the full converse of interlacing). For arbitrary ψ, a homotopy argument is used to reduce the question to the ordinary (Euclidean) case. The only problem (as the reader of [6] will notice) is the fulfillment of a technical condition involving the s-numbers of a direct sum. For $\psi = \chi$ the condition is automatic. For general ψ its study leads to a new set of problems.

Details will appear elsewhere.

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OBSERVABILITY OF LINEAR POSITIVE DYNAMIC SYSTEMS

by M. D. SOTO TORRES and R. FERNANDEZ LECHON[33]

1. *Introduction*

The input and output structure of a system can significantly influence the available means for control. Two fundamental concepts characterizing the dynamic implications of input and output structure are the dual concepts of controllability and observability. Controllability concerns the possibility of steering the state from the input, while observability analyzes the possibility of estimating the state from the output.

Various authors [for example, Kalman (1960, 1963), Silverman (1971)] have found necessary and sufficient conditions for controllability and observability of dynamic systems without constraints, that is, neither the variables nor the parameters of the system have to satisfy conditions. Also, various

[33] Departamento de Economía Aplicada, Facultad de Ciencias Económicas y Empresariales, Universidad de Valladolid, Avda, Valle Esgueva 6, 47011 Valladolid, Spain

researchers have examined controllability of a dynamic system with different constraints, for example, Evans and Murthy (1978) where the inputs can assume only nonnegative values, and Murthy (1986) for a linear positive dynamic system. However, the conditions found in Murthy (1986) cannot be transferred to analyze observability of a linear positive dynamic system, on account of the differences between the concepts.

The purpose of this paper, considering a positive dynamic system and observable in the usual sense with linear structure and discrete time, is to explore necessary and/or sufficient conditions that determine the nonnegativity of the initial state.

2. *Conditions for Nonnegativity*

Consider a discrete-time linear system of the form

$$\begin{aligned} x(t+1) &= Ax(t) + bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{1}$$

where $t = 0, 1, 2, \ldots$; $x(t)$ is the n-dimensional state vector; $y(t)$ is the output; $u(t)$ is the input; and A, B, C, and D are $n \times n$, $n \times 1$, $1 \times n$, and 1×1 matrices, respectively. The system (1) is positive if and only if $A \geqslant 0$, $A \neq 0$, $B \geqslant 0$, $C \geqslant 0$, $C \neq 0$, $D \geqslant 0$, $u(t) \in \mathbb{R}^+$, $y(t) \in \mathbb{R}^+$. Then, in positive dynamic systems, if the initial state $x(0)$ is nonnegative, the following states will also be nonnegative, and if a state $x(t)$, $t > 0$, is negative, all previous states are negative, too.

Assume that the system (1) is positive and observable. Then if the system is observable, the state vector at $t = 0$, $x(0)$, is uniquely determined from the outputs $y(0), y(1), \ldots, y(N)$ and from the inputs $u(0), u(1), \ldots, u(N)$ for some $N > 0$.

Consider the state equation (1) for $t = 0, 1, \ldots, N-1$. We have

$$Y_N = \mathrm{OB}_N x(0) + U_N, \tag{2}$$

where

$$Y_N^t = \big(y(0), y(1), \ldots, y(N-1)\big),$$

the matrix OB_N is the observability matrix of the dynamic system, and

$$\begin{aligned} U_N^t = \big(&Du(0), CBu(0) + Du(1), \ldots, \\ &CA^{N-2}Bu(0) + CA^{N-3}Bu(1) + \cdots + Du(N-1)\big). \end{aligned}$$

Define $Z_N = Y_N - U_N$. Then if the matrices B and D are null, the vector Z_N is equal to Y_N.

The following propositions are immediate consequences of the definition of positive system and of observable system.

PROPOSITION 1. *If the initial state $x(0)$ is nonnegative, then the vector Z_n is nonnegative.*

PROPOSITION 2. *If the inverses of the observability matrix and the vector Z_n are nonnegative, then the initial state is nonnegative.*

Note that if the matrices B and D are null, the condition $Z_n \geqslant 0$ always holds.

Proposition 2 is not a sufficient condition, since the inverse of the observability matrix can be of arbitrary sign and the initial state can be nonnegative.

If the observability matrix of the dynamic system is a monomial matrix, then its inverse is nonnegative. In this case, the convex cone generated by the columns of the matrix OB_n and the convex cone generated by the columns of the matrix $(\mathrm{OB}_n^{-1})^t$ are identical, and both are equal to $\mathbb{R}_n^+$.

Now, we establish one necessary and sufficient condition for the nonnegativity of the initial state. For this purpose the concept of convex cone is essential.

PROPOSITION 3. *Assume that the dynamical system* (1) *is positive and observable. Then the initial state $x(0)$ is nonnegative if and only if Z_n is in the intersection of the convex cone generated by the columns of the observability matrix and $\mathbb{R}_n^+$.*

Proof. The vector Z_n satisfies

$$Z_n = \mathrm{OB}_n x(0). \tag{3}$$

Thus Z_n belongs to the convex cone generated by the columns of the matrix OB_n, since $x(0)$ is nonnegative; moreover Z_n is nonnegative from Proposition 1.

Conversely, if Z_n belongs to the convex cone, then it satisfies

$$Z_n = \mathrm{OB}_n a, \qquad a \geqslant 0;$$

but the system is observable, so the initial state is uniquely determined and it satisfies [3]. Thus, we get

$$Z_n = \mathrm{OB}_n x(0) = \mathrm{OB}_n a,$$

or equivalently

$$x(0) = a \geqslant 0. \qquad \blacksquare$$

Let us see now an example of this proposition. Consider the following linear difference equation with constant coefficients:

$$y(t+2) - 2y(t+1) - 5y(t) = 0,$$

and let the initial conditions be $y(0) = 1$, $y(1) = 2$. If we choose as state variables

$$x_1(t) = \tfrac{1}{5}y(t+1) - \tfrac{2}{5}y(t),$$

$$x_2(t) = \tfrac{9}{5}y(t) - \tfrac{2}{5}y(t+1),$$

we have a dynamical equation of the form

$$\begin{pmatrix} x_1(t+1) \\ x_2(t+1) \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},$$

$$y(t) = (2,1) \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}.$$

The system is positive and observable, since the observability matrix

$$\mathrm{OB}_2 = \begin{pmatrix} 2 & 1 \\ 9 & 2 \end{pmatrix}$$

has rank two. The vector Z_2 can be expressed

$$Z_2 = (1,2)^t = \mathrm{OB}_2(0,1)^t.$$

Thus all the states of the system are nonnegative.

3. *Concluding Remarks*

We have considered a positive dynamic system with linear structure and discrete time which is observable, and have obtained necessary conditions and/or sufficient conditions for the nonnegativity of the initial state. Also, we assume a single input and a single output. The study of observability in positive systems with these assumptions relaxed is an open problem and worthy of attention.

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LINEAR MODELS AND PROGRAMS FOR THE OPTIMIZATION OF THE MECHANICS OF ASSISTED BREATHING

by MARCEL STAROSWIECKI,[34] PIERRE VANPEENE,[34] and MARIE CHRISTINE CHAMBRIN[35]

1. *Introduction*

This study takes place in a biomedical context. A much discussed problem in mechanical ventilation is that of modifying the inspiratory flow pattern. Most of the existing breathing machines operate by blowing a constant, increasing, or decreasing output. No reference exists in order to justify the use of such a blowing curve during the inspiration cycle. The blowing output

[34] Département d'Automatique, UA CNRS 370, Université des Sciences et Techniques de Lille Flandres Artois, 59655 Villeneuve d'Ascq Cedex, France.

[35] C.T.B. INSERM U 279 13-17, Rue Camille Guerin, 59000 Lille, France.