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Quantum gravitational dust collapse does not result in a black hole

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Abstract

Quantum gravity suggests that the paradox recently put forward by Almheiri et al. (AMPS) can be resolved if matter does not undergo continuous collapse to a singularity but condenses on the apparent horizon. One can then expect a quasi-static object to form even after the gravitational field has overcome any degeneracy pressure of the matter fields. We consider dust collapse. If the collapse terminates on the apparent horizon, the Misner–Sharp mass function of the dust ball is predicted and we construct static solutions with no tangential pressure that would represent such a compact object. The collapse wave functions indicate that there will be processes by which energy extraction from the center occurs. These leave behind a negative point mass at the center which contributes to the total energy of the system but has no effect on the energy density of the dust ball. The solutions describe a compact object whose boundary lies outside its Schwarzschild radius and which is hardly distinguishable from a neutron star.

1. Introduction

The final fate of gravitational collapse has long been a mystery. Classical collapse models suggest that a star that is massive enough to overcome all degeneracy pressures will undergo collapse beyond the apparent horizon [1,2] eventually forming a naked or covered singularity of spacetime, depending on the initial conditions. But there is something deeply unsatisfying

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about this picture since it does not take into account quantum gravity, which is expected to play a significant role in the final stages of collapse.

If the collapse begins with initial data that lead to the formation of a naked singularity [3] then a semi-classical treatment of the radiation (assuming the validity of effective field theory) from the singularity suggests that the final stages will be catastrophic [4,5]. It is not known what the final fate of such a collapse is: either the collapsing star will dissipate entirely or a remnant will attempt to form a covered singularity. However, if the initial data are such as to lead to the formation of a covered singularity and an event horizon forms then, as Hawking pointed out [6], the semi-classical theory would yield thermal radiation from the point of view of the observer who remains outside the black hole provided that the freely falling observer detects nothing unusual while crossing the horizon. The semi-classical analysis would seem to suggest that information is lost if the black hole evaporates completely, since what is left is a density matrix and not a wave function. But if the quantum theory is unitary then either (a) the evaporation is not in fact thermal and the Hawking radiation is pure or (b) the thermal evaporation process should, by an as yet unknown mechanism, leave behind a stable remnant that contains all the information that fell into the hole. The second option is difficult to imagine since a relatively small object would be required to possess a huge degeneracy while remaining stable. Moreover, it is ruled out if we assume that quantum gravity is CPT invariant.

This leaves just the first option, that the Hawking radiation is pure. In 1993, Susskind et al. [7], building on the work of 't Hooft [8] and Preskill [9], proposed that the unitarity of the Hawking radiation could be preserved if information is both emitted at the horizon and passes through it, so an observer outside would see it in the Hawking radiation and an observer who falls into the black hole would see it inside but no single observer would be able to confirm both pictures. Although there is no precise mechanism by which it can be said to occur, thought experiments that appear to support this picture of "Black Hole Complementarity" rely on three fundamental assumptions, *viz.*, (a) the unitarity of the Hawking radiation, (b) the validity of effective field theory outside a "stretched" horizon and (c) the equivalence principle. Recently, however, Almheiri et al. (AMPS) have argued that the three assumptions are logically inconsistent and would lead to a violation of the strong subadditivity of the entanglement entropy [10–12]. To resolve the paradox the authors suggested giving up the third assumption, *i.e.*, the equivalence principle.

But Hawking has proposed an intriguing alternative, suggesting that no event horizon would form in the first place if somehow the collapse did not continue beyond the apparent horizon [13]. If indeed no event horizon is formed, the entire discussion about information loss becomes moot. Yet, one is left with the question of how the system evolves after the formation of the apparent horizon. There appears to be ample experimental evidence supporting the existence of very massive, quasi-stable, compact objects located in galactic centers that are consistent with black holes, although it is not known for certain if these supermassive configurations are indeed black holes with event horizons. In this paper, we will examine Hawking's conjecture as it relates to dust collapse, by re-examining some results of an exact quantization [14,15] of the LeMaître–Tolman–Bondi family of solutions [16].

In previous work [17] we have shown that two kinds of functional solutions (analogous to plane waves) of the Wheeler–DeWitt equation for dust collapse may be given. In one, dust shells coalesce onto the apparent horizon on both sides of it. Exterior, infalling waves representing the collapsing shells of dust are accompanied by interior, outgoing waves, which are produced with a relative probability given by the Boltzmann factor at the Hawking temperature of the shells. These interior waves, which are of quantum origin, represent an interior Unruh radiation. In the other solution, dust moves away from the apparent horizon on both sides of it. Interior, infalling waves representing the continued collapse of the dust shells across the apparent horizon are accompanied by exterior, outgoing waves, which are produced with a relative probability again given by the Boltzmann factor at the Hawking temperature. These latter outgoing waves represent the exterior Unruh radiation.

Continued collapse across the apparent horizon from an initial diffuse state and to a central singularity can be achieved by combining the two solutions and requiring the net flux to vanish at the apparent horizon as in [17]. The net effect is that the collapse is accompanied by Unruh radiation in the exterior, as is well known [18], but ends in a central singularity. However, if the collapse does not continue past the apparent horizon, there will be no exterior radiation during the collapse. Furthermore, as the shells coalesce on the apparent horizon, no event horizon will form and the AMPS paradox is resolved. This picture is captured by the first of the exact solutions of the Wheeler–DeWitt equation discussed in the previous paragraph [19].

In this paper, we will examine the consequences of taking seriously the possibility that continued collapse does not occur, *i.e.*, that *quantum* collapse is described by the first solution described above. The collapsing matter is then accompanied by Unruh radiation in the interior of the apparent horizon. In this case, we expect to end up with a spherically symmetric, quasi-static configuration of finite extension and with a specific mass function as the end state of the collapse. Even though no *classical*, static, extended dust configuration can exist, we will show that the interior Unruh radiation that accompanies the infalling dust shells during the collapse will generate the conditions appropriate for a quasi-static configuration to exist. In effect it creates a negative mass point source at the center of the star, which is enveloped by the collapsed matter. We allow for radial but no tangential pressure. This is in keeping with the midi-superspace quantization that informs our construction [14,15]. With the inclusion of a constant negative vacuum energy and radial pressure, unique static solutions exist. There are no horizons and the matter itself extends to twice the Schwarzschild radius.

In Section 2 we briefly summarize our previous work on the wave functionals describing the collapse. In Section 3 we construct the static, spherically symmetric solutions described above and analyze the solutions. In Section 4 we estimate the size of the central negative mass. We conclude with a discussion on our results and possible implications for future observations in Section 4. We take $\hbar = c = 1$ in what follows.

2. Quantum dust collapse

Dust collapse in any dimension, with or without a cosmological constant, is described by the LeMaître–Tolman–Bondi family of solutions [16]. In comoving and synchronous coordinates, (t, ρ, θ, ϕ), one has

$$ds^2 = d\tau^2 - \frac{R'(\tau,\rho)^2}{1+f(\rho)} d\rho^2 - R^2(\tau,\rho) d\Omega^2, \tag{1}$$

where the area radius, $R(\tau, \rho)$ obeys the Einstein equation

$$\dot{R}(\tau,\rho) = \sqrt{f(\rho) + \frac{2GF(\rho)}{R(\tau,\rho)} + \frac{1}{3}\Lambda R^2(\tau,\rho)} \tag{2}$$

and the energy density is given by

$$\epsilon(\tau,\rho) = \frac{F'(\rho)}{R^2(\tau,\rho)R'(\tau,\rho)}. \tag{3}$$

Λ is the cosmological constant. There are two integration functions, $F(\rho)$ and $f(\rho)$, that are interpreted as the twice the gravitational (Misner–Sharp) mass contained within a shell located at ρ and the total energy contained within the same shell respectively. They are the "mass" and "energy" functions of the collapse [3,20].

By considering the expansion of an outgoing, radial null congruence,

$$\Theta = \frac{2R'(\tau,\rho)}{R(t,\rho)}\left[1 - \sqrt{\frac{2GF(\rho)}{R(\tau,\rho)} + \frac{1}{3}\Lambda R^2(\tau,\rho)}\right], \tag{4}$$

one sees that the condition for trapping is met when

$$\frac{2GF(\rho)}{R(\tau,\rho)} + \frac{1}{3}\Lambda R^2(\tau,\rho) = 1, \tag{5}$$

which can be used to determine the time of formation, $\tau_{\text{ah}}(\rho)$, of the apparent horizon once a solution of (2) is determined.

The canonical dynamics of the collapsing dust shells is described by embedding the spherically symmetric ADM metric in the LTB spacetime of (1). After a series of canonical transformations [14,21,22], they are described in a phase space consisting of the dust proper time, $\tau(t,r)$, the area radius, $R(t,r)$, the mass density, $\Gamma(r) = F'(r)$, and their conjugate momenta, $P_\tau(t,r)$, $P_R(t,r)$ and $P_\Gamma(t,r)$ respectively by two constraints,

$$\begin{aligned} \mathcal{H}_r &= \tau' P_\tau + R' P_R - \Gamma P_\Gamma' \approx 0 \\ \mathcal{H} &= P_\tau^2 + \mathcal{F} P_R^2 - \frac{\Gamma^2}{\mathcal{F}} \approx 0, \end{aligned} \tag{6}$$

where the prime denotes a derivative with respect to the ADM radial label coordinate, r, and

$$\mathcal{F} \stackrel{\text{def}}{=} 1 - \frac{2GF}{R} - \frac{1}{3}\Lambda R^2.$$

The apparent horizon occurs when $\mathcal{F} = 0$. In the absence of a cosmological constant, this says that on the apparent horizon the physical radius of each shell is given by

$$R(\tau_{\text{ah}},\rho) = 2GF(\rho). \tag{7}$$

Dirac's quantization procedure may be employed to turn the classical constraints in (6) into quantum constraints, which act on wave functionals. The Hamiltonian constraint then yields a formal Wheeler–DeWitt equation and the momentum constraint imposes diffeomorphism invariance. We begin with an ansatz for the wave functional [14],

$$\Psi[\tau,R,\Gamma] = \exp\left[-\frac{i}{2}\int dr\,\Gamma(r)\mathcal{W}\big(\tau(r),R(r),\Gamma(r)\big)\right], \tag{8}$$

which automatically satisfies the momentum constraint if $\mathcal{W}$ has no explicit dependence on r. The Wheeler–DeWitt equation must be regularized before solutions can be obtained. This regularization was performed on a one dimensional lattice [23,24] given by a discrete set of points, r_i, representing dust shells and separated by a spacing σ. One then finds that the ansatz in (8) yields a product of what may be thought of as shell wave functions,

$$\Psi = \lim_{\sigma\to 0}\prod_i \psi_i(\tau_i,R_i,\Gamma_i) = \lim_{\sigma\to 0}\prod_i e^{\omega_i b_i} \times \exp\left\{-i\omega_i\left[a_i\tau_i \pm \int^{R_i} dR_i \frac{\sqrt{1-a_i^2\mathcal{F}_i}}{\mathcal{F}_i}\right]\right\}, \tag{9}$$

with a well defined continuum limit ($\sigma \to 0$), where $a_i = 1/\sqrt{1+f_i}$ is related to the energy function and $\omega_i = \sigma\Gamma_i/2$. Diffeomorphism invariance also requires that both a_i and b_i depend on r via the mass function, *i.e.*, $a_i = a_i(F_i)$ and $b_i = b_i(F_i)$.

These solutions are defined everywhere except at the apparent horizon. Thus there are “exterior” wave functions that must be matched to “interior” wave functions across the horizon. As can be seen, however, the phases of the interior and exterior wave functions diverge there. A standard technique used in such cases is to analytically continue the solutions into the complex plane. This technique was used to derive the Hawking radiation as a tunneling process in [25]. Thus, analytically continuing into the complex R plane, taking $\mathcal{F}_i = \epsilon\exp[i\varphi]$, with $\epsilon > 0$, and comparing them at $\varphi = \pi/2$. One then finds two sets of matched solutions, with support everywhere; the first is given by [17]

$$\psi^{(1)}_{i,\mathrm{col}}(\tau_i, R_i, F_i) = \begin{cases} e^{\omega_i b_i} \times \exp\left\{-i\omega_i\left[a_i\tau_i + \int^{R_i} dR_i \frac{\sqrt{1-a_i^2\mathcal{F}_i}}{\mathcal{F}_i}\right]\right\} & \mathcal{F}_i > 0 \\ e^{-\frac{\pi\omega_i}{g_{i,h}}} \times e^{\omega_i b_i} \times \exp\left\{-i\omega_i\left[a_i\tau_i + \int^{R_i} dR_i \frac{\sqrt{1-a_i^2\mathcal{F}_i}}{\mathcal{F}_i}\right]\right\} & \mathcal{F}_i < 0 \end{cases} \tag{10}$$

and the second by

$$\psi^{(2)}_{i,\mathrm{col}}(\tau_i, R_i, F_i) = \begin{cases} e^{-\frac{\pi\omega_i}{g_{i,h}}} \times e^{\omega_i b_i} \times \exp\left\{-i\omega_i\left[a_i\tau_i - \int^{R_i} dR_i \frac{\sqrt{1-a_i^2\mathcal{F}_i}}{\mathcal{F}_i}\right]\right\} & \mathcal{F}_i > 0 \\ e^{\omega_i b_i} \times \exp\left\{-i\omega_i\left[a_i\tau_i - \int^{R_i} dR_i \frac{\sqrt{1-a_i^2\mathcal{F}_i}}{\mathcal{F}_i}\right]\right\} & \mathcal{F}_i < 0, \end{cases} \tag{11}$$

where $g_{i,h} = \partial_R\mathcal{F}(R)|_{R_{i,h}}/2$ is the surface gravity of the ith shell at the apparent horizon.

These are the shell wave functions we described in the introduction. The first (in (10)) represents a flow toward the apparent horizon on both sides of it: an infalling shell in the exterior is accompanied by an interior, outgoing shell, produced with a relative probability determined by the Boltzmann factor at the Hawking temperature of the shell. The second (in (11)) describes a flow away from the apparent horizon: an infalling shell in the interior, which represents its continued collapse past the apparent horizon and to a central singularity is accompanied by an exterior, outgoing shell, with a relative probability also determined by the Boltzmann factor. It represents the thermal radiation in the exterior.

One might in principle be interested in constructing wave packets that represent an evolution from a configuration in which the dust cloud begins far from the apparent horizon. Such a wave packet would serve to clarify the semi-classical description of the collapsing ball and would be constructed by superposing the solutions given above with different energies, $a_i(F_i)$. This difficult problem, which is currently under investigation, does not seem feasible at present as both the factor ordering and the diffeomorphism invariance depend on the energy function [24]. Nevertheless, some useful conclusions can be drawn from the “plane wave” solutions we have presented above, as one does, for example, in ordinary quantum scattering theory. We notice that if we take the wave functions in (10) to form the basis for the *quantum* collapse of dust then there will be thermal Unruh radiation inside the apparent horizon but no thermal radiation outside, accompanying the collapse. There will also be no continued collapse to a central singularity; the collapse would terminate at the apparent horizon ($\mathcal{F}_i = 0$), which agrees with Hawking's proposal [13,19]. These conclusions would hold true even if one could find a way to construct diffeomorphism invariant wave packets from (10) representing the collapse.

3. A quasi-classical configuration

As there is good experimental evidence for the existence of very massive, quasi-stable compact objects, we look for static, spherically symmetric solutions of Einstein's equations satisfying the following criteria:

- the collapsed dust ball should occupy a finite region and possesses an energy density that is characteristic of a dust cloud that has condensed onto its apparent horizon, *i.e.*, given by (7),
- the solutions should incorporate the effect of the internal Unruh radiation that has occurred during the collapse phase and
- they must match smoothly to the Schwarzschild vacuum at the boundary.

Within the dust ball the metric will be of the form

$$ds^2 = e^{2A}dt^2 - e^{2B}dr^2 - r^2 d\Omega^2, \tag{12}$$

where $A = A(r)$, $B = B(r)$ and r represents the physical radius. In this coordinate system, if we take the components of the stress-energy to be $T^\mu{}_\nu = \text{diag}(-\varepsilon(r), p_r(r), p_\theta(r), p_\theta(r))$ but impose no equations of state, the field equations are

$$\begin{aligned}
&1 - e^{2B} - 2rB' = -8\pi G r^2 e^{2B}\varepsilon \\
&1 - e^{2B} + 2rA' = 8\pi G r^2 e^{2B} p_r \\
&rA'^2 - B' + A'(1 - rB') + rA'' = 8\pi G r e^{2B} p_\theta,
\end{aligned} \tag{13}$$

where a prime indicates a derivative with respect to the radius, r. The conservation of energy-momentum gives a constraint,

$$\varepsilon A' + p_r' + p_r\left[\frac{2}{r} + A'\right] - \frac{2}{r}p_\theta = 0, \tag{14}$$

which represents the condition for static equilibrium. Two of the stress-energy components may be chosen arbitrarily and then the third is determined by either Einstein's equations or by the conservation law. Below we will choose the energy density and set the tangential pressure to zero.

The first equation in (13) may be re-expressed as

$$\left[r\left(1 - e^{-2B}\right)\right]' = 8\pi G r^2 \varepsilon, \tag{15}$$

which is straightforwardly integrated to give

$$r\left(1 - e^{-2B}\right) = 8\pi G \int^r dr' r'^2 \varepsilon\left(r'\right) - r_0, \tag{16}$$

where r_0 is an integration constant. This is usually set to zero in stellar models to avoid a central singularity, but we will not do so here for reasons that will become clear in the following. The Misner–Sharp mass function of the dust is to be identified with the integral on the right,

$$F(r) = 4\pi \int^r dr' r'^2 \varepsilon(r). \tag{17}$$

Now, according to (7), the mass function that may be expected of a dust ball whose collapse has terminated at the apparent horizon is

$$F(r) = \frac{r}{2G}, \tag{18}$$

for a total gravitational mass of $M_{ms} = F(r_b) = r_b/2G$, where r_b denotes its boundary. It corresponds to an energy density of

$$\varepsilon(r) = \frac{1}{8\pi G r^2} \tag{19}$$

and (16) gives

$$e^{2B} = r/r_0. \tag{20}$$

We see that the constant $r_0 > 0$ is essential and cannot be discarded. Without it there do not exist real solutions for $B(r)$ with the desired mass function, even if pressure is included. Strictly it describes a negative mass point source the center. Such a negative mass source is actually predicted by the wave functions in (10) to form *during* the collapse as energy is extracted from the center by the interior, outgoing Unruh radiation that accompanies the exterior, collapsing shells. This process of energy extraction from the center continues until the collapse terminates. In the next section we will estimate its size.

With $B(r)$ given in (20) and no tangential pressure, we solve the Riccati equation in (13) for $A(r)$ and find

$$ds^2 = r^2\left(1 + \frac{\gamma}{r^{3/2}}\right)^2 dt^2 - \frac{r}{r_0}dr^2 - r^2 d\Omega^2, \tag{21}$$

where γ is another integration constant. There are curvature singularities at $r = 0$ and at $r = (-\gamma)^{2/3}$. To avoid the singularity at $r = (-\gamma)^{2/3}$, either γ must be positive or $(-\gamma)^{2/3}$ must lie outside the outer boundary of the collapsed star, where the solution no longer applies. We will soon show that the second condition cannot be met.

We determine the pressure directly from the second equation in (13)

$$p_r(r) = -\frac{1}{8\pi G r^2}\left[1 - \frac{3r_0}{r}\left(1 + \frac{\gamma}{r^{3/2}}\right)^{-1}\right], \tag{22}$$

so with $\gamma \geq 0$ our solutions are well behaved except at the singular center and they obey the weak energy conditions.

If r_b denotes the outer boundary of the collapsed star, we want to match the interior geometry to an external vacuum, described by the Schwarzschild metric

$$ds^2 = f(R)dT^2 - f^{-1}(R)dR^2 - R^2 d\Omega^2, \tag{23}$$

where

$$f(R) = \left(1 - \frac{2GM_s}{R}\right)$$

and M_s is the Schwarzschild mass of the dust ball. The junction conditions require that

$$R_b = r_b, \qquad T_b = \frac{e^{A(r_b)}}{\sqrt{f(r_b)}}t$$

$$e^{-B(r_b)} = \sqrt{f(R_b)}, \qquad 2A'(r_b) = (\ln f)'|_{R_b} \tag{24}$$

and therefore

$$r_0 = r_b - r_s, \qquad \gamma = 2r_b^{3/2}\left(1 - \frac{3r_s}{2r_b}\right), \tag{25}$$

where we have let $r_s = 2GM_s$ be the Schwarzschild radius.

The first condition says that the physical radius of the boundary must lie outside its Schwarzschild radius. Therefore, as expected, the Schwarzschild mass of the star is less than the Misner–Sharp mass of the dust,

$$M_s = \frac{r_s}{2G} = \frac{r_b - r_0}{2G} = M_{ms} - M_0, \tag{26}$$

by precisely the negative central mass, $-M_0$. If $\gamma \geq 0$, the second condition requires that $r_b \geq 3r_s/2$. But, for $\gamma < 0$ a regular solution is obtained only if we require the singularity to lie outside the boundary of the star. According to (25), this can happen if

$$2\left(\frac{3r_s}{2r_b} - 1\right) > 1, \tag{27}$$

but this would imply that $r_s > r_b$. As this is not possible, the star will be singularity free (except at the center) only if $r_b \geq 3r_s/2$. This implies that $r_b \leq 3r_0$ and $r_s \leq 2r_0$.

4. Estimating r_0

We can provide a simple estimate of the radius, r_0, as follows. The energy extraction from the center occurs during the collapse because every collapsing shell is accompanied by an interior, outgoing wave, which will extract energy from the center. We want to estimate how much energy is extracted in this process. For the given mass function, the energy density of the dust is constant, $\Gamma = 1/2G$. If σ represents the shell thickness, the average energy, ω_i, of each shell will also be constant, $\omega_i = \omega = \sigma\Gamma/2 = \sigma/4G$.

The collapse of the ith shell will have been accompanied by the emission in the interior of an outgoing wave of the same frequency, with a probability that is given by the Boltzmann factor, $e^{-\beta_i\omega}$, at the Hawking temperature, $\beta_i = 2\pi r_i$, of the shell. It follows that the average energy of the outgoing shell is $\langle\omega\rangle = \omega e^{-\beta_i\omega}$ and, to get the total energy extracted, we must sum over all collapsed shells,

$$M_0 = \frac{1}{\sigma}\int_0^{r_b} dr\omega e^{-2\pi\omega r} = \frac{1}{2\pi\sigma}\left[1 - e^{-2\pi\omega r_b}\right]. \tag{28}$$

Replacing ω by $\sigma/4G$ and taking the limit as the shell spacing approaches zero then gives

$$M_0 = \frac{r_b}{4G} = \frac{1}{2}M_{ms}, \tag{29}$$

which implies that $r_0 = r_b/2$. By the matching conditions, it follows that $r_0 = r_s$, therefore the region of negative energy occupies the Schwarzschild radius of the star. Although it extends to half of the boundary radius of the collapsed dust ball and is necessarily surrounded by a cloud of ordinary matter, this is a surprisingly large length scale over which quantum gravitational effects should predominate. There is no event horizon. A photon, emitted near the boundary of this cloud, would experience a relatively tame redshift of

$$z = \sqrt{\frac{r_b}{r_0}} - 1 = \sqrt{2} - 1 \approx 0.414, \tag{30}$$

which is compatible with the gravitational redshift of neutron stars of low core densities [26], suggesting that, in a collapse of realistic matter, quantum gravity could "kick in" much before previously imagined, very near the time at which nuclear densities are achieved. This is consistent with the idea that in extreme conditions quantum gravity may be relevant on distance scales much larger than previously anticipated.

5. Discussion

In this paper we have speculated on the consequences of a simple quantum model of dust collapse. We have argued that the AMPS paradox is avoided if continued collapse does not occur and all dust shells coalesce onto the apparent horizon. We showed that the collapse process is then accompanied by Unruh radiation within the apparent horizon. We argued that the effect of the interior Unruh radiation is energy extraction from the center of the cloud, leaving behind a negative mass singularity as the cloud settles into a quasi-stable equilibrium.

Stable classical solutions, with the given mass function and including pressure were determined. The solutions are governed by two parameters, the Schwarzschild radius, r_s, of the dust ball, equivalently its mass as measured by a distant observer, and the boundary radius, r_b. The difference between the two is the radius, r_0, of a region inside of which the total energy is negative. There are strong constraints on the parameters r_0, r_s and r_b if the interior geometry is required to be well behaved everywhere (except at the center). We have shown that the r_0 should extend to more than one half the Schwarzschild radius and more than one third the radius of the entire star, so it will occupy a significant fraction of the star. A more detailed analysis of the Unruh radiation from the center during the collapse indicated that $r_0 = r_s = r_b/2$.

The effective energy momentum tensor which sources the Einstein equations in (13) is presumed to contain quantum gravitational corrections incorporating the back reaction of the Unruh radiation. Beginning, to the best of our knowledge, with [27], in which it was argued that quantum gravitational corrections can make gravity repulsive at very high densities, many attempts have been made at modeling the radiation back reaction via an effective action for the gravitational field, but this approach has proved to be challenging and remains poorly understood. More recently, an interesting model of homogeneous, perfect fluid collapse was studied in [28], where it was argued that the repulsive nature of the quantum corrections at short distances would cause the collapse to bounce. A general action modeling dimensionally reduced, spherical gravity with a radiation term taken from the two dimensional conformal anomaly was examined in [29], but collapse was not discussed there. In a different approach, the Unruh radiation was included explicitly in a numerical study of dust collapse in [30]. The authors concluded that the collapse results in a bounce prior to crossing the apparent horizon and found, in the cases addressed, that the effective mass of the star is reduced through Unruh radiation by a factor of two, while the star shrinks preserving $r_b/r_s \gtrsim 2$. The latter conclusions are strikingly similar to our own findings, although in this paper we have asked for static solutions and there is a difference between the predictions of our wave functional (10) and the model of [30] in that the Unruh radiation in our model is confined to the interior of the apparent horizon. It therefore remains to show how the object we have described and in particular the negative energy central singularity, which weakens gravity, may form as the end state of the collapse of matter obeying reasonable energy conditions with regular initial data. This is necessary for the solution proposed above to constitute a resolution of the information paradox and we hope to report on it in the future.

Even in this simple dust model the picture that emerges is somewhat different from the traditional view of a black hole. The collapsed dust object occupies twice its Schwarzschild radius, there is no horizon and the spacetime geometry is regular everywhere except at the center. What is being encountered has more in common with other Compact Stars (CSs) such as neutron stars, except that what holds the system up is not the matter equation of state (EOS) but vacuum energy. The traditional radius of the hole (the Schwarzschild radius) is in fact surrounded by a matter cloud. Outgoing radiation from the close to boundary of this cloud should not suffer a gravitational redshift much larger than from very compact neutron stars. Observationally, however, assuming that the general conclusions of our model continue to hold in the presence of more realistic matter, there will be some differences.

An obvious difference between the two types of compact objects is that while the mass of an ordinary CS is limited by the matter EOS, there is no limit to how massive a quantum black hole may be. If the matter EOS is assumed to be solely responsible for holding up the star, then the recently discovered CSs of mass $\sim 2M_\odot$ [31,32] appear to rule out exotic matter EOSs, which tend to become soft at high densities. Under the same assumption, this leaves "ordinary" (nucleonic) matter EOSs with comparatively large radii >11 km for a $2M_\odot$ CS [31]. Quantum black holes would be both more massive and possess smaller radii than neutron stars, but larger radii than classical black holes of the same mass. Therefore it is necessary to measure the radius of CSs in a precise and model independent way to provide this information. While this has proved difficult so far, the proposed Large Observatory for X-ray timing (LOFT) has claimed to be able to measure the radii of some CSs with a precision of up to 1 km [33].

Another difference between them will be their luminosities in the presence of accretion flows such as would occur in X-ray binaries or in galactic centers where supermassive black holes are thought to exist. One may expect accretion onto the surface of an ordinary neutron star to lead to higher luminosities than accretion onto the surface of a quantum black hole because an accreting shell of matter encounters a hard surface as it collapses onto an ordinary neutron star, but the quantum theory dictates that it should slow down and coalesce onto the apparent horizon as it approaches the "surface" of a quantum black hole. Accreting quantum black holes will therefore look fainter than accreting neutron stars. The reason for the darkness of the quantum black hole is quantum mechanics and not the absence of a surface, but the outcome agrees qualitatively with the predictions of [34,35].

Very large compact objects, such as the supermassive black holes that are thought to exist at the centers of galaxies make excellent candidates for verifying or falsifying the existence of quantum black holes, if their radius can be determined accurately. In the near future, observations of the supermassive black hole Sgr A* by the Event Horizon Telescope (EHT) are expected to be sensitive to distance scales of better than a horizon length in the 1 mm range and direct measurements of Sgr A*'s size are expected to become possible [36,37].

Finally, we also mention that a recent study of the periodic modulation in the intensity vs. frequency spectrum of galactic centers seems to support the *similarity* between behaviors of certain pulsars and supermassive black holes [38]. These issues are under investigation, as is also the problem of constructing wave packets representing a collapsing dust ball.

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References

[1] S.W. Hawking, R. Penrose, Proc. Phys. Soc. Lond., Sect. A 300 (1967) 182;
S.W. Hawking, R. Penrose, Proc. Phys. Soc. Lond., Sect. A 314 (1970) 529.

[2] S.W. Hawking, G.F.R. Ellis, The Large Scale Structure of Spacetime, Cambridge University Press, Cambridge, 1973.

[3] P.S. Joshi, Global Aspects in Gravitation and Cosmology, Clarendon Press, Oxford, 1993.

[4] C. Vaz, L. Witten, Phys. Lett. B 325 (1994) 27;
C. Vaz, L. Witten, Phys. Lett. B 13 (1996) L59;
C. Vaz, L. Witten, Phys. Lett. B 487 (1997) 409.

[5] S. Barve, T.P. Singh, C. Vaz, L. Witten, Nucl. Phys. B 532 (1998) 361;
S. Barve, T.P. Singh, C. Vaz, L. Witten, Nucl. Phys. B 58 (1998) 104018.

[6] S.W. Hawking, Commun. Math. Phys. 43 (1975) 199;
S.W. Hawking, Commun. Math. Phys. 14 (1976) 2460.

[7] L. Susskind, L. Thorlacius, J. Uglum, Phys. Rev. D 48 (1993) 3743.

[8] G. 't Hooft, Nucl. Phys. B 256 (1985) 727;
G. 't Hooft, Nucl. Phys. B 335 (1990) 138.

[9] J. Preskill, arXiv:hep-th/9209058.

[10] A. Almheiri, D. Marolf, J. Polchinski, J. Sully, arXiv:1207.3123;
A. Almheiri, D. Marolf, J. Polchinski, D. Stanford, J. Sully, arXiv:1304.6483.

[11] A precursor to the idea of a firewall was proposed by S. Braunstein, S. Pirandola, K. Życzkowski, Phys. Rev. Lett. 110 (2013) 101301, arXiv:0907.1190.

[12] Inconsistencies in the Black Hole Complimentarity picture were noted even earlier by D.-H. Yeom, H. Zoe, Phys. Rev. D 78 (2008) 104008;
D.-H. Yeom, H. Zoe, Int. J. Mod. Phys. A 26 (2011) 3287.

[13] S.W. Hawking, arXiv:1401.5761.

[14] C. Vaz, T.P. Singh, L. Witten, Phys. Rev. D 63 (2001) 104020.

[15] C. Vaz, L. Witten, T.P. Singh, Phys. Rev. D 69 (2004) 104029;
C. Vaz, R. Tibrewala, T.P. Singh, Phys. Rev. D 78 (2008) 024019.

[16] G. LeMaître, Ann. Soc. Sci. Brux. A 53 (1933) 51;
for an English translation see Gen. Relativ. Gravit. 29 (1997) 641;
R. Tolman, Proc. Natl. Acad. Sci. USA 20 (1934) 410;
H. Bondi, Mon. Not. R. Astron. Soc. 107 (1947) 343.

[17] C. Vaz, L.C.R. Wijewardhana, Phys. Rev. D 82 (2010) 084018;
C. Vaz, K. Lochan, Phys. Rev. D 87 (2013) 024045.

[18] W.G. Unruh, Phys. Rev. D 14 (1976) 870;
L. Parker, in: L. Witten, F.P. Esposito (Eds.), Asymptotic Structure of Spacetime, 1976;
S.M. Christensen, S.A. Fulling, Phys. Rev. D 15 (1977) 2088;
N.D. Birrell, P.C.W. Davies, Quantum Fields in Curved Space, Cambridge Mono. Math. Phys., 1981.

[19] C. Vaz, Int. J. Mod. Phys. D 23 (2014) 1441002.

[20] J. Plebansky, A. Krasinski, An Introduction to General Relativity and Cosmology, Cambridge University Press, Cambridge, 2006.

[21] K. Kuchař, Phys. Rev. D 50 (1994) 3961.

[22] J. Brown, K. Kuchař, Phys. Rev. D 51 (1995) 5600.

[23] C. Vaz, T.P. Singh, L. Witten, Phys. Rev. D 69 (2004) 104029.

[24] Claus Kiefer, J. Mueller-Hill, C. Vaz, Phys. Rev. D 73 (2006) 044025.

[25] M. Parikh, F. Wilczek, Phys. Rev. Lett. 85 (2000) 5042.

[26] L. Lindblom, Astrophys. J. 278 (1984) 364.

[27] V.P. Frolov, G.A. Vilkovisky, Phys. Lett. B 106 (1981) 307.

[28] C. Bambi, D. Malafarina, L. Modesto, Eur. Phys. J. C 74 (2014) 2767.

[29] T. Taves, G. Kunstater, arXiv:1408.1444.

[30] L. Mersini-Houghton, H.P. Pfeiffer, arXiv:1409.1837.

[31] P. Demorest, et al., Nature 467 (2010) 1081.

[32] J. Antoniadis, et al., Science 340 (6131) (26 April, 2013).

[33] M. Feroci, et al., Exp. Astron. 34 (2012) 415.

[34] R. Narayan, J. McClintock, New Astron. Rev. 51 (2008) 733.

[35] A. Broderick, A. Loeb, R. Narayan, Astrophys. J. 701 (2009) 1357.
[36] J.A. Hodgson, et al., in: P. Charlot, et al. (Eds.), 11th European VLBI Network Symposium, Bordeaux, France, 9–12 October, 2012, arXiv:1407.8112.
[37] T.P. Krichbaum, et al., in: P. Charlot, et al. (Eds.), 11th European VLBI Network Symposium, Bordeaux, France, 9–12 October, 2012, arXiv:1305.2811.
[38] E.F. Borra, Astrophys. J. 774 (2013) 142.