Delta x, y) = d_1^\eta + d_2^\eta y + d_3^\eta y^2, \]

(12)

where \( a_1^\eta, b_1^\eta, c_1^\eta, d_1^\eta \) are constants determined from the three nodal points on each boundary side.

The stream function equation is linear and possesses a nonhomogeneous term, \(-\zeta(x, y)\), which couples the stream function to the vorticity. To solve equation (3b) subject to the boundary conditions specified in equation (12), it is divided into two component problems. One problem has a homogeneous equation with nonhomogeneous boundary conditions, whereas the second problem has a nonhomogeneous equation with homogeneous boundary conditions

\[
\Psi = \Psi^a + \Psi^b. \]

(13)

**Problem 1**

\[ \nabla^2 \Psi^a = 0, \]
\[ \Psi^a(x, y_0 + \Delta y) = a_1^\eta + a_2^\eta x + a_3^\eta x^2, \]
\[ \Psi^a(x_0 + \Delta x, y) = b_1^\eta + b_2^\eta y + b_3^\eta y^2, \]
\[ \Psi^a(x, y_0 - \Delta y) = c_1^\eta + c_2^\eta x + c_3^\eta x^2, \]
\[ \Psi^a(x_0 - \Delta x, y_0) = d_1^\eta + d_2^\eta x + d_3^\eta x^2, \]

(14)
Problem 2

\[ \nabla^2 \Psi^b = -\zeta(x, y), \]

\[ \Psi^b(x_0 + \Delta x, y) = 0, \]

\[ \Psi^b(x_0 - \Delta x, y) = 0, \]

\[ \Psi^b(x, y_0 + \Delta y) = 0, \]

\[ \Psi^b(x, y_0 + \Delta y) = 0. \]

(15)

The solutions for \( \Psi^a \) and \( \Psi^b \) are then determined by separation of variables

\[ \Psi(x, y) = \sum_{n=1}^{\infty} \left( \left[ B^s_n \sinh(\lambda^s_n x) + B^c_n \cosh(\lambda^s_n x) \right] \sin(\lambda^s_n y + \Delta y) + \right. \]

\[ + \left. \left[ B^s_n \sinh(\lambda^s_n y) + B^c_n \cosh(\lambda^s_n y) \right] \sin(\lambda^s_n x + \Delta x) + \right. \]

\[ + \left. \left[ G^s_n \sinh(\lambda^s_n y) + G^c_n \cosh(\lambda^s_n y) + G^s_n + G^c_n \right] \right) \sin(\lambda^s_n (x + \Delta x)). \]

(16)

The stream function is continuously differentiable across the grid element. Hence the \( U \) and \( V \) velocity components can be obtained analytically by differentiating the stream function solution. The solutions for \( \Psi, \zeta, U \) and \( V \) are then used to determine the pressure in the flow field and on the boundaries. Thus, the locally analytical solutions for the velocity components and the pressure are performed as post processes.

UNSTEADY FLOW FIELD

The unsteady vorticity is described by a linear partial differential equation with nonconstant coefficients, equation (4). In particular, the unsteady perturbation velocity coefficients \( u \) and \( v \) vary across the typical computational grid element. However, the steady velocity coefficients \( U \) and \( V \) are known from the previously determined steady state solution and are constant in the typical grid element, as specified in equation (8).

To determine the locally analytical solution to the unsteady perturbation vorticity equation, it is approximated as a constant coefficient partial differential equation in individual grid elements. This is accomplished by assuming that the perturbation velocities \( u \) and \( v \) are constant in each element

\[ u = \frac{2A'}{\text{Re}}, \quad v = \frac{2B'}{\text{Re}}, \]

(17)

where \( A' \) and \( B' \) are constant in each individual grid element, taking on different values in different grid elements.

Thus, the following linear coefficient partial differential equation defines the unsteady perturbation vorticity in an individual computational grid element

\[ k \cdot i \cdot \text{Re} \zeta + 2A\xi_x + 2B\xi_y + (2A'\zeta_x + 2B'\zeta_y) = \xi_{xx} + \xi_{yy}. \]

(18)

To determine the analytical solution in the typical grid element, equation (18) is rewritten as a homogeneous equation

\[ \nabla^2 \xi = 2A\xi_x + 2B\xi_y, \]

(19)

where

\[ \xi(x, y) = \xi(x, y) + \frac{S \left( Ax + \frac{B}{A} y \right)}{2 \left( A^2 + \frac{B^2}{A} \right)} \]

and

\[ S(x, y) = (2A'\zeta_x + 2B'\zeta_y + k \cdot \text{Re} \cdot i\zeta). \]
This equation is of the same form as that for the steady linearized vorticity, equation (9). Thus, the solution for $\zeta$ is obtained in a manner exactly analogous to that for the steady vorticity, $\zeta$, and is given in equation (20)

$$
\zeta(x_0, y_0) = z_1(x_0 + \Delta x, y_0 + \Delta y)\zeta(x_0 + \Delta x, y_0 + \Delta y) + z_2(x_0 + \Delta x, y_0)\zeta(x_0 + \Delta x, y_0)
+ z_3(x_0 + \Delta x, y_0 - \Delta y)\zeta(x_0 + \Delta x, y_0 - \Delta y) + z_4(x_0, y_0 - \Delta y)\zeta(x_0, y_0 - \Delta y)
+ z_5(x_0 - \Delta x, y_0 - \Delta y)\zeta(x_0 - \Delta x, y_0 - \Delta y) + z_6(x_0 - \Delta x, y_0)\zeta(x_0 - \Delta x, y_0)
+ z_7(x_0 - \Delta x, y_0 + \Delta y)\zeta(x_0 - \Delta x, y_0 + \Delta y) + z_8(x_0, y_0 + \Delta y)\zeta(0, \Delta y),
$$

where the coefficients $z_i$ are dependent on the steady state velocity components, $U$ and $V$.

The unsteady stream function is described by equation (4b). This equation is identical to that for the steady stream function, equation (3b). Hence, the solution procedure is identical to that for the steady stream function. As the coefficients for the stream function are only a function of their position in the grid element, that is, $\Delta x$ and $\Delta y$, the unsteady coefficients remain the same as those found previously for the steady stream function $\Psi(x_0, y_0)$. Thus, the solution for the unsteady stream function is determined from the steady stream function solution, equation (16), by replacing $\Psi$ by $\psi$ and the steady vorticity $\zeta$ by the vorticity $\xi$. The algebraic equation for the value of the unsteady stream function at the center of the typical element in terms of the values of the unsteady stream function and vorticity at its eight neighboring values is given in equation (21)

$$
\psi(x_0, y_0) = p_{\xi} \psi(x_0 + \Delta x, y_0 + \Delta y) + p_{\xi} \psi(x_0 + \Delta x, y_0) + p_{\xi} \psi(x_0 + \Delta x, y_0 - \Delta y)
+ p_{\xi} \psi(x_0 - \Delta x, y_0 + \Delta y) + p_{\xi} \psi(x_0 - \Delta x, y_0) + p_{\xi} \psi(x_0, y_0 - \Delta y)
+ q_{\xi} \xi(x_0 + \Delta x, y_0 + \Delta y) + q_{\xi} \xi(x_0 + \Delta x, y_0) + q_{\xi} \xi(x_0 + \Delta x, y_0 - \Delta y)
+ q_{\xi} \xi(x_0 - \Delta x, y_0 + \Delta y) + q_{\xi} \xi(x_0 - \Delta x, y_0) + q_{\xi} \xi(x_0, y_0 - \Delta y)
+ q_{\xi} \xi(x_0 - \Delta x, y_0 + \Delta y) + q_{\xi} \xi(x_0, y_0 + \Delta y) + q_{\xi} \xi(x_0, y_0).
$$

The unsteady velocity components $u$ and $v$ are determined by differentiating the unsteady stream function, with the locally analytical solution for the unsteady pressure determined by a post process.

**RESULTS**

The unsteady viscous flow model and the locally analytical solutions developed herein are utilized to demonstrate the effects of moderate Reynolds number and reduced frequency on the convected gust generated unsteady aerodynamics of a flat plate airfoil at zero mean flow incidence. These results are presented in the format of the real and imaginary components of the unsteady airfoil surface pressure difference across the chordline of the airfoil, and the corresponding gust generated complex unsteady lift, as defined in equation (22)

$$
C_L = \frac{L}{\rho c U \pi} = \int_{0}^{c} \frac{(p_{\text{lower}} - p_{\text{upper}})}{\rho c U} \, dx.
$$

The locally analytical predictions are obtained on a $65 \times 35$ rectangular grid with $\Delta x = 0.025$ and $\Delta y = 0.025$. Forty-one points are located on the flat plate airfoil. The convergence criteria for the internal and external iterations for the stream function are both $10^{-4}$, with the vorticity tolerance being $5 \cdot 10^{-2}$. The tolerances for the pressure iterations are $10^{-6}$ and $10^{-5}$ for the internal
and external iterations, respectively. Starting the computation with a converged solution from a similar baseline flow case greatly decreases the computational time required, with the computational time dependent on the size of the change from the baseline in the Reynolds number and the reduced frequency.

The steady flow model and its locally analytical solution are first utilized to predict the steady viscous flow past the airfoil. For example, Fig. 3 shows the predicted steady vorticity on the airfoil surface at Reynolds numbers of 100 and 1,000 and zero incidence. The small perturbation gust generated unsteady viscous aerodynamics which are coupled to the steady viscous flow are then analyzed with the unsteady flow model and its locally analytical solution.

The significant effect of viscosity on the chordwise variation of the complex unsteady pressure difference and the corresponding unsteady aerodynamic lift are demonstrated in Figs 4-8. In particular, the transverse gust generated complex unsteady pressure differences across the chordline of the airfoil in a viscous flow characterized by Reynolds numbers ranging from 100 to 5,000 are presented in Figs 4 and 5 for a reduced frequency of 1.0, and in Figs 6 and 7 for a reduced frequency of 3.0. The resulting variation of the unsteady aerodynamic lift with Reynolds number and reduced frequency, calculated by the integration of these pressure difference predictions, is shown in Fig. 8. Also presented as a reference in each of these figures is the corresponding inviscid prediction determined from the classical model of Sears [1].
In the airfoil leading edge region, the viscous predictions are nonsingular, in contrast to the inviscid predictions, thereby leading to the large differences between the complex viscous and inviscid results. Also, as the chordwise position increases, both the real and imaginary components of the viscous predictions first increase, then decrease, and finally increase near the trailing edge. These viscous chordwise variations become more pronounced as the Reynolds number is increased. In comparison, the corresponding inviscid pressure difference predictions are monotonic.

The effect of viscosity on the unsteady aerodynamic lift at these reduced frequency values, Fig. 8, results from the above noted chordwise variations of the viscous and inviscid solutions. At reduced frequencies of 1.0 and 3.0, the real part of the inviscid chordwise pressure difference predictions are always positive, with the imaginary part very small but negative only near to the airfoil leading edge at a reduced frequency of 1.0, and always positive at a reduced frequency of 3.0. The corresponding viscous predictions, both the real and imaginary components, have large chordwise regions with negative pressure difference values, with these negative regions becoming larger as the Reynolds number is increased. Hence, the real and imaginary components of the complex unsteady aerodynamic lift decrease both with increasing Reynolds number and as compared to the inviscid predictions at the reduced frequency values considered.

SUMMARY AND CONCLUSIONS

A mathematical model has been developed to analyze the effects of Reynolds number and reduced frequency on the convected transverse gust unsteady aerodynamics of a flat plate airfoil in an incompressible laminar flow. The unsteady viscous flow is assumed to be a small perturbation to the steady viscous flow which is described by the Navier-Stokes equations. The small perturbation unsteady viscous flow field is described by a system of linear partial differential equations that are coupled to the steady flow field.

Solutions for both the steady and the unsteady viscous flow fields were obtained by developing locally analytical solutions. In this approach, the discrete algebraic equations which represent the flow field equations are obtained from analytical solutions in individual grid elements. For the steady viscous flow, this was accomplished by first locally linearizing the convective terms in the Navier-Stokes equations. The complete flow field solutions are then obtained through the application of the global boundary conditions and the assembly of the local grid element solutions.

This model and locally analytical solutions were then utilized to demonstrate the significant effect of viscosity on the transverse gust generated unsteady aerodynamics of a flat plate airfoil. In particular, the transverse gust generated complex unsteady chordwise pressure differences and resulting unsteady lift on the airfoil in a viscous flow characterized by Reynolds numbers ranging from 100 to 5,000 were analyzed and correlated with classical inviscid results for reduced frequency values of 1.0 and 3.0.


REFERENCES