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# Correspondence

"Long-term consolidation behavior interpreted with isotache concept for worldwide clays" by Watabe, Y., Udaka, K., Nakatani, Y., Leroueil, S. [Soils and Foundations 52 (3) (2012) 449–464]

#### 1. Introduction

The authors would like to acknowledge the discusser for his interest in this technical paper. The discusser says that the objective of his discussion is to point out that the authors' parameter  $\alpha$  (= $\Delta \log p'_c/\Delta \log \dot{\epsilon}_{vp}$ ) is not necessarily equal to  $C_{\alpha e}/C_c$  (= $C_{\alpha e}/C_{c\epsilon}$ ), where  $C_{c\epsilon}$  is the compression index in strain. As shown in the following section, this is incorrect. It can first be noted that incremental strain  $\Delta \epsilon$ and strain rate  $\dot{\epsilon}$  after the end of primary consolidation are essentially equal to the incremental visco-plastic strain  $\Delta \epsilon_{vp}$ and visco-plastic strain rate  $\dot{\epsilon}_{vp}$ , respectively.

The discusser also claims that the applicability of authors' model to worldwide clays is not surprising because he and his co-workers have already illustrated the applicability of the constant  $C_{\alpha c}/C_c$  concept to various kinds of soils. In this closure, the authors would like to discuss the applicability of these models, considering that the strain rates generally observed in the laboratory are different from those observed in the field. The authors will also reply to the other criticisms from the discusser.

# 2. $C_{\alpha e}/C_{\rm c}$ and $\alpha (=\Delta \log p_{\rm c}'/\Delta \log \dot{\epsilon}_{\rm vp})$

Because the slope of the compression curves in Fig. 19 is defined as  $C_{c\varepsilon}$ , the relationship between  $x_{\varepsilon}$ ,  $x_{\log p}$ , and  $C_{c\varepsilon}$  is geometrically expressed as Eq. (11).

$$\frac{x_{\varepsilon}}{x_{\log p}} = \frac{C_{c\varepsilon}}{1} \tag{11}$$

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The distances  $x_{\varepsilon}$  and  $x_{\log p}$  are derived as follows:

From the definition of secondary compression index in strain  $C_{\alpha\varepsilon}$ , incremental strain  $\Delta\varepsilon$  for an incremental logarithmic elapsed time  $\Delta \log t$  is expressed as Eq. (12).

$$\Delta \varepsilon = C_{\alpha \varepsilon} \Delta \log t \tag{12}$$

Using Eq. (12), strain rate is calculated as below:

$$\dot{\varepsilon} = \frac{\Delta\varepsilon}{\Delta t} = \frac{C_{\alpha\varepsilon}\Delta\log t}{\Delta t} = \frac{C_{\alpha\varepsilon}}{\ln 10}\frac{\Delta\ln t}{\Delta t} = \frac{C_{\alpha\varepsilon}}{2.303}\frac{1}{t}$$
(13)

Eq. (13) means that the strain rate decreases with elapsed time in inverse proportion, indicating that the strain rate decreases by a factor of ten when elapsed time increases by 10 times (i.e.,  $\Delta \log t=1$ ) if  $C_{\alpha\varepsilon}$  is constant during the time increment. Therefore, from Eq. (12),  $\Delta\varepsilon$  (corresponding to the distance  $x_{\varepsilon}$  in Fig. 19), equals to  $C_{\alpha\varepsilon}$  as expressed in Eq. (14).

$$\kappa_{\varepsilon} = C_{\alpha\varepsilon} \tag{14}$$

The distance  $x_{\log p}$  in Fig. 19 equals to increment of  $\log p'_{\rm c}$  for one log cycle of strain rate, and can be expressed as Eq. (15).

$$x_{\log p} = \frac{\Delta \log p'_{\rm c}}{\Delta \log \dot{\varepsilon}} = \alpha \tag{15}$$

This equation is the exact definition of the parameter  $\alpha$ .

Eqs. (14) and (15) are substituted into Eq. (11), and then Eq. (16) is obtained.

$$\frac{C_{\alpha\varepsilon}}{C_{c\varepsilon}} = \frac{\Delta \log p'_{c}}{\Delta \log \dot{\varepsilon}} = \alpha$$
(16)

Consequently, it can be said that the discusser's  $C_{\alpha e}/C_c$ (= $C_{\alpha e}/C_{ce}$ ) and the authors' parameter  $\alpha$  (= $\Delta \log p'_c/\Delta \log \dot{\epsilon}_{vp}$ ) are mathematically the same.

The discusser says that Terzaghi et al. (1996) proposed the following empirical equation.

$$p'_{cEOP} = p'_{cCRS} \left(\frac{\dot{\varepsilon}_{EOP}}{\dot{\varepsilon}_{CRS}}\right)^{C_{ac}/C_c}$$
(17)

This equation can be transformed into Eq. (18).

$$\ln\left(\frac{p_{cEOP}}{p_{cCRS}'}\right) = \frac{C_{\alpha e}}{C_{c}} \ln\left(\frac{\dot{\varepsilon}_{EOP}}{\dot{\varepsilon}_{CRS}}\right)$$
(18)

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Fig. 19. Illustration of geometrical relationship between the parameters.

Eq. (18) is equivalent to Eq. (16) and is thus not empirical. Eqs. (17) and (18) mean that the relationship between log  $p'_{\rm c}$  and log  $\dot{\varepsilon}_{\rm vp}$  can be expressed as a line with a slope equal to  $C_{\alpha e}/C_c$ . From Eq. (16), this slope  $C_{\alpha e}/C_c$ equals to the authors' parameter  $\alpha$  (= $\Delta \log p'_c / \Delta \log \dot{\epsilon}_{vp}$ ), but the discusser's one is considered to be constant. According to the discusser and co-workers  $C_{\alpha e}/C_{c}$  is approximately equal to  $0.04 \pm 0.01$  for inorganic clays. The authors show that the parameter  $\alpha$  (i.e.,  $C_{\alpha e}/C_c$ ) is not constant but decreases with decreasing strain rate. From Fig. 17, it can be seen that  $C_{\alpha e}/C_c = 0.04 \pm 0.01$  is essentially valid for strain rates between  $1 \times 10^{-5}$  and  $1 \times 10^{-8}$  s<sup>-1</sup>, i.e. for the range of strain rates generally observed in the laboratory. However, it does not seem to be general, particularly at high strain rates and for in situ strain rates, which are generally much smaller than  $1 \times 10^{-8} \text{ s}^{-1}$ . In the authors' modeling, the strain rate decreases to an infinitesimal value with the lower limit of consolidation yield stress  $p'_{cL}$ , i.e., it results in a certain ultimate creep strain. Actually, the model equation between  $\ln p_{\rm c}$  and  $\ln \dot{\varepsilon}_{\rm vp}$ expressed as Eq. (4) can be transformed to a power function expressed as Eq. (19).

$$\dot{\varepsilon}_{\rm vp} = c_3 \left( \frac{p'_{\rm c} - p'_{\rm cL}}{p'_{\rm cL}} \right)^{c_4} \tag{19}$$

where  $c_3$  and  $c_4$  equal to  $\exp(-c_1/c_2)$  and  $1/c_2$ , respectively. Eq. (19) is exactly the same as Perzyna's (1963) overstress viscoplastic theory, which is often used to model the creep behavior of materials such as concrete and steel. Consequently, it can be said that the authors' creep model is very natural as a rheological approach to material mechanics. This model is also consistent with the observation made by several authors, including the discusser (Mesri and Feng, 1991), that significant unloading (to below  $p'_{cL}$  in the authors' model) does not produce secondary settlement but secondary swelling.

# 3. Applicability of discusser's concept and authors' model

The discusser says that he and co-workers have already illustrated the applicability of the constant  $C_{\alpha e}/C_c$  concept for various soils. The authors highly appreciate these

works, because the discusser's concept and the authors' model are essentially consistent in a range of strain rates generally observed in the laboratory (from  $1 \times 10^{-5}$  to  $1 \times 10^{-8} \text{ s}^{-1}$ ) and during secondary consolidation. It is thought, however, that the discusser's model overestimate strains at smaller strain rates. More fundamentally, the discusser's model considers that soils have two rheological models, one that applies during primary consolidation (no viscous component) and one that applies during secondary consolidation (with a viscous component), which is surprising in itself. This concept corresponds to Hypothesis A. The authors do not agree with this Hypothesis since all well documented embankment cases show that Hypothesis A is not valid, and that it is rather Hypothesis B, which includes viscosity during primary consolidation, that is valid (Kabbaj et al., 1988; Larsson and Mattsson, 2003; Leroueil, 2006). The authors' model corresponds to Hypothesis B.

### 4. Reply to the other criticisms

The discusser points out high pressure increments for some specimens. After the long-term consolidation test, all the data were plotted on the relationship between viscoplastic strain and normalized pressure, such as Fig. 10 in Watabe et al. (2008) and Fig. 9 in this paper. Then, the authors confirmed that there was no evidence of soil extrusion under the specified target pressures for the long-term consolidation tests conducted in this study. In fact, when the authors conducted a long-term consolidation test under a significantly large target pressure of 4.5 times  $p'_c$  for some Osaka Bay Pleistocene clay and Louiseville clay, the test results were below the isotache compression curves, probably because of soil extrusion. Those data resulting in unexpected large settlements were not used in this study.

The discusser commented on the compression curve of the reconstituted Ma13 (Ma13Re) specimen. The undisturbed sample Ma13 with a liquidity index of 0.7 was remolded at a water content of  $2w_L$  and then preliminarily consolidated under 98 kPa. Therefore, the bi-linear compression curve is a very normal result.

The discusser points out that EOP is indicated at a strange point in some cases. As clearly described in the paper, the authors defined the EOP as the point where the strain corresponds to  $1.11 \ (=10/9)$  times the strain obtained at 90% degree of consolidation as evaluated by the square root *t* method. For overconsolidated clays, this kind of strange EOP is sometimes defined. In this study, however, these EOP points do not affect the isotache concept because the authors only took small strain rates obviously in the secondary consolidation stage into consideration.

The discusser says that the isotache lines indicated in Fig. 5 are not consistent with the compression curve for which the compression index decreases with consolidation pressure. This is incorrect. The authors drew lines in Fig. 5

to indicate isotache tendency for the data obtained from the long-term consolidation tests, but used the reference compression curve for fitting. This is clearly illustrated in Fig. 6. In addition, the discusser claims that the authors' data are consistent with the constant  $C_{\alpha e}/C_c$  concept and the concave shape compression curve. However, as mentioned above, the authors' results do not support the discussers' concept in the range of the small strain rates generally observed in the field.

The discusser says that the authors' reference strain rate of  $1 \times 10^{-7}$  s<sup>-1</sup> is close to the typical value of EOP strain rate determined by the discusser and co-workers. In fact, it can be demonstrated (Leroueil, 1988) that, assuming that primary consolidation is completed at that time, the strain rate after 24 h (t=86,400 s) of loading is equal to:

$$\dot{\varepsilon} = \frac{0.434}{t} \frac{C_{\alpha e}}{C_{\rm c}} \frac{C_{\rm c}}{1+e_0} = 2 \times 10^{-7} \frac{C_{\rm c}}{1+e_0} \tag{20}$$

This equation shows that the strain rate is equal to  $1 \times 10^{-7} \text{ s}^{-1}$  when  $C_c/(1+e_0)=0.5$ ; it is slightly smaller in less compressible soils. The EOP strain rate depends on the compressibility of the soil, the coefficient of consolidation and the drainage length, and is, consequently, highly variable.

### 5. Concluding remarks

Most of the discusser's criticisms are based on misunderstandings. The authors' model is more general than the discusser's concept. The constant  $C_{\alpha e}/C_c$  concept is approximately valid only in large strain rates generally observed in the laboratory; it is thought that the authors' model is valid over a wider range of strain rates observed in both the laboratory and field. To predict long-term consolidation field settlement, very small strain rates which are much smaller than those generally observed in the laboratory have to be considered.

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