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Multifractal analysis of China's agricultural commodity futures markets

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Abstract

We investigate the multifractal characteristics of the volatility time series from China's agricultural commodity futures markets, using Multifractal Detrended Fluctuation Analysis and multifractal spectrum analysis. We find that prominent multifractal features exist in China's major agricultural commodity futures markets, including the Hard Winter Wheat (HW) futures, the Strong Gluten Wheat (SG) futures, Soy Bean (SB) futures and corn futures. Furthermore, the multifractality strength and multifractal spectrum width of HW futures are both bigger than that of SG, SB and corn futures, implying that the market risk for HW futures might be the strongest among all the four futures contracts. Finally, comparing empirical results of shuffling and surrogate data, we also find that nonlinear temporal correlations instead of non-Gaussian distribution constitute the major contributions in the formation of multifractal features in these four agricultural commodity futures markets.

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1. Introduction

A fractal is a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole (Mandelbrot, 1982) [1]. Numerous researchers found financial markets exhibit complex dynamics features and have attempted to apply physical theories and methods to analyze these economic and financial problems (e.g. Peters, 1991; Peng et al., 1994; Gu et al., 2010; Alvarez-Ramirez et al., 2002). As a result of that, an interdisciplinary science called Econophysics is created. Mandelbrot (1963, 1967) [2,3], the pioneer of Econophysics, applied fractal

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geometry to US agricultural commodity spot prices to demonstrate that these market prices do not obey “random-walk” hypothesis and they display different properties, that is, long-term correlation or memory, chaos, fractal, etc. From then on, fractal phenomena have been widely confirmed in various financial markets, commodity markets and capital markets by means of fractal geometry (Peters, 1991, 1994) [4,5].

Prior researches have tried various methodological approaches in studying fractal phenomena in complex systems. Hydrologist H.E. Hurst (1951) [6] proposed Rescaled Range Analysis (R/S), which is the most popular analysis of fractal features. However, the results of the R/S analysis depend on the extreme values of the selected samples and are sensitive to the abnormal values of series. Thus, the R/S analysis cannot be used to analyze the long-range correlations of non-stationary series. In order to overcome the drawbacks, Peng et al. (1994)[7] proposed Detrended Fluctuation Analysis (DFA) in the procedure of studying interior correlation of DNA molecular chain. This method applies long-range-power-law correlation to analysis procedure to overcome the requirement of strict short-range correlation in R/S analysis and is widely used to determine the mono-fractal scaling properties. Both R/S and DFA analysis can only be used to analyze properties of mono-fractal, which cannot describe the multiscale and subtle substructures of fractals in complex systems. So Kantelhardt and Zschiegner (2002) [8] combined DFA analysis with multifractal system and proposed Multifractal Detrended Fluctuation Analysis (MF-DFA). Many scholars studied multifractal properties in crude oil market (Gu et al., 2010; Wang et al., 2010; Alvarez-Ramirez et al., 2002) [9-11], stock markets (Zunino et al., 2008; Yuan et al., 2009) [12,13], international exchange rate markets (Norouzzadeh et al., 2006) [14] and gold markets (Wang et al., 2010) [15]. However, few studies can be found in the area of econophysics to test the efficiency of agricultural commodity futures markets and there has no empirical evidence provided by the existing literature that can describe multifractal features of China’s agricultural commodity futures and explain why there exist such multifractality in China’s agricultural commodity futures markets.

This paper seeks to answer these questions by the means of MF-DFA and multifractal spectra. We choose Hard Winter Wheat and Strong Gluten Wheat futures from Dalian Commodity Exchange and Soy Bean and corn futures from Zhengzhou Commodity Exchange as the representatives of China’s agricultural commodity futures markets, we then apply MF-DFA to analyze the strength of multifractality and risk in China’s agricultural commodity futures markets and find the main contributions to information of multifractality by means of shuffled and surrogate data.

The paper is organized as follows: Section 2 presents the MF-DFA procedure. Section 3 describes the data for four representative agricultural commodity futures prices and the summary statistics of their returns. Section 4 applies the MF-DFA method to China’s agricultural commodity futures markets and analyzes the multifractal properties and sources of multifractality in the time series. Section 5 outlines the conclusions.

2. Methodology

Let us suppose $P(i)$, $i=1,2,\dots,N$ to be the time series of four representative agricultural commodity futures prices, where N is the length of the series. Firstly, define their prices returns as

$$r_i = \ln P_{i+1} - \ln P_i \quad (1)$$

Then the “profile” is given by

$$y_i = \sum_{k=1}^i (r_k - \bar{r}) \quad (2)$$

where \bar{r} denotes the averaging returns over the whole time series.

Next, divide the profile $Y(i)$ into $N_s=[N/S]$ non-overlapping segments of equal length s . Since the length N of the series is often not a multiple of the considered time scale s , a short part at the end of the

profile $Y(i)$ may remain. In order not to discard this part of the series, the same procedure is repeated starting from the opposite end. Thereby, $2N_s$ segments are obtained altogether. And then calculate the local trends for each of the $2N_s$ segments by m th order polynomial fit. Then the variance is determined by

$$F^2(s, \lambda) = \frac{1}{s} \sum_{j=1}^s [y_{(\lambda-1)s+j}(j) - \tilde{y}_\lambda(j)]^2 \tag{3}$$

for each segment $\lambda, \lambda=1, 2, \dots, N_s$ and

$$F^2(s, \lambda) = \frac{1}{s} \sum_{j=1}^s [y_{N-(\lambda-N_s)s+j}(j) - \tilde{y}_\lambda(j)]^2 \tag{4}$$

for each segment $\lambda, \lambda=N_s+1, N_s+2, \dots, 2N_s$. Here, $\tilde{y}_\lambda(j)$ is the fitting polynomial with order m in segment λ (generally, called m th order MF-DFA and wrote MF-DFAm).

Then let us average over all segments to obtain the q th order fluctuation function

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\lambda=1}^{2N_s} [F^2(s, \lambda)]^{q/2} \right\}^{1/q} \tag{5}$$

for any real value $q \neq 0$ and

$$F_0(s) = \exp \left\{ \frac{1}{4N_s} \sum_{\lambda=1}^{2N_s} \ln [F^2(s, \lambda)] \right\} \tag{6}$$

Repeating Eq(2)-(6) above for different time scale s . It is apparent that $F_q(s)$ will increase with increasing s . At the same time, $F_q(s)$ depends on the MF-DFA order m with $s \geq m+2$.

By analyzing log-log plots $F_q(s)$ versus s for each value of q , we determine the scaling behavior of the fluctuations. If the time series r_t are long-range power-law correlated, $F_q(s)$ will increase for large values of s as a power-law

$$F_q(s) \propto s^{h(q)} \tag{7}$$

Here, the scaling exponent $h(q)$ can be obtained by observing the slope of the log-log plots of $F_q(s)$ versus s through the method of ordinary least squares (OLS).

Another way to confirm multifractality in time series is using the singularity spectrum $f(\alpha)$, which can be obtained by multifractal spectrum analysis based on the relationship

$$\tau(q) = qh(q) - 1 \tag{8}$$

and then the Legendre transform

$$\alpha = h(q) + qh'(q) \tag{9}$$

$$f(\alpha) = q[\alpha - h(q)] + 1 \tag{10}$$

where α is the Holder exponent and $f(\alpha)$ indicates the dimension of the subset of the series that is characterized by α .

3. Data analysis

The data of this research are the time series of daily closing price for Hard Winter Wheat futures contract from March 28, 2003 to November 12, 2010 (L=1810) and Strong Gluten Wheat futures contract from November 1, 1999 to November 12, 2010 (L=2448) from Zhengzhou Commodity Exchange, Soy Bean futures contracts from July 15, 2002 to November 12, 2010 (L=1963) and corn futures contracts from September 22, 2004 to November 12, 2010 (L=1492) from Dalian Commodity Exchange. All these data are taken from wind@database. In the following discussions, we define size s ranges from 10 to $N/6$ with the computation interval 5, where N is the length of the agricultural commodity futures return series; the degree of polynomial $m=1,2,3$; the rang of q varies from -5 to 5 with the step 0.1.

Table 1. The summary statistics of SB, corn, HW and SG futures contracts

Commodity Exchange	Kind of futures	mean	Std.dev	Kurtosis	Skewness	JB
Dalian	SB	0.000399	0.013869	25.36428	0.290936	40895*
	corn	0.000377	0.009591	23.64962	2.251746	27750.49*
Zhengzhou	HW	0.000220	0.017038	62.53799	3.120718	364644.3*
	SG	0.000341	0.013930	74.43165	4.008425	389443.8*

* indicates the number of JB reject the null hypothesis that the sample comes from a normal distribution at the significance of 0.01.

According to Table 1, we can see a large positive skew and the probability distribution function of variations also show a high degree of peakness and fat-tails instead of a normal distribution. Thus there is a clear departure from Gaussian distribution.

4. Empirical results

4.1. Multifractal characteristics of agricultural commodity futures markets returns

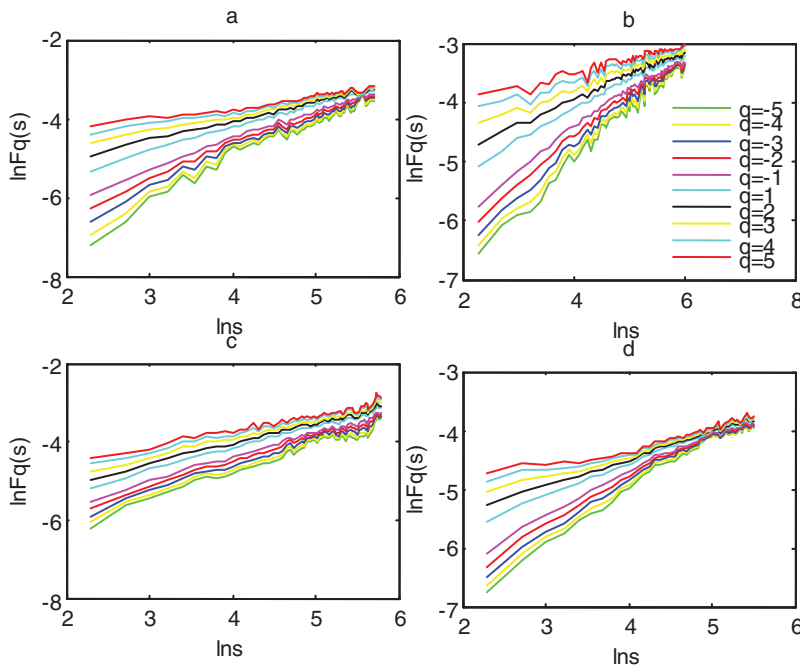


Fig. 1. The relationships between $\ln F_q(s)$ and $\ln s$ in Strong Gluten Wheat(a), Hard Winter Wheat(b), Soy Bean(c) and corn(d) futures contracts respectively where q varies from -5 to 5 from bottom to top.

First of all, based on MF-DFA model, we obtained the relationships between $\ln F_q(s)$ and $\ln s$ in agricultural commodity futures markets (see Fig. 1) for $q=-5, q=-4, \dots, q=5$, where the upper and the lower curves are the curves of $q=5$ and $q=-5$. From the figures we can see the relationships are linear. It shows the return series obey power-law and agricultural commodity futures markets in China display fractal

characteristics. Furthermore, according to the relationships of $h(q)$ and q (see Fig.2), we can see that the exponent $h(q)$ is not a constant, which decreases with the rising q . These results imply China’s agricultural commodity futures markets are multifractal.

In order to further investigate the existence of multifractality in the agricultural commodity futures markets, we got the relationships of Renyi exponent $\tau(q)$ and q (see Fig.3), which are nonlinear and the figures of four representative agricultural commodity futures are all convex to the horizontal axis. It is another evidence of multifractality in agricultural commodity futures markets. At the same time, as the higher nonlinearity of the spectrum has the stronger multifractality in time series, we can obtain the order of multifractal strength in China’s agricultural commodity futures markets from the biggest to the smallest one is Hard Winter Wheat, Strong Gluten Wheat, corn and Soy Bean futures according to the different convexity of figures.

Table 2 also shows that the exponent $h(q)$ depends on q for all different orders, which is another piece of empirical evidence for multifractality in agricultural commodity futures markets. Take order $m=3$ for instance, when q varies from -5 to 5 , $h(q)$ of Hard Winter Wheat futures return series decreases from 0.838 to 0.2238 ; $h(q)$ of Strong Gluten Wheat futures return series decreases from 0.9415 to 0.3053 ; $h(q)$ of corn futures return series decreases from 0.8516 to 0.3436 ; $h(q)$ of Soy Bean futures return series decreases from 0.6929 to 0.3968 . All of these are apparently not constant, indicating agricultural commodity futures markets are multifractal. Especially, when $q=2$, all Hurst exponent in these commodity futures markets are less than 0.5 for all different orders except Soy Bean futures. It shows China’s agricultural commodity futures markets are inefficient and Hard Winter Wheat, Strong Gluten wheat and corn futures markets display anti-persistent properties, while Hurst exponent in Soy Bean futures market is more than 0.5 in the same circumstances, which implies Soy Bean futures market displays persistent properties.

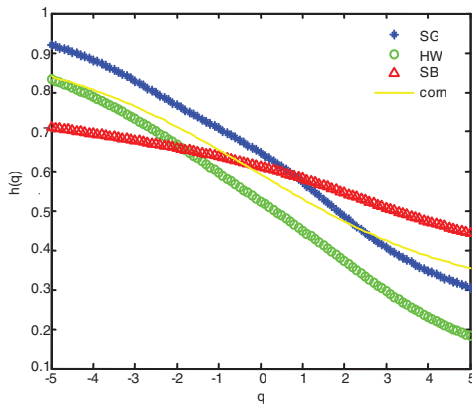


Fig.2. The relationships of $h(q)$ and q in Strong Gluten Wheat(SG), Hard Winter Wheat(HW), Soy Bean(SB) and corn futures contracts respectively where $m=3$ and $-5 \leq q \leq 5$ with the step 0.1

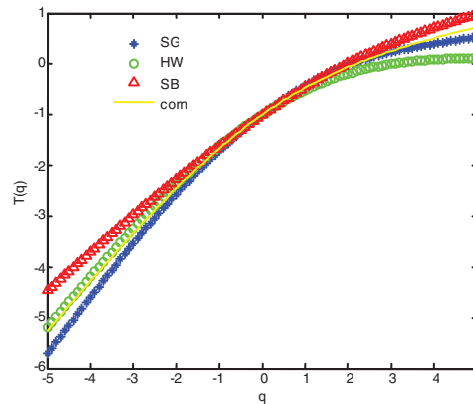


Fig.3. The curves of $\tau(q) \sim q$ in Strong Gluten Wheat(SG), Hard Winter Wheat(HW), Soy Bean(SB) and corn futures contracts when $m=3$ and $-5 \leq q \leq 5$ with the step 0.1

In addition, the variation range of Hurst exponent (see Δh in Table 2) in the four agricultural commodity futures markets for the same order m is different. The variation range of Hurst exponent in Hard Winter Wheat futures market is greater than that of other three futures contracts, suggesting that the multifractal features of Hard Winter Wheat futures is stronger than that other three futures and the risk (Yuan et al., 2009) [13] in Hard Winter Wheat futures markets is also higher than that of three futures.

4.2. Sources of multifractality

In General, there are two major sources for multifractality in various time series. One is different long-range temporal correlations for small and large fluctuations, the other is fat-tailed probability distribution of increments (Norouzzade et al., 2006)[14]. Usually we apply two procedures, that is, shuffling procedure and phase-randomization, to distinguish the contribution of two sources. The shuffling procedure destroys any temporal correlations for small and large fluctuations but preserves the distribution of the variations. In other words, the shuffled data have the same fluctuation distributions with the original data without memory. In contrast, since memory in the time series can be retained and non-Gaussianity of the distributions can be weakened by creating the phase-randomized surrogates, we introduce the surrogate data to explore the contribution of the fat-tailed probability distribution on the multifractality. Both procedures were given by Norouzzade and Rahmani (2006)[14].

Table 2. The Hurst exponent for original, shuffled and surrogate data

	original				shuffled				surrogate			
	SG	HW	SB	corn	SG	HW	SB	corn	SG	HW	SB	corn
<i>m=1</i>												
<i>h</i> (-5)	0.8991	0.8066	0.6943	0.7593	0.6492	0.7026	0.647	0.7081	0.4601	0.4582	0.5597	0.4762
<i>h</i> (-1)	0.6917	0.5689	0.652	0.6191	0.5368	0.5855	0.5828	0.6232	0.4791	0.4287	0.5366	0.4597
<i>h</i> (2)	0.4667	0.359	0.6155	0.4861	0.5051	0.4415	0.5324	0.5197	0.4758	0.4082	0.522	0.4679
<i>h</i> (5)	0.3038	0.2048	0.5455	0.402	0.3861	0.3071	0.4985	0.4254	0.4581	0.3862	0.5111	0.4743
Δh	0.5953	0.6018	0.1488	0.3573	0.2831	0.3955	0.1485	0.2827	0.002	0.072	0.0486	0.0019
<i>m=2</i>												
<i>h</i> (-5)	0.9197	0.8325	0.7114	0.8398	0.6913	0.7486	0.6424	0.7206	0.4881	0.5011	0.5526	0.5083
<i>h</i> (-1)	0.7080	0.5959	0.6376	0.6542	0.6542	0.641	0.5469	0.5898	0.4958	0.4613	0.5323	0.4825
<i>h</i> (2)	0.4853	0.3717	0.5459	0.4727	0.5377	0.5312	0.4875	0.4916	0.4952	0.4366	0.5179	0.4816
<i>h</i> (5)	0.3051	0.1821	0.4442	0.3542	0.3434	0.4245	0.4268	0.4045	0.4795	0.4139	0.5018	0.4832
Δh	0.6146	0.6504	0.2672	0.4856	0.2631	0.3241	0.2156	0.3215	0.0086	0.0872	0.0508	0.0251
<i>m=3</i>												
<i>h</i> (-5)	0.9415	0.868	0.6929	0.8516	0.7233	0.7820	0.6387	0.748	0.499	0.5287	0.5853	0.5492
<i>h</i> (-1)	0.7203	0.6178	0.6094	0.6668	0.6414	0.6387	0.5649	0.6263	0.5019	0.482	0.5446	0.5014
<i>h</i> (2)	0.4886	0.4036	0.5093	0.4699	0.4732	0.4641	0.5092	0.5120	0.5018	0.458	0.5199	0.4842
<i>h</i> (5)	0.3053	0.2238	0.3968	0.3436	0.2934	0.2922	0.4456	0.3981	0.4947	0.4377	0.4984	0.4708
Δh	0.6362	0.6442	0.2961	0.508	0.4299	0.4898	0.1931	0.1217	0.0043	0.091	0.0869	0.0784

Notes: SG, HW, SB stands for Hard Winter Wheat futures, Strong Gluten Wheat futures and Soy Bean futures respectively.

According to the results in Table 2, we find that the Hurst exponents $h(q)$ of original agricultural commodity futures price returns for order $m=1,2,3$ do change and become weaker after we shuffled or randomized the time series, which indicates that both nonlinear temporal correlation and non-Gaussianity of the distributions make major contributions to the multifractality formation. Especially, when $q=2$, for the shuffled data, all the Hurst exponent in China’s agricultural commodity futures markets are approximate to 0.5, for example, $h(2)=0.5053\pm 0.032$ (Strong Gluten Wheat futures), $h(2)=0.4789\pm 0.0374$ (Hard Winter Wheat futures), $h(2)=0.5097\pm 0.0227$ (Soy Bean futures) and $h(2)=0.5078\pm 0.0119$ (corn futures). These results clearly display the shuffled returns series are close to random walk and nonlinear temporal correlations make much larger contribution to the information of multifractality than non-Gaussianity of the distributions.

5. Conclusions

In this paper, we have tested the multifractality properties and the causes in four representative agricultural commodity futures markets in China, including Hard Winter Wheat futures, Strong Gluten

Wheat futures, Soy Bean and corn futures. Our findings are as follows.

First, empirical evidence from MF-DFA confirms that there exist multifractality in China's agricultural commodity futures markets.

Second, by shuffling the original time series, we eliminated market memories and remained the distribution of price fluctuations, while by phase randomization of original time series, we weakened the non-Gaussianity distributions of price fluctuations. Comparing the empirical results from MF-DFA of shuffling data with surrogate one, we can identify that most multifractality of agricultural commodity futures prices variations is due to different long-range correlations for small and large fluctuations; at the same time, the non-Gaussianity distribution also contributes to multifractal behavior of time series.

Finally, the multifractality properties of the four representative agricultural commodity futures prices are different. Compared to Strong Gluten Wheat, Soy Bean and corn futures markets, Hard Winter Wheat futures market indicates much richer multifractality and wider singularity spectrum, implying there exist higher risk in Hard Winter Wheat futures market.

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