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Three-loop universal anomalous dimension of the Wilson operators in $\mathcal{N}=4$ SUSY Yang–Mills model

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Abstract

We present results for the three-loop universal anomalous dimension $\gamma_{\rm uni}(j)$ of Wilson twist-2 operators in the $\mathcal{N}=4$ Supersymmetric Yang–Mills model. These expressions are obtained by extracting the most complicated contributions from the three loop non-singlet anomalous dimensions in QCD which were calculated recently. Their singularities at $j=1$ coincide with the predictions obtained from the BFKL equation for $\mathcal{N}=4$ SYM in the next-to-leading order. The asymptotics of $\gamma_{\rm uni}(j)$ at large j is in an agreement with the expectations based on an interpolation between week and strong coupling regimes in the framework of the AdS/CFT correspondence.

1. Introduction

The anomalous dimensions (AD) of the twist-two Wilson operators govern the Bjorken scaling violation for parton distributions in a framework of Quantum Chromodynamics (QCD) [1]. These quantities are expressed through the Mellin transformation

$$\gamma_{ab}(j)=\int_0^1 dx\, x^{j-1} W_{b\to a}(x)$$

of the splitting kernels $W_{b\to a}(x)$ for the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation [2] which relates the parton densities $f_a(x,Q^2)$ (hereafter $a=\lambda, g, \phi$ for the spinor, vector and scalar particles, respectively)

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with different values of Q^2 as follows

$$\frac{d}{d\ln Q^2} f_a\bigl(x, Q^2\bigr) = \int_x^1 \frac{dy}{y} \sum_b W_{b\to a}\left(\frac{x}{y}\right) f_b\bigl(y, Q^2\bigr).$$

The anomalous dimensions and splitting kernels in QCD are well known up to the next-to-leading order (NLO) of the perturbation theory [1].

The QCD expressions for AD can be transformed to the case of the $\mathcal{N}=1$ supersymmetric Yang–Mills theories (SYM) if one will use for the Casimir operators C_A, C_F, T_f the following values $C_A = C_F = N_c$, $T_f = N_c/2$ (the last substitution follows from the fact, that each gluino λ_i being a Majorana particle gives a half of the contribution for the Dirac spinor). For extended supersymmetric theories the anomalous dimensions cannot be obtained in this simple way, because additional contributions coming from scalar particles should be also taken into account [3]. Recently these anomalous dimensions were calculated in the next-to-leading approximation [4] for the $\mathcal{N}=4$ supersymmetric Yang–Mills theory.

It turns out, that the expressions for eigenvalues of the AD matrix in the $\mathcal{N}=4$ SYM can be derived directly from the QCD anomalous dimensions without tedious calculations by using a number of plausible arguments. The method elaborated in Ref. [5] for this purpose is based on special properties of solutions of the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation [6,7] in this model and a new relation between the BFKL and DGLAP equations (see [3]). In the NLO approximation this method gives the correct results for AD eigenvalues, which was checked by direct calculations in [4]. Its properties will be reviewed below only shortly and a more extended discussion can be found in [5].

Next-to-next-to-leading order (NNLO) corrections to AD in QCD were calculated recently [8][1] in the non-singlet case. Using these results and the method of Ref. [5] we derive in this Letter the eigenvalues of the anomalous dimension matrix for the $\mathcal{N}=4$ SYM in the NNLO approximation.

2. Evolution equation in $\mathcal{N}=4$ SYM

The reason to investigate the BFKL and DGLAP equations in the case of supersymmetric theories is based on a common belief, that the high symmetry may significantly simplify the structure of these equations. Indeed, it was found in the leading logarithmic approximation (LLA) [10], that the so-called quasi-partonic operators in $\mathcal{N}=1$ SYM are unified in supermultiplets with anomalous dimensions obtained from universal anomalous dimensions $\gamma_{\mathrm{uni}}(j)$ by shifting its arguments by an integer number. Further, the anomalous dimension matrices for twist-2 operators are fixed by the superconformal invariance [10]. Calculations in the maximally extended $\mathcal{N}=4$ SYM, where the coupling constant is not renormalized, give even more remarkable results. Namely, it turns out, that here all twist-2 operators enter in the same multiplet, their anomalous dimension matrix is fixed completely by the super-conformal invariance and its universal anomalous dimension in LLA is proportional to $\Psi(j-1) - \Psi(1)$, which means, that the evolution equations for the matrix elements of quasi-partonic operators in the multicolor limit $N_c \to \infty$ are equivalent to the Schrödinger equation for an integrable Heisenberg spin model [11,12]. In QCD the integrability remains only in a small sector of the quasi-partonic operators [13]. In the case of $\mathcal{N}=4$ SYM the equations for other sets of operators are also integrable [14–16]. Evolution equations for quasi-partonic operators are written in an explicitly super-conformal form in Ref. [17].

Similar results related to the integrability of the multi-color QCD were obtained earlier in the Regge limit [18]. Moreover, it was shown [3], that in the $\mathcal{N}=4$ SYM there is a deep relation between BFKL and DGLAP evolution equations. Namely, the j-plane singularities of AD of the Wilson twist-2 operators in this case can be obtained

[1] See also Ref. [9] for the singlet case.

from the eigenvalues of the BFKL kernel by their analytic continuation. The NLO calculations in $\mathcal{N}=4$ SYM demonstrated [5], that some of these relations are valid also in higher orders of perturbation theory. In particular, the BFKL equation has the property of the Hermitian separability, the linear combinations of the multiplicatively renormalized operators do not depend on the coupling constant, the eigenvalues of the anomalous dimension matrix are expressed in terms of the universal function $\gamma_{\mathrm{uni}}(j)$ which can be obtained also from the BFKL equation [5]. The results for $\gamma_{\mathrm{uni}}(j)$ were checked by the direct calculations in Ref. [4].

In the $\mathcal{N}=4$ SYM theory [19] we have the following field content: one gluon g, four Majorana fermions λ and three complex scalars ϕ. All particles belong to the adjoint representation of the gauge group $SU(N_c)$. This model possesses an internal $SU(4)$ symmetry. In the $\mathcal{N}=4$ SYM theory one can introduce the following color and $SU(4)$ singlet local Wilson twist-2 operators [10,20–22]:

$$\mathcal{O}^g_{\mu_1,\ldots,\mu_j} = \hat{S} G^a_{\rho\mu_1} \mathcal{D}_{\mu_2} \mathcal{D}_{\mu_3} \cdots \mathcal{D}_{\mu_{j-1}} G^a_{\rho\mu_j}, \tag{1}$$

$$\tilde{\mathcal{O}}^g_{\mu_1,\ldots,\mu_j} = \hat{S} G^a_{\rho\mu_1} \mathcal{D}_{\mu_2} \mathcal{D}_{\mu_3} \cdots \mathcal{D}_{\mu_{j-1}} \tilde{G}^a_{\rho\mu_j}, \tag{2}$$

$$\mathcal{O}^\lambda_{\mu_1,\ldots,\mu_j} = \hat{S} \bar{\lambda}^a_i \gamma_{\mu_1} \mathcal{D}_{\mu_2} \cdots \mathcal{D}_{\mu_j} \lambda^{a\,i}, \tag{3}$$

$$\tilde{\mathcal{O}}^\lambda_{\mu_1,\ldots,\mu_j} = \hat{S} \bar{\lambda}^a_i \gamma_5 \gamma_{\mu_1} \mathcal{D}_{\mu_2} \cdots \mathcal{D}_{\mu_j} \lambda^{a\,i}, \tag{4}$$

$$\mathcal{O}^\phi_{\mu_1,\ldots,\mu_j} = \hat{S} \bar{\phi}^a_r \mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} \cdots \mathcal{D}_{\mu_j} \phi^a_r, \tag{5}$$

where $\mathcal{D}_\mu$ are covariant derivatives. The spinors λ_i and field tensor $G_{\rho\mu}$ describe gluinos and gluons, respectively, and ϕ_r is the complex scalar fields appearing in the $\mathcal{N}=4$ supersymmetric model. Indices $i=1,\ldots,4$ and $r=1,\ldots,3$ refer to $SU(4)$ and $SO(6)\simeq SU(4)$ groups of inner symmetry, respectively. The symbol $\hat{S}$ implies a symmetrization of each tensor in the Lorentz indices $\mu_1,\ldots,\mu_j$ and a subtraction of its traces. The anomalous dimension matrices can be written for unpolarized and polarized cases, respectively, as follows

$$\gamma_{\mathrm{unpol}} = \begin{vmatrix} \gamma_{gg} & \gamma_{g\lambda} & \gamma_{g\phi} \\ \gamma_{\lambda g} & \gamma_{\lambda\lambda} & \gamma_{\lambda\phi} \\ \gamma_{\phi g} & \gamma_{\phi\lambda} & \gamma_{\phi\phi} \end{vmatrix}, \qquad \gamma_{\mathrm{pol}} = \begin{vmatrix} \tilde{\gamma}_{gg} & \tilde{\gamma}_{g\lambda} \\ \tilde{\gamma}_{\lambda g} & \tilde{\gamma}_{\lambda\lambda} \end{vmatrix}. \tag{6}$$

Note, that in the super-multiplet of twist-2 operators there are also operators with fermion quantum numbers and operators anti-symmetric in two Lorentz indices [10,22]. For the case $\mathcal{N}=4$ the multiplicatively renormalized operators were found in an explicit way and their universality properties for all orders of perturbation theory were formulated in Refs. [5,12].

After their diagonalization, the new unpolarized γ and polarized $\tilde{\gamma}$ AD matrices have the following form

$$\gamma = V^{-1}\gamma_{\mathrm{unpol}}V = \begin{vmatrix} \gamma_+(j) & \gamma_{+0}(j) & \gamma_{+-}(j) \\ \gamma_{0+}(j) & \gamma_0(j) & \gamma_{0-}(j) \\ \gamma_{-+}(j) & \gamma_{-0}(j) & \gamma_-(j) \end{vmatrix}, \qquad \tilde{\gamma} = \tilde{V}^{-1}\gamma_{\mathrm{pol}}\tilde{V} = \begin{vmatrix} \tilde{\gamma}_+(j) & \tilde{\gamma}_{+-}(j) \\ \tilde{\gamma}_{-+}(j) & \tilde{\gamma}_-(j) \end{vmatrix} \tag{7}$$

which corresponds to AD matrices for multiplicatively renormalizable linear combinations of operators (1)–(5). Here, the matrices V, V^{-1}, $\tilde{V}$ and $\tilde{V}^{-1}$ were calculated in [5] and in LO we have $\gamma_{lm}(j)=0$, $\tilde{\gamma}_{lm}(j)=0$ for $l,m=+,0,-$. In NLO the AD matrices become triangle [4] due to superconformal invariance breaking [23], similar to the case of $\mathcal{N}=1$ SYM [21]. The eigenvalues $\gamma_l(j)$ and $\tilde{\gamma}_l(j)$ govern the power-like violation of the Bjorken scaling for the parton distributions.

Due to the fact that all twist-2 operators belong to the same supermultiplet the anomalous dimensions $\gamma_l(j)$ and $\tilde{\gamma}_l(j)$ $(l=+,0,-)$ have the properties [5,12]

$$\gamma_+(j) = \tilde{\gamma}_+(j-1) = \gamma_0(j-2) = \tilde{\gamma}_-(j-3) = \gamma_-(j-4) = \gamma_{\mathrm{uni}}(j), \tag{8}$$

where $\gamma_{\mathrm{uni}}(j)$ is the universal anomalous dimension.

3. Method of obtaining AD eigenvalues in $\mathcal{N}=4$ SYM

As it was already pointed out in Introduction, the universal anomalous dimension can be extracted directly from the QCD results without finding the scalar particle contribution. This possibility is based on deep relation between DGLAP and BFKL dynamics in the $\mathcal{N}=4$ SYM [3,5].

To begin with, the eigenvalues of the BFKL kernel turn out to be analytic functions of the conformal spin $|n|$ at least in two first orders of perturbation theory [5]. Further, in the framework of the $\overline{\mathrm{DR}}$-scheme [24] one can obtain from the BFKL equation (see [3]), that there is no mixing among the special functions of different transcendentality levels i,[2] i.e., all special functions at the NLO correction contain only sums of the terms $\sim 1/j^i$ $(i=3)$. More precisely, if we introduce the transcendentality level for the eigenvalues of integral kernels of the BFKL equations as functions of γ and appearing in the perturbation theory in an accordance with the complexity of the terms in the corresponding sums

$$\Psi \sim \frac{1}{\gamma}, \qquad \Psi' \sim \beta' \sim \zeta(2) \sim \frac{1}{\gamma^2}, \qquad \Psi'' \sim \beta'' \sim \zeta(3) \sim \frac{1}{\gamma^3},$$

then for the BFKL kernel in the leading order (LO) and in NLO the corresponding levels are $i=1$ and $i=3$, respectively.

Because in $\mathcal{N}=4$ SYM there is a relation between the BFKL and DGLAP equations (see [3,5]), the similar properties should be valid for the anomalous dimensions themselves, i.e., the basic functions $\gamma_{\mathrm{uni}}^{(0)}(j)$, $\gamma_{\mathrm{uni}}^{(1)}(j)$ and $\gamma_{\mathrm{uni}}^{(2)}(j)$ are assumed to be of the types $\sim 1/j^i$ with the levels $i=1$, $i=3$ and $i=5$, respectively. An exception could be for the terms appearing at a given order from previous orders of the perturbation theory. Such contributions could be generated and/or removed by an approximate finite renormalization of the coupling constant. But these terms do not appear in the $\overline{\mathrm{DR}}$-scheme.

It is known, that at the LO and NLO approximations the most complicated contributions (with $i=1$ and $i=3$, respectively) are the same for all LO and NLO anomalous dimensions in QCD [1] (with the SUSY relation for the QCD color factors $C_F=C_A=N_C$)[3] and for the LO and NLO scalar-scalar anomalous dimensions [4]. This property allows one to find the universal anomalous dimensions $\gamma_{\mathrm{uni}}^{(0)}(j)$ and $\gamma_{\mathrm{uni}}^{(1)}(j)$ without knowing all elements of the anomalous dimension matrix [5], which was verified by the exact calculations in [4].

Using above arguments, we conclude, that at the NNLO level there is only one possible candidate for $\gamma_{\mathrm{uni}}^{(2)}(j)$. Namely, it is the most complicated part of the non-singlet QCD anomalous dimension matrix (with the SUSY relation for the QCD color factors $C_F=C_A=N_c$). Indeed, after the diagonalization of the anomalous dimension matrix the eigenvalues $\gamma_l(j)$ and $\tilde{\gamma}_l(j)$ in Eq. (7) should have this most complicated part as a common contribution because they differ each from others only by a shift of the argument (see Eq. (8)) and their differences are constructed from less complicated terms. The non-diagonal matrix elements of $\gamma_{ab}(j)$ in Eq. (7) contain also only less complicated terms (see, for example, AD exact expressions at LO and NLO approximations in Ref. [1] for QCD and Ref. [4] for $\mathcal{N}=4$ SYM) and therefore they cannot generate the most complicated contributions to $\gamma_l(j)$ and $\tilde{\gamma}_l(j)$.

Thus, the most complicated part of the non-singlet NNLO QCD anomalous dimension should coincide (up to color factors) with the universal anomalous dimension $\gamma_{\mathrm{uni}}^{(2)}(j)$.

[2] Note that similar arguments were used also in [25] to obtain analytic results for contributions of some complicated massive Feynman diagrams without direct calculations.

[3] The property is valid also at NNLO approximation: see results of Ref. [9].

4. NNLO anomalous dimension for $\mathcal{N}=4$ SYM

The final three-loop result[4] for the universal anomalous dimension $\gamma_{\mathrm{uni}}(j)$ for $\mathcal{N}=4$ SYM is

$$\gamma(j) \equiv \gamma_{\mathrm{uni}}(j) = \hat{a}\gamma_{\mathrm{uni}}^{(0)}(j) + \hat{a}^2\gamma_{\mathrm{uni}}^{(1)}(j) + \hat{a}^3\gamma_{\mathrm{uni}}^{(2)}(j) + \cdots, \quad \hat{a} = \frac{\alpha N_c}{4\pi}, \tag{9}$$

where

$$\frac{1}{4}\gamma_{\mathrm{uni}}^{(0)}(j+2) = -S_1, \tag{10}$$

$$\frac{1}{8}\gamma_{\mathrm{uni}}^{(1)}(j+2) = (S_3 + \bar{S}_{-3}) - 2\bar{S}_{-2,1} + 2S_1(S_2 + \bar{S}_{-2}), \tag{11}$$

$$\begin{aligned}\frac{1}{32}\gamma_{\mathrm{uni}}^{(2)}(j+2) = {} & 2\bar{S}_{-3}S_2 - S_5 - 2\bar{S}_{-2}S_3 - 3\bar{S}_{-5} + 24\bar{S}_{-2,1,1,1} + 6(\bar{S}_{-4,1} + \bar{S}_{-3,2} + \bar{S}_{-2,3}) \\ & - 12(\bar{S}_{-3,1,1} + \bar{S}_{-2,1,2} + \bar{S}_{-2,2,1}) - \big(S_2 + 2S_1^2\big)(3\bar{S}_{-3} + S_3 - 2\bar{S}_{-2,1}) \\ & - S_1\big(8\bar{S}_{-4} + \bar{S}_{-2}^2 + 4S_2\bar{S}_{-2} + 2S_2^2 + 3S_4 - 12\bar{S}_{-3,1} - 10\bar{S}_{-2,2} + 16\bar{S}_{-2,1,1}\big)\end{aligned} \tag{12}$$

and $S_a \equiv S_a(j)$, $S_{a,b} \equiv S_{a,b}(j)$, $S_{a,b,c} \equiv S_{a,b,c}(j)$ are harmonic sums

$$S_a(j) = \sum_{m=1}^{j} \frac{1}{m^a}, \qquad S_{a,b,c,\dots}(j) = \sum_{m=1}^{j} \frac{1}{m^a} S_{b,c,\dots}(m), \tag{13}$$

$$\begin{aligned} & S_{-a}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{m^a}, \qquad S_{-a,b,c,\dots}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{m^a} S_{b,c,\dots}(m), \\ & \bar{S}_{-a,b,c,\dots}(j) = (-1)^j S_{-a,b,c,\dots}(j) + S_{-a,b,c,\dots}(\infty)\big(1-(-1)^j\big). \end{aligned} \tag{14}$$

The expression (14) is defined for all integer values of arguments (see [5,26]) but can be easily analytically continued to real and complex j by the method of Refs. [5,27].

The limit $j \to 1$ is important for the investigation of the small-x behavior of parton distributions (see review [31] and references therein). Especially it became popular recently because there are new experimental data at small x produced by the H1 and ZEUS Collaborations in HERA [32].

Using asymptotic expressions for harmonic sums at $j = 1 + \omega \to 1$ we obtain for the $\mathcal{N}=4$ universal anomalous dimension $\gamma_{\mathrm{uni}}(j)$ in Eq. (9)

$$\gamma_{\mathrm{uni}}^{(0)}(1+\omega) = \frac{4}{\omega} + O\big(\omega^1\big), \tag{15}$$

$$\gamma_{\mathrm{uni}}^{(1)}(1+\omega) = -32\zeta_3 + O\big(\omega^1\big), \tag{16}$$

$$\gamma_{\mathrm{uni}}^{(2)}(1+\omega) = 32\zeta_3\frac{1}{\omega^2} - 232\zeta_4\frac{1}{\omega} - 1120\zeta_5 + 256\zeta_3\zeta_2 + O\big(\omega^1\big) \tag{17}$$

in an agreement with the predictions for $\gamma_{\mathrm{uni}}^{(0)}(1+\omega)$, $\gamma_{\mathrm{uni}}^{(1)}(1+\omega)$ and also for the first term of $\gamma_{\mathrm{uni}}^{(2)}(1+\omega)$ coming from an investigation of BFKL equation at NLO accuracy in [3].[5]

[4] Note, that in an accordance with Ref. [7] our normalization of $\gamma(j)$ contains the extra factor $-1/2$ in comparison with the standard normalization (see [1]) and differs by sign in comparison with Vermaseren–Moch–Vogt one [8].

[5] Unfortunately, the results of Refs. [3,5] contain a misprint. Namely, the coefficient in front of $\hat{a}^3$ obtained in the limit $j \to 1$ in Eq. (39) of Ref. [5] should be multiplied by a factor 4.

5. Resummation of $\gamma_{\rm uni}$ and the AdS/CFT correspondence

In the limit $j \to \infty$ the AD results (10)–(12) are simplified significantly. Note, that this limit is related to the study of the asymptotics of structure functions and cross-sections at $x \to 1$ corresponding to the quasi-elastic kinematics of the deep-inelastic ep scattering.

We obtain the following asymptotics for the $\mathcal{N}=4$ universal anomalous dimension $\gamma_{\rm uni}(j)$ in Eq. (9) with

$$\gamma_{\rm uni}^{(0)}(j) = -4(\ln j + \gamma_e) + O\big(j^{-1}\big), \tag{18}$$

$$\gamma_{\rm uni}^{(1)}(j) = 8\zeta_2(\ln j + \gamma_e) + 12\zeta_3 + O\big(j^{-1}\big), \tag{19}$$

$$\gamma_{\rm uni}^{(2)}(j) = -88\zeta_4(\ln j + \gamma_e) - 16\zeta_2\zeta_3 - 80\zeta_5 + O\big(j^{-1}\big). \tag{20}$$

Recently there was a great progress in the investigation of the $\mathcal{N}=4$ SYM theory in a framework of the AdS/CFT correspondence [28] where the strong-coupling limit $\alpha_s N_c \to \infty$ is described by a classical supergravity in the anti-de Sitter space $AdS_5 \times S^5$. In particular, a very interesting prediction [29] (see also [30]) was obtained for the large-j behavior of the anomalous dimension for twist-2 operators

$$\gamma(j) = a(z)\ln j, \quad z = \frac{\alpha N_c}{\pi} = 4\hat{a} \tag{21}$$

in the strong coupling regime (see Ref. [33] for asymptotic corrections)[6]

$$\lim_{z\to\infty} a = -z^{1/2} + \frac{3\ln 2}{4\pi} + \mathcal{O}\big(z^{-1/2}\big). \tag{22}$$

On the other hand, all anomalous dimensions $\gamma_i(j)$ and $\tilde{\gamma}_i(j)$ $(i=+,0,-)$ coincide at large j and our results for $\gamma_{\rm uni}(j)$ in Eq. (9) allow one to find three first terms of the small-z expansion of the coefficient $a(z)$

$$\lim_{z\to 0} a = -z + \frac{\pi^2}{12}z^2 - \frac{11}{720}\pi^4 z^3 + \cdots. \tag{23}$$

For resummation of this series we suggest the following equation for $\tilde{a}$ [4][7]

$$z = -\tilde{a} + \frac{\pi^2}{12}\tilde{a}^2 \tag{24}$$

interpolating between its weak-coupling expansion up to NNLO

$$\tilde{a} = -z + \frac{\pi^2}{12}z^2 - \frac{1}{72}\pi^4 z^3 + O\big(z^4\big) \tag{25}$$

and strong-coupling asymptotics

$$\tilde{a} \approx -1.1026 z^{1/2} + 0.6079 + \mathcal{O}\big(z^{-1/2}\big). \tag{26}$$

It is remarkable, that the prediction for NNLO based on the above simple equation is valid with the accuracy $\sim 10\%$. It means, that this extrapolation seems to be good for all values of z.

Further, for $j \to 2$ due to the energy–momentum conservation

$$\gamma(j) = (j-2)\gamma'(2) + \cdots, \tag{27}$$

[6] Here we took into account, that in our normalization $\gamma(j)$ contains the extra factor $-1/2$ in comparison with that in Ref. [29].
[7] Note, that we use the $\overline{\rm DR}$-scheme for coupling constant which removes $-1/12$ from coefficients of $\tilde{a}$ in Eq. (28) of Ref. [4] (see [5,34]).

where the coefficient $\gamma'(2)$ can be calculated from our above results in three first orders of the perturbation theory:

$$\gamma'(2) = -\frac{\pi^2}{6}z + \frac{\pi^4}{72}z^2 - \frac{\pi^6}{540}z^3 + \cdots. \tag{28}$$

We use the following interpolating equation for $\tilde{\tilde{a}} = \gamma'(2)$

$$\frac{\pi^2}{6}z = -\tilde{\tilde{a}} + \frac{1}{2}\tilde{\tilde{a}}^2. \tag{29}$$

Its solution at small z up to NNLO is

$$\tilde{\tilde{a}} = -\frac{\pi^2}{6}z + \frac{\pi^4}{72}z^2 - \frac{\pi^6}{432}z^3 + \cdots. \tag{30}$$

Note, that similar to the case of large j, the prediction for NNLO based on the above simple equation is valid with the accuracy $\sim 20\%$.

On the other hand using the same method of resummation as we used above for $\tilde{a}$, we obtain for large z

$$\gamma'(2) = -0.8597z^{1/2} + 0.6079 + \cdots. \tag{31}$$

Let us take into account, that in this limit $\gamma = 1/2 + i\nu + (j-1)/2 \to 1 + (j-2)/2$ for the principal series of unitary representations of the Möbius group appearing in the BFKL equation [7]. Therefore we obtain for large z

$$j = 2 - 1.1632z^{-1/2} - 0.1460z^{-1}, \tag{32}$$

in agreement with the result that the Pomeron in the strong coupling regime coincides with the graviton [36]. The correction $\sim z^{-1/2}$ to the graviton spin $j=2$ coincides in form with that obtained in Ref. [36] from the AdS/CFT correspondence but the coefficient in front of $z^{-1/2}$ was not calculated yet.

6. Conclusion

Thus, in this Letter we constructed the anomalous dimension $\gamma_{\text{uni}}(j)$ for the $\mathcal{N}=4$ supersymmetric gauge theory in the next-to-next-to-leading approximation and verified its self-consistency in the Regge ($j \to 1$) and quasi-elastic ($j \to \infty$) regimes. Our result for universal anomalous dimension at $j=4$ could be used to determine anomalous dimension of Konishi operator [35] up to 3-loops. It is remarkable, that our results coincide[8] with corresponding predictions from dilatation operator approach and integrability [37,38]. The method, developed for this construction, can be applied also to less symmetric cases of $\mathcal{N}=1,\ 2$ SYM and QCD, which are very important for phenomenological applications. For the verification of the AdS/CFT correspondence the calculations of the various physical quantities in $\mathcal{N}=4$ SYM attract a great interest due to a possibility to develop non-perturbative approaches to QCD.

We demonstrated above that the expressions interpolating between the week and strong regime work remarkably well both in limit $j \to \infty$ and $j \to 2$. The integrability of the evolution equations for the quasi-partonic operators in LLA [11,12] is an interesting property of $\mathcal{N}=4$ SYM which should be verified on NLO and NNLO level. We hope to discuss these problems in our future publications.

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References

[1] D.J. Gross, F. Wilczek, Phys. Rev. D 8 (1973) 3633;
H. Georgi, H.D. Politzer, Phys. Rev. D 9 (1974) 416;
E.G. Floratos, D.A. Ross, C.T. Sachrajda, Nucl. Phys. B 129 (1977) 66;
E.G. Floratos, D.A. Ross, C.T. Sachrajda, Nucl. Phys. B 139 (1978) 545, Erratum;
E.G. Floratos, D.A. Ross, C.T. Sachrajda, Nucl. Phys. B 152 (1979) 493;
A. Gonzalez-Arroyo, C. Lopez, F.J. Yndurain, Nucl. Phys. B 153 (1979) 161;
A. Gonzalez-Arroyo, C. Lopez, Nucl. Phys. B 166 (1980) 429;
G. Gurci, W. Furmanski, R. Petronzio, Nucl. Phys. B 175 (1980) 27;
W. Furmanski, R. Petronzio, Phys. Lett. B 97 (1980) 437;
E.G. Floratos, C. Kounnas, R. Lacage, Nucl. Phys. B 192 (1981) 417;
C. Lopes, F.J. Yndurain, Nucl. Phys. B 171 (1980) 231;
C. Lopes, F.J. Yndurain, Nucl. Phys. B 183 (1981) 157;
R. Hamberg, W.L. van Neerven, Nucl. Phys. B 379 (1992) 143;
R.K. Ellis, W. Vogelsang, hep-ph/9602356;
R. Mertig, W.L. van Neerven, Z. Phys. C 70 (1996) 637;
W. Vogelsang, Nucl. Phys. B 475 (1996) 47.
[2] V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438;
V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 675;
L.N. Lipatov, Sov. J. Nucl. Phys. 20 (1975) 94;
G. Altarelli, G. Parisi, Nucl. Phys. B 126 (1977) 298;
Yu.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641.
[3] A.V. Kotikov, L.N. Lipatov, Nucl. Phys. B 582 (2000) 19.
[4] A.V. Kotikov, L.N. Lipatov, V.N. Velizhanin, Phys. Lett. B 557 (2003) 114.
[5] A.V. Kotikov, L.N. Lipatov, hep-ph/0112346;
A.V. Kotikov, L.N. Lipatov, Nucl. Phys. B 661 (2003) 19.
[6] L.N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338;
V.S. Fadin, E.A. Kuraev, L.N. Lipatov, Phys. Lett. B 60 (1975) 50;
E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 44 (1976) 443;
E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 45 (1977) 199;
I.I. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822;
I.I. Balitsky, L.N. Lipatov, JETP Lett. 30 (1979) 355.
[7] V.S. Fadin, L.N. Lipatov, Phys. Lett. B 429 (1998) 127;
G. Camici, M. Ciafaloni, Phys. Lett. B 430 (1998) 349.
[8] S. Moch, J.A.M. Vermaseren, A. Vogt, hep-ph/0403192.
[9] A. Vogt, S. Moch, J.A.M. Vermaseren, hep-ph/0404111.
[10] A.P. Bukhvostov, E.A. Kuraev, L.N. Lipatov, G.V. Frolov, JETP Lett. 41 (1985) 92, Pis'ma Zh. Eksp. Teor. Fiz. 41 (1985) 77;
A.P. Bukhvostov, G.V. Frolov, L.N. Lipatov, E.A. Kuraev, Nucl. Phys. B 258 (1985) 601.
[11] L.N. Lipatov, Perspectives in hadronic physics, in: Proceedings of the ICTP Conference, World Scientific, Singapore, 1997.
[12] L.N. Lipatov, in: Proceedings of the International Workshop on Very High Multiplicity Physics, Dubna, 2000, pp. 159–176;
L.N. Lipatov, Nucl. Phys. B (Proc. Suppl.) 99 (2001) 175.
[13] V.M. Braun, S.E. Derkachov, A.N. Manashov, Phys. Rev. Lett. 81 (1998) 2020;
A.V. Belitsky, Phys. Lett. B 453 (1999) 59.
[14] N. Beisert, M. Staudacher, Nucl. Phys. B 670 (2003) 439.
[15] L. Dolan, C.R. Nappi, E. Witten, JHEP 0310 (2003) 017.
[16] J.A. Minahan, K. Zarembo, JHEP 0303 (2003) 013.
[17] R. Kirschner, hep-th/0404060.
[18] L.N. Lipatov, Preprint DFPD/93/TH/70;

L.N. Lipatov, hep-th/9311037;
L.N. Lipatov, JETP Lett. 59 (1994) 596, Pis'ma Zh. Eksp. Teor. Fiz. 59 (1994) 571;
L.D. Faddeev, G.P. Korchemsky, Phys. Lett. B 342 (1995) 311.
[19] L. Brink, J.H. Schwarz, J. Scherk, Nucl. Phys. B 121 (1977) 77;
F. Gliozzi, J. Scherk, D.I. Olive, Nucl. Phys. B 122 (1977) 253.
[20] Y.M. Makeenko, Sov. J. Nucl. Phys. 33 (1981) 440, Yad. Fiz. 33 (1981) 842;
T. Ohrndorf, Nucl. Phys. B 198 (1982) 26.
[21] A.V. Belitsky, D. Müller, A. Schafer, Phys. Lett. B 450 (1999) 126;
A.V. Belitsky, D. Müller, Nucl. Phys. B (Proc. Suppl.) 79 (1999) 576;
A.V. Belitsky, D. Müller, Phys. Rev. D 65 (2002) 054037.
[22] A.I. Onishchenko, V.N. Velizhanin, hep-ph/0309222.
[23] A.I. Onishchenko, V.N. Velizhanin, JHEP 0402 (2004) 036.
[24] W. Siegel, Phys. Lett. B 84 (1979) 193.
[25] J. Fleischer, A.V. Kotikov, O.L. Veretin, Nucl. Phys. B 547 (1999) 343;
J. Fleischer, A.V. Kotikov, O.L. Veretin, Acta Phys. Pol. B 29 (1998) 2611, hep-ph/9808243.
[26] D.I. Kazakov, A.V. Kotikov, Nucl. Phys. B 307 (1988) 721;
D.I. Kazakov, A.V. Kotikov, Nucl. Phys. B 345 (1990) 299, Erratum;
D.I. Kazakov, A.V. Kotikov, Phys. Lett. B 291 (1992) 171.
[27] A.V. Kotikov, Phys. At. Nucl. 57 (1994) 133.
[28] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231;
J. Maldacena, Int. J. Theor. Phys. 38 (1998) 1113;
S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428 (1998) 105;
E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253.
[29] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Nucl. Phys. B 636 (2002) 99.
[30] M. Kruczenski, JHEP 0212 (2002) 024;
Yu. Makeenko, JHEP 0301 (2003) 007;
M. Axenides, E. Floratos, A. Kehagias, Nucl. Phys. B 662 (2003) 170.
[31] Bo Andersson, et al., Eur. Phys. J. C 25 (2002) 77.
[32] H1 Collaboration, C. Adloff, et al., Eur. Phys. J. C 21 (2001) 33;
ZEUS Collaboration, S. Chekanov, et al., Eur. Phys. J. C 21 (2001) 943.
[33] S. Frolov, A.A. Tseytlin, JHEP 0206 (2002) 007.
[34] A.V. Belitsky, A.S. Gorsky, G.P. Korchemsky, Nucl. Phys. B 667 (2003) 3.
[35] K. Konishi, Phys. Lett. B 135 (1984) 439.
[36] J. Polchinski, M.J. Strassler, JHEP 0305 (2003) 012.
[37] N. Beisert, C. Kristjansen, M. Staudacher, Nucl. Phys. B 664 (2003) 131.
[38] N. Beisert, hep-th/0310252.