Computers and Mathematics with Applications 60 (2010) 2666–2672

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

On soft mappings

Pinaki Majumdar [a,*], S.K. Samanta [b]

[a] Department of Mathematics, M.U.C Women's College, Burdwan, West-Bengal, Pin-713104, India

[b] Department of Mathematics, Visva-Bharati, Santiniketan, West-Bengal, Pin-731235, India

ARTICLE INFO

Article history:
Received 28 March 2010
Received in revised form 31 August 2010
Accepted 1 September 2010

Keywords:
Soft mapping
Fuzzy mapping
Fuzzifying function
Fuzzy bunch of functions
Soft set
Image of a soft set
Soft group

ABSTRACT

In this paper an idea of soft mappings is given and some of their properties are studied. Images and inverse images of crisp sets and soft sets under soft mappings are also studied. An application of soft mapping in medical diagnosis has been shown.

0. Introduction

Molodtsov [1] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. In recent years the development in the fields of soft set theory and its application has been taking place in a rapid pace. This is because of the general nature of parametrization expressed by a soft set. Later other authors like Maji et al. [2–4] further studied the theory of soft sets and used this theory to solve some decision making problems. They also introduced the concept of fuzzy soft set, a more generalised concept, which is a combination of fuzzy set and soft set and studied its properties. In 2007, Aktaş and Cağman [5] introduced a notion of soft group. Recently Kong et al. [6,7] applied the soft set theoretic approach in decision making problems. The idea of soft semirings was introduced by Feng et al. [8]. In 2009, Ali et al. [9] defined some new operations on soft sets. Majumdar and Samanta [10,11] studied the problem of similarity measurement between soft sets and fuzzy soft sets and they also introduced the concept of generalised fuzzy soft sets [12].

In the first section of this paper some preliminary results are given. In Section 2, a definition of soft mapping is given and some of its properties are studied. It has also been shown that a fuzzy mapping is a special case of soft mapping. Also it has been shown that some classes of soft mappings can form a soft monoid and an example is given when a soft mapping can also be considered as a soft group. The image of a set and a soft set under a soft mapping and their respective inverse images are defined and some of their properties are studied in Sections 3 and 4. An application of this soft mapping in medical diagnosis problem has been shown in Section 5. Section 6 concludes the paper.

1. Preliminaries

In this section we recollect a few definitions and properties regarding soft sets.

* Corresponding author.
E-mail addresses: pmajumdar2@rediffmail.com (P. Majumdar), syamal.samanta@visva-bharati.ac.in (S.K. Samanta).

0898-1221/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2010.09.004

Definition 1.1 ([1]). Let U be an initial universal set and let E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subset E$. A pair (F, A) is called a soft set over U iff F is a mapping given by $F : A \to P(U)$.

Example 1.2. As an illustration, consider the following example.

Suppose a soft set (F, E) describes the attractiveness of the shirts which the authors are going to wear.

U = the set of all shirts under consideration = $\{x_1, x_2, x_3, x_4, x_5\}$
E = {colorful, bright, cheap, warm} = $\{e_1, e_2, e_3, e_4\}$.
Let $F(e_1) = \{x_1, x_2\}, F(e_2) = \{x_1, x_2, x_3\}, F(e_3) = \{x_4\}, F(e_4) = \{x_2, x_5\}$.
So, the soft set (F, E) is a family $\{F(e_i), i = 1, 2, 3, 4\}$ of $P(U)$.

Definition 1.3 ([1]). A soft set (F, A) over U is said to be a null soft set denoted by $\tilde{\Phi}$, if $\forall\, e \in A,\ F(e) = \varphi$.

Definition 1.4 ([1]). A soft set (F, A) over U is said to be an absolute soft set denoted by $\tilde{A}$, if $\forall\, e \in A,\ F(e) = U$.

Definition 1.5 ([13]). Let (F, A) and (G, B) be two soft sets over U. Then (F, A) is called a soft subset of (G, B), denoted by $(F, A) \subseteq (G, B)$, if (i) $A \subset B$, (ii) $\forall \varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$.

Definition 1.6 (*[13] Equality of Two Soft Sets*). Two soft sets (F, A) and (G, B) over U are said to be equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 1.7 (*[10] Complement of a Soft Set*). The complement of a soft set (F, A) is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^C, A)$, where $F^C : A \to P(U)$ is a mapping given by $F^C(\alpha) = U - F(\alpha), \forall \alpha \in A$.

Definition 1.8 ([9]). Let (F, A) and (G, B) be two soft sets over a common universe U.

(1) The extended union of (F, A) and (G, B), denoted here by $(F, A) \cup (G, B)$, is defined as the soft set (H, C), where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = F(e), \quad \text{if } e \in A - B; \ = G(e), \quad \text{if } e \in B - A; = F(e) \cup G(e), \quad \text{if } e \in A \cap B.$$

(2) The restricted intersection of (F, A) and (G, B), denoted here by $(F, A) \cap (G, B)$, is defined as the soft set (H, C), where $C = A \cap B$ and $\forall e \in C, H(e) = F(e) \cap G(e)$.

Throughout this paper we have taken extended union as union of soft sets and restricted intersection as our intersection of soft sets.

Theorem 1.9 ([14]). *Let $f : A \to B$ be a mapping and P, Q be non-empty subsets of A. Then the following holds:*

(a) $P \subset Q \Rightarrow f(P) \subset f(Q)$
(b) $f(P \cup Q) = f(P) \cup f(Q)$
(c) $f(P \cap Q) \subset f(P) \cap f(Q)$
(d) $f(P \cap Q) = f(P) \cap f(Q)$, *if f is injective*
(e) $(f(P))^c \subset f(P^c)$, *if f is surjective*
(f) $(f(P))^c = f(P^c)$, *if f is bijective.*

Theorem 1.10 ([14]). *Let $f : A \to B$ be an onto mapping and S, T be non-empty subsets of B. Then the following holds:*

(i) $S \subset T \Rightarrow f^{-1}(S) \subset f^{-1}(T)$
(ii) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
(iii) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$
(iv) $f^{-1}(S^c) = (f^{-1}(S))^c$.

Definition 1.11 ([5]). Let G be a group and A be a non-empty set. Let (F, A) be a soft set over G. Then (F, A) is said to be a soft group over G if $F(x)$ is a subgroup of $G\, \forall\, x \in A$.

Definition 1.12 ([9]). A soft set (F, A) over U is called a full soft set if $\bigcup_{a \in U} F(a) = U$.

2. Soft mappings

In this section we introduce a notion of soft mappings and study its properties. Let X be the universal set and E be a parameter set. Then the pair (X, E) will be called a soft universe. Throughout this section we assume that (X, E) is our soft universe.

Definition 2.1. Let A, B be two non-empty sets and E' be a parameter set. Then the mapping $F : E' \to P(B^A)$ is called a soft mapping from A to B under E', where B^A is the collection of all mappings from A to B.

Actually a soft mapping F from A to B under E' is a soft set over B^A.

Example 2.2. Let $X = \{x_1, x_2\}$ and $I = [0, 1]$. Then I^X is the collection of all fuzzy subsets of X.
Let $C = \{f_i,\ i = 1, 2, \dots, 5\} \subseteq I^X$,
where $f_1 = \{\frac{x_1}{0.8}, \frac{x_2}{0.4}\}, f_2 = \{\frac{x_1}{0.7}, \frac{x_2}{0.0}\}, f_3 = \{\frac{x_1}{0.1}, \frac{x_2}{0.8}\}, f_4 = \{\frac{x_1}{0.5}, \frac{x_2}{0.5}\}$ and $f_5 = \{\frac{x_1}{0.2}, \frac{x_2}{0.3}\}$.
Let us define a function $F : E = (0, 1) \to P(I^X)$ as follows:
For $\alpha \in E = (0, 1)$, $F(\alpha)$ is the collection of members of C having same strong α-cuts (i.e. $\{x \in X; f(x) > \alpha,\ \text{for } f \in I^X\}$).
For example, $F(0.6) = \{f_1, f_2\}$, $F(0.1) = \{f_1, f_4, f_5\}$, $F(0.9) = \phi$, etc.
Then F is a soft mapping from X to I under E.

Example 2.3. Let $E = \{e_1, e_2\},\ A = \{x_1, x_2\},\ B = \{x_3, x_4, x_5\}$.
Let $f_1, f_2, f_3, f_4 : A \to B$ be defined as follows:

$$f_1(x_1) = x_3, \quad f_1(x_2) = x_4, \quad f_2(x_1) = x_4, \quad f_2(x_2) = x_3, \quad f_3(x_1) = x_4,$$
$$f_3(x_2) = x_5 \quad \text{and} \quad f_4(x_1) = x_5, \quad f_4(x_2) = x_4.$$

Let $F : E \to P(B^A)$ be defined as follows: $F(e_1) = \{f_1, f_4\},\ F(e_2) = \{f_1, f_2, f_3\}$.
Then F is a soft mapping from A to B under E.

Remark 2.4. It should be noted that if we consider the fuzzy functions of a non-fuzzy variable [15], then a fuzzy function is a special kind of soft function.

Fuzzy functions of a non-fuzzy variable can be of two types namely (i) fuzzifying function and (ii) fuzzy bunch of crisp functions. We will consider both the cases in detail.

(i) A fuzzifying function $\tilde{f}$ from X to Y is an ordinary function from X to I^Y, $\tilde{f} : X \to I^Y$, $I = [0, 1]$.
Now $\tilde{f}$ can be thought of as a soft set over (I^Y, X) and also as a soft mapping $F : X \to P(I^Y)$ from Y to I under X such that for each $x \in X,\ F(x)$ is a singleton set.

(ii) Again a fuzzy bunch F of functions from X to Y is a fuzzy set in Y^X, that is, each function f from X to Y has a membership value in F. Again Aktaş and Cağman [5] have shown that every fuzzy set in X can be considered as a soft set over X with parameter set [0,1] by means of its α-level sets. Therefore here a fuzzy bunch F can be considered as a soft set G over Y^X, with parameter set [0, 1]. And in Definition 2.1 we have shown that this type of soft set can be considered as a soft mapping from X to Y under [0, 1]. Hence F is a soft mapping.

Definition 2.5. Let A, B, C be non-empty sets and E', E_1, E_2 be parameter sets. Then

(i) the soft mapping $F : E' \to P(A^A)$, defined by $F(e) = \{i_A\}\ \forall\, e \in E'$, where $i_A : A \to A$ is the identity mapping in A, is called the identity soft mapping on A under E'.

(ii) A soft mapping $F : E' \to P(B^A)$ is said to be a constant soft mapping under E' if $\forall\, e \in E'$, $F(e)$ is a collection of constant mappings from A to B.

(iii) For two soft mappings $F_1 : E_1 \to P(B^A)$ and $F_2 : E_2 \to P(B^A)$ over (U, E), they are said to be equal (denoted by $F_1 = F_2$) if (i) $E_1 = E_2$ and (ii) $F_1(e) = F_2(e)$, $\forall e \in E_1 = E_2$.

(iv) For a soft mapping $F : E' \to P(B^A)$ and for $E'' \subseteq E'$, the soft mapping $F_{E''} : E'' \to P(B^A)$, defined by $F_{E''}(e) = F(e),\ \forall e \in E''$, is called the restriction of F to E''.

(v) For two soft mappings with $F : E_1 \to P(B^A)$ and $G : E_2 \to P(C^B)$ with $E_1 \cap E_2 \neq \phi$, their composition $G * F : E_1 \cap E_2 \to P(C^A)$ is defined by:

$$(G * F)(e) = \{h^e = g^e \circ f^e : A \to C;\ g^e \in G(e), f^e \in F(e)\}, \quad e \in E_1 \cap E_2.$$

Example 2.6. Let $A = \{x_1, x_2\}$, $B = \{x_3, x_4, x_5\}$, $C = \{x_5, x_6, x_7\}$.
Also let $E_1 = \{e_1, e_2\} = E_2$. Let $f_1, f_2 : A \to B$ and $g_1, g_2 : B \to C$ be ordinary mappings.
Let $F : E_1 \to P(B^A)$ be a soft mapping defined as: $F(e_1) = \{f_1\}$ and $F(e_2) = \{f_2\}$.
Again let $G : E_2 \to P(C^B)$ be another soft mapping defined as: $G(e_1) = \{g_1, g_2\}$ and $G(e_2) = \{g_2\}$.
Then the composition of F and G is possible and let $H = G * F : E_1 \cap E_2 \to P(C^A)$,
where $H(e_1) = \{g_1 \circ f_1, g_2 \circ f_1\}$ and $H(e_2) = \{g_2 \circ f_2\}$.

Definition 2.7. A soft mapping $F : E' \to P(B^A)$ is said to be weakly injective if $\forall\, e, f \in E', e \neq f \Rightarrow F(e) \neq F(f)$. Again F is said to be strongly injective if $\forall\, e, f \in E', e \neq f \Rightarrow F(e) \cap F(f) = \phi$.

Notes 2.8. This is clear from the above definition that a strongly injective soft mapping is also a weakly injective soft mapping. Also a soft mapping is weakly surjective iff it is a full soft set.

Definition 2.9. A soft mapping $F : E' \to P(B^A)$ is said to be weakly surjective if for any $f \in B^A, \exists\, e \in E'$, such that $f \in F(e)$. F is said to be strongly surjective if $f \in B^A \Rightarrow f \in F(e)\ \forall\, e \in E'$.

Definition 2.10. A soft mapping $F : E' \to P(B^A)$ is said to be weakly (strongly) bijective if F is both weakly (strongly) injective and weakly (strongly) surjective.

Example 2.11. In Example 2.6, F is a strongly injective soft mapping but G is weakly injective.

Theorem 2.12. *If F is strongly surjective and G is strongly surjective then $G * F$ is strongly surjective.*

Proof. Let $F : E_1 \to P(B^A)$ and $G : E_2 \to P(C^B)$. Then $H = G * F : E(=E_1 \cap E_2) \to P(C^A)$.

Let $f \in C^A$, then f is a mapping from A to C.

Next we take a function $g : A \to B$. Define a function $h : B \to C$ by $h(g(x)) = f(x)$, $\forall x \in A$ and $h(y) = f(x_0)$, where x_0 is a fixed element of A, if $y \in B \setminus g(A)$.

Then $g \in F(e)$ and $h \in G(e)\ \forall e \in E$ since F and G are strongly surjective. Hence $f = h \circ g \in (G * F)(e)\ \forall e \in E$. Also f is an arbitrary member ofC^A.

Thus $H = G * F$ is also strongly surjective. □

Corollary 2.13. *Let $E_1 = E_2 = E$. Then if F is weakly surjective and G is strongly surjective, then $G * F$ is weakly surjective.*

Proof. Clear from the proof of Theorem 2.12. □

Notes 2.14. Composition of two weakly (strongly) injective soft mappings may not be weakly (strongly) injective.

To show this we present the following example:

Example 2.15. Let $E = \{e_1, e_2\}, A = \{x_1, x_2\}, B = \{y_1, y_2\}, C = \{z_1, z_2\}$. Let $f_1, f_2 \in B^A$ be defined as follows: $f_1(x_1) = y_1, f_1(x_2) = y_2; f_2(x_1) = y_2, f_2(x_2) = y_1$.

Also let $g_1, g_2 \in C^B$ be defined as follows : $g_1(y_1) = z_1, g_1(y_2) = z_2; g_2(y_1) = z_2, g_2(y_2) = z_1$.

Further let $F : E \to P(B^A)$ be a strongly injective soft mapping defined as follows: $F(e_1) = \{f_1\}$, $F(e_2) = \{f_2\}$. Again let $G : E \to P(C^B)$ be another strongly injective soft mapping defined as follows: $G(e_1) = \{g_1\}, G(e_2) = \{g_2\}$. Then $G * F : E \to P(C^A)$ is given by: $(G * F)(e_1) = \{g_1 \circ f_1\}$ and $(G * F)(e_2) = \{g_2 \circ f_2\}$. But $(G * F)(e_1) = (G * F)(e_2)$ although $e_1 \neq e_2$.

Theorem 2.16. *The composition operation of soft mappings is associative.*

Proof. Let $e \in E$, then $((F * G) * H)(e) = \{t \circ h;\ t = f \circ g \in (F * G)(e),\ h \in (H)(e)\} = \{(f \circ g) \circ h;\ f \in F(e), g \in G(e), h \in H(e)\} = \{f \circ (g \circ h); f \in F(e), g \in G(e), h \in H(e)\}$ (since crisp function composition is associative)

$$= \{(f \circ s); f \in F(e),\ s \in (G * H)(e)\} \subseteq (F * (G * H))(e).$$

Similarly it can be shown that $(F * (G * H))(e) \subseteq ((F * G) * H)(e)$.

Hence the theorem. □

Definition 2.17. A soft mapping $F : E \to P(B^A)$ is said to be naturally injective (surjective) if $F(e)$ is a collection of injective (surjective) mappings from A to $B\ \forall e \in E$. If F is both naturally injective and naturally surjective then F will be called naturally bijective.

Let (X, E) be the soft universe. Let A be a non-empty subset of X. Let $\Im_E$ be the collection of all soft mappings from A to A. Now let '*', the composition of two soft mappings, be our binary operation. Then $(\Im_E, *)$ forms a monoid.

For, (i) by Theorem 2.16 it follows that $*$ is associative,

(ii) the identity soft mapping I_A, over A exists. Hence $I_A \in \Im_E$ and for any $F \in \Im_E$, $F * I_A = I_A * F = F$. Hence an identity element exists in $(\Im_E, *)$.

Again sometimes a soft mapping can form a soft group. Consider the following example:

Example 2.18. Consider the symmetry group $S_3 = \{\rho_0, \rho_1, \rho_2, \rho_3, \rho_4, \rho_5\}$. Then $\{\rho_0\}, \{\rho_0, \rho_3\}, \{\rho_0, \rho_4\}, \{\rho_0, \rho_5\}, \{\rho_0, \rho_1, \rho_2\}$ and S_3 are subgroups of S_3 where ρ_i is a permutation on A, for $i = 0, 1, 2, 3, 4, 5$.

Next let $A = \{1, 2, 3\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Let $F : E \to P(A^A)$ be a soft mapping defined as follows:

$$F(e_1) = \{\rho_0\}, \quad F(e_2) = \{\rho_0, \rho_3\}, \quad F(e_3) = \{\rho_0, \rho_4\}, \quad F(e_4) = \{\rho_0, \rho_5\},$$
$$F(e_5) = \{\rho_0, \rho_1, \rho_2\}, \quad F(e_6) = S_3.$$

Then F is a soft group.

3. Image of a set under a soft mapping

Definition 3.1. Let $F : E \to P(B^A)$ be a soft mapping. Let $T \subseteq A$. Then $F(T)$ is a mapping from E to $P(B)$ such that $F(T)(e) = \bigcap_{f_e \in F(e)} f_e(T) = \bigcap_{f_e \in F(e)} \{f_e(t) : t \in T\}$. Thus $F(T)$is soft set over (B, E).

Example 3.2. Let $X = \{x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4\}$, $E = \{e_1, e_2\}$, $A = \{x_1, x_2, x_3\}$, $B = \{y_1, y_2, y_3\}$. Let $F : E \to P(B^A)$ is a soft mapping defined as follows: $F(e_1) = \{f_1, f_2\}$, $F(e_2) = \{f_3\}$, where $f_1, f_2, f_3 : A \to B$ are given as:

$$f_1(x_1) = y_1, \quad f_1(x_2) = y_2, \quad f_1(x_3) = y_3; \quad f_2(x_1) = y_2, \quad f_2(x_2) = y_3, \quad f_2(x_3) = y_1;$$
$$f_3(x_1) = y_3, \quad f_3(x_2) = y_1, \quad f_3(x_3) = y_2.$$

Let $T = \{x_1, x_2\} \subseteq A$.
Then $F(T) : E \to P(B)$ is such that $F(T)(e_1) = \{y_2\}$.

$$F(T)(e_2) = \{y_1, y_3\}.$$

Theorem 3.3. *Let $F : E \to P(B^A)$ be a soft mapping and T_1, T_2 be two non-empty subsets of A, then the following holds:*

(i) $T_1 \subset T_2 \Rightarrow F(T_1) \subset F(T_2)$
(ii) $F(T_1 \cup T_2) = F(T_1) \cup F(T_2)$
(iii) $F(T_1 \cap T_2) \subset F(T_1) \cap F(T_2)$ *and* $F(T_1 \cap T_2) = F(T_1) \cap F(T_2)$ *if F is naturally injective.*
(iv) $F(T_1^c) \subset (F(T_1)^c)$, *if F is naturally injective.*

Proof. Let $F : E \to P(B^A)$ be a soft mapping. Then for $e \in E$, $F(e) = \{f_e : f_e : A \to B\}$.

(i) Following Theorem 1.9(a) we have for each mapping $f_e : A \to B$,

$$T_1 \subset T_2 \Rightarrow f_e(T_1) \subset f_e(T_2). \therefore \bigcap_{f_e \in F(e)} f_e(T_1) \subset \bigcap_{f_e \in F(e)} f_e(T_2) \Rightarrow F(T_1)(e) \subset F(T_2)(e).$$

Thus the first result holds.

(ii) Also from Theorem 1.9(b) for each mapping $f_e : A \to B$, we have $f_e(T_1 \cup T_2) = f_e(T_1) \cup f_e(T_2)$,

$$\therefore F(T_1 \cup T_2) \supset F(T_1) \cup F(T_2).$$

Thus the second result holds.

(iii) From Theorem 1.9(c and d) we have the following:

$$f_e(T_1 \cap T_2) \subset f_e(T_1) \cap f_e(T_2).$$
$$\therefore \bigcap_{f_e \in F(e)} f_e(T_1 \cap T_2) \subset \bigcap_{f_e \in F(e)} (f_e(T_1) \cap f_e(T_2)) = F(T_1) \cap F(T_2)(e).$$
$$\therefore F(T_1 \cap T_2) \subset F(T_1) \cap F(T_2).$$

Hence the third result also holds

(iv) Now from Definition 3.1 and De Morgan's law we have:

$$(F(T_1))^c = \left(\bigcap_{f_e \in F(e)} f_e(T_1) \right)^c = \bigcup_{f_e \in F(e)} (f_e(T_1))^c.$$

Again from Theorem 1.9(e) we have:

$$F(T_1^c) = \bigcap_{f_e \in F(e)} f_e(T_1^c) = \cap_{f_e \in F(e)} (f_e(T_1))^c = \left(\bigcup_{f_e \in F(e)} f_e(T_1) \right)^c = \bigcap_{f_e \in F(e)} (f_e(T_1))^c \subset \bigcup_{f_e \in F(e)} (f_e(T_1))^c = (F(T_1))^c.$$

Thus the fourth result also holds and hence the theorem is proved. □

Definition 3.4. Let $F : E \to P(B^A)$ be a soft mapping. Let $T \subset B$. Then the inverse image of T under F is defined as follows: For $e \in E$, $F^{-1}(T)(e) = \bigcup_{f_e \in F(e)} \{S : f_e(S) \subset T\} = \bigcup_{f_e \in F(e)} f_e^{-1}(T)$.

Thus the inverse image $F^{-1}(T)$ is a soft set over A.

Theorem 3.5. *Let $F : E \to P(B^A)$ be a naturally surjective soft mapping. Let S, T be non-empty subsets of B. Then the following holds:*

(i) $S \subset T \Rightarrow F^{-1}(S) \subset F^{-1}(T)$
(ii) $F^{-1}(S \cup T) = F^{-1}(S) \cup F^{-1}(T)$
(iii) $F^{-1}(S \cap T) = F^{-1}(S) \cap F^{-1}(T)$
(iv) $F^{-1}(S^c) \supset (F^{-1}(S))^c$.

Proof. The proofs are straightforward. □

4. Image of a soft set under a soft mapping

Definition 4.1. Let $F : E \to P(B^A)$ be a soft mapping. Let $S = (P, E_1)$ be a soft set over A, where $E_1 \subset E$. Then the image of S under F, denoted by $F(S)$, is defined as follows:

$$\text{For } e \in E_1, \quad F(S)(e) = \bigcap_{f_e \in F(e)} f_e(P(e)) = \bigcap_{f_e \in F(e)} \{f_e(t);\ t \in P(e)\}, \quad \text{if } P(e) \neq \phi$$
$$= \phi, \quad \text{if } P(e) = \phi.$$

Hence $F(S)$ is a soft set over B.

Example 4.2. Let $E = E_1 = \{e_1, e_2\}$, $A = \{x_1, x_2\}$, $B = \{y_1, y_2, y_3\}$. Let $f_1, f_2 : A \to B$ be defined as follows: $f_1(x_1) = y_1$, $f_1(x_2) = y_2$; $f_2(x_1) = y_3, f_2(x_2) = y_2$. Also let $P : E \to P(A)$ be defined as follows:

$P(e_1) = \{x_2\}$, $P(e_2) = \{x_1\}$. Then $S = (P, E)$ is a soft set over A.

Let $F : E \to P(B^A)$ be defined as follows: $F(e_1) = f_1$, $F(e_2) = f_2$.

Then $F(S)(e_1) = f_1(P(e_1)) = \{y_2\}$ and $F(S)(e_2) = f_2(P(e_2)) = \{y_3\}$.

$\therefore F(S) = \{\{y_2\}, \{y_3\}\}$, which is a soft set over B.

The following theorem holds here.

Theorem 4.3. *Let $F : E \to P(B^A)$ be a soft mapping and $S_1 = (P_1, E_1), S_2 = (P_2, E_2)$ be two soft sets over A then the following holds:*

(i) $S_1 \subset S_2 \Rightarrow F(S_1) \subset F(S_2)$
(ii) $F(S_1 \cup S_2) = F(S_1) \cup F(S_2)$
(iii) $F(S_1 \cap S_2) \subset F(S_1) \cap F(S_2)$ *and* $F(S_1 \cap S_2) = F(S_1) \cap F(S_2)$ *if* F *is naturally injective.*
(iv) $F(S_1^C) \subset F(S_1)^C$.

Proof. The results follow from Theorem 3.3 and Definition 4.1 and the definition of soft set. Because for each $e \in F(e)$, $P_1(e)$ and $P_2(e)$ are crisp sets, the result follow from Theorem 3.3. □

5. An application of soft mapping in medical diagnosis

A soft mapping can be used to model a disease–symptom relationship in connection with medical diagnosis problem.

The problem: In interior places of a third world country, a patient comes with certain symptoms to a physician and he often has to diagnose the disease by studying the symptoms only because in most of the cases proper clinical facilities are not available. So there is a possibility for human errors.

We want to device a mathematical system based on soft mapping which will help the physicians to diagnose the disease correctly.

For that we first have to construct a model soft mapping indicating the disease–symptom relationship. This model may be different depending upon different geographical regions, etc.

Let $E = \{d_1, d_2, d_3, \ldots, d_m\}$ denote the set of all diseases; $A = \{s_1, s_2, \ldots, s_n\}$ denote the set of all known symptoms; $J = [0, 1]$. Then we construct a soft mapping $F : E \to P(J^A)$ such that $e \in E$, $F(e)$ is a singleton set $\{f_e\}$, where $f_e : A \to J$ is an injective function. This function f_e assigns a numeric value to each symptom with respect to a particular disease. Thus the soft mapping F represents a model of diseases and the occurring symptoms with a weight given to each symptom. The weights indicate the possibility of a particular disease with respect to a symptom. This model soft mapping can be constructed by consulting a group of specialist physicians.

Now suppose that a patient comes with certain symptoms. We then construct the set S of his symptoms. Then to determine his disease we find $F(S)$.

Corresponding to each $d_j \in E$, we form the set $F(S)(d_j) \subset J$. Now we calculate the score of a particular disease with respect to the symptoms S. The score of $d_j \in E$, is defined as follows:

Score $(d_j) = \sum_{s_i \in F(S)(d_j)} f_{d_j}(s_i)$.

We conclude that the person suffers from disease $d_k \in E$, if Score (d_k) is maximum.

We further illustrate the process with an example: Here let $E = \{d_1, d_2, d_3\}$, where d_1 is influenza, d_2 is asthma and d_3 is pneumonia. We consider the following symptom set $A = \{s_1, s_2, \ldots, s_n\}$, where s_1 is mild fever, s_2 is high fever, s_3 is acute breathing trouble, s_4 is sneezing, s_5 is loose motion, s_6 is body ache, s_7 is head ache, s_8 is wheeze cough, s_9 is mucus in lungs, s_{10} is whistle sound while breathing, s_{11} is running nose. We define three functions f_1, f_2 and f_3 in the form of a table as follows:

$$f_1 = \left| \begin{array}{ccccccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} \\ 0.8 & 0.2 & 0.1 & 0.7 & 0 & 0.4 & 0.9 & 0 & 0 & 0 & 1.0 \end{array} \right.$$

$$f_2 = \left| \begin{array}{ccccccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} \\ 0.1 & 0 & 0.9 & 0.6 & 0.1 & 0 & 0.1 & 1.0 & 0.7 & 0.9 & 0.3 \end{array} \right.$$

$$f_3 = \left| \begin{array}{ccccccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} \\ 0.3 & 0.6 & 0.5 & 0.1 & 0 & 0.8 & 0.6 & 0.4 & 1.0 & 0.4 & 0.1 \end{array} \right.$$

We construct the model soft mapping $F : E \to P(J^A)$ defined as follows: $F(d_1) = \{f_1\}, F(d_2) = \{f_2\}, F(d_3) = \{f_3\}$.

A patient comes with the following symptoms $S = \{s_2, s_4, s_6, s_7, s_{11}\}$. We find the image of S under F and find the respective scores:

Score $(d_1) = 3.2$, Score $(d_2) = 1.0$, Score $(d_3) = 2.2$.

Hence we conclude that the patient is suffering from d_1, i.e. from influenza.

This is a very preliminary model which may be improved by incorporating detailed disease–symptoms information and also clinical results.

6. Conclusion

In this paper the idea of soft mapping is introduced. Mapping is a fundamental mathematical concept which is used in many fundamental areas of science and mathematics and has numerous applications. The authors are hopeful that this paper will serve the need for an appropriate soft function and will be used in many areas of application. One can also investigate the theory of fuzzy soft and intuitionistic fuzzy soft mappings.

Acknowledgements

The authors are grateful to the anonymous reviewers for their valuable comments which helped in rewriting the paper in its present form. The authors are also grateful to the editor of this journal for his kind help.

The present work is supported by Minor Research Project (MRP) of UGC, New Delhi, India [grant No. F.PSW-018/08–09(ERO)].

References

[1] D. Molodtsov, Soft set theory–first results, Computers & Mathematics with Applications 37 (1999) 19–31.
[2] P.K. Maji, et al., Soft set theory, Computers & Mathematics with Applications 45 (2003) 555–562.
[3] P.K. Maji, et al., An application of soft sets in a descision making problem, Computers & Mathematics with Applications 44 (2002) 1077–1083.
[4] P.K. Maji, et al., Fuzzy soft-sets, The Journal of Fuzzy Mathematics 9 (3) (2001) 589–602.
[5] H. Aktaş, N. Cağman, Soft sets and soft groups, Information Science 177 (2007) 2726–2735.
[6] Z. Kong, et al., The normal parameter reduction of soft sets and its algorithm, Computers & Mathematics with Applications 56 (2008) 3029–3037.
[7] Z. Kong, et al., Comment on "A fuzzy soft set theoretic approach to decision making problems", Journal of Computational & Applied Mathematics 223 (2009) 540–542.
[8] F. Feng, Y.B. Jun, X.Z. Zhao, Soft semirings, Computers and Math with Appl. 56 (2008) 2621–2628.
[9] M.I. Ali, et al., On some new operations in soft set theory, Computers and Math with Appl. 57 (2009) 1547–1553.
[10] P. Majumdar, S.K. Samanta, Similarity measure of soft sets, New Mathematics & Natural Computation 4 (1) (2008) 1–12.
[11] P. Majumdar, S.K. Samanta, On similarity measure of fuzzy soft sets, in: Proc. of the International Conf. on Soft Computing & Intelligent Systems, ICSCIS-07, Jabalpur, India, 27–29 December 2007, vol. 1, pp. 40–44.
[12] P. Majumdar, S.K. Samanta, Generalised fuzzy soft sets, Computers and Math with Appl. 59 (2010) 1425–1432.
[13] F. Feng, et al., Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Computing 14 (2010) 8999–9911.
[14] S.K. Mapa, Higher Algebra, Sarat Book Distributors, 2005.
[15] D. Dubois, H. Prade, Fuzzy Sets and Systems — Theory and Applications, Academic Press, 1980.