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PHYSICS LETTERS B

Physics Letters B 575 (2003) 131–136

www.elsevier.com/locate/physletb

Massless $N = 1$ super-sinh-Gordon: form factors approach

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Received 25 August 2003; accepted 11 September 2003

Editor: L. Alvarez-Gaumé

Abstract

The $N = 1$ super-sinh-Gordon model with spontaneously broken supersymmetry is considered. Explicit expressions for form-factors of the trace of the stress energy tensor Θ , the energy operator ϵ , as well as the order and disorder operators σ and μ are proposed.

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PACS: 11.25.Hf; 11.55.Ds

1. Introduction

The SShG model can be considered as a perturbed super Liouville field theory, which Lagrangian density is given by

$$\mathcal{L} = \frac{1}{8\pi}(\partial_a\phi)^2 - \frac{1}{2\pi}(\bar{\psi}\partial\bar{\psi} + \psi\bar{\partial}\psi) + i\mu b^2\psi\bar{\psi}e^{b\phi} + \frac{\pi\mu^2b^2}{2}e^{2b\phi}$$

with the background charge $Q = b + 1/b$. This model is a CFT with central charge

$$c_{\text{SL}} = \frac{3}{2}(1 + 2Q^2).$$

The super-sinh-Gordon model is $(1 + 1)$ -dimensional integrable quantum field theory with $N = 1$ supersymmetry. We consider the Lagrangian

$$\mathcal{L} = \frac{1}{8\pi}(\partial_a\phi)^2 - \frac{1}{2\pi}(\bar{\psi}\partial\bar{\psi} + \psi\bar{\partial}\psi) + 2i\mu b^2\psi\bar{\psi}\sinh b\phi + 2\pi\mu^2b^2\cosh^2 b\phi.$$

In this model the supersymmetry is spontaneously broken [1]: the bosonic field becomes massive, but the Majorana fermion stays massless and plays the role of goldstino. In the IR limit, the effective theory for the goldstino is to the lowest order the Volkov–Akulov Lagrangian [2]

$$\mathcal{L}_{\text{IR}} = (\bar{\psi}\partial\bar{\psi} + \psi\bar{\partial}\psi) - \frac{4}{M^2}(\psi\partial\psi)(\bar{\psi}\bar{\partial}\bar{\psi}) + \dots, \quad (1)$$

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where supersymmetry is realized nonlinearly. The irrelevant operator along which the super-Liouville theory flows into Ising is the product of stress-energy tensor $T\bar{T} = (\psi\partial\psi)(\bar{\psi}\bar{\partial}\bar{\psi})$, which is the lowest dimension nonderivative operator allowed by the symmetries. The dots include higher-dimensional irrelevant operators.

The scattering in the left–left and right–right subchannels is trivial, but not in the right–left channel. The following scattering matrices were proposed in [1]

$$S_{RR}(\theta) = S_{LL}(\theta') = -1, \quad S_{RL}(\theta - \theta') = -\frac{\sinh(\theta - \theta') - i \sin \pi \nu}{\sinh(\theta - \theta') + i \sin \pi \nu}, \quad \nu \equiv b/Q.$$

For the right and left movers, the energy momentum is parametrized in terms of the rapidity variables θ and θ' by $p^0 = p^1 = \frac{M}{2}e^\theta$ (and $p^0 = -p^1 = \frac{M}{2}e^{-\theta'}$). The mass scale of the theory M^{-2} is equal to $2 \sin \pi \nu$. The form factors¹ $F_{r,l}(\theta_1, \theta_2, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l)$ are defined to be matrix elements of an operator between the vacuum and a set of asymptotics states. The form factor bootstrap approach [3–5] (developed originally for massive theories, but that turned out to be also an effective tool for massless theories [6,7]) leads to a system of linear functional relations for the matrix elements $F_{r,l}$; let us introduce the minimal form factors which have neither poles nor zeros in the strip $0 < \text{Im} \theta < \pi$ and which are solutions of the equations $f_{\alpha_1 \alpha_2}(\theta) = f_{\alpha_1 \alpha_2}(\theta + 2i\pi) S_{\alpha_1 \alpha_2}(\theta)$, $\alpha_i = R, L$.

Then the general form factor is parametrized as follows:

$$F_{r,l}^\alpha(\theta_1, \theta_2, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l) = \prod_{1 \leq i < j \leq r} f_{RR}(\theta_i - \theta_j) \prod_{i=1}^r \prod_{j=1}^l f_{RL}(\theta_i - \theta'_j) \prod_{1 \leq i < j \leq l} f_{LL}(\theta'_i - \theta'_j) Q_{r,l}(\theta_1, \theta_2, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l),$$

and the function $Q_{r,l}$ depends on the operator considered. The RR and LL scattering formally behave as in the massive case, so annihilation poles occur in the RR and LL subchannel. This leads to the residue formula

$$\begin{aligned} & \text{Res}_{\theta_{12}=i\pi} F_{r,l}(\theta_1, \theta_2, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l) \\ &= 2F_{r-2,l}(\theta_3, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l) \left(1 - \prod_{j=3}^r S_{RR}(\theta_{2j}) \prod_{k=1}^l S_{RL}(\theta_2 - \theta'_k) \right), \end{aligned} \quad (2)$$

and a similar expression in the LL subchannel. It is important to note that these equations *do not* refer to any specific operator.

2. Expression for form factors

The minimal form factors read explicitly:

$$f_{RR}(\theta) = \sinh \frac{\theta}{2}, \quad f_{LL}(\theta') = \sinh \frac{\theta'}{2},$$

and

$$f_{RL}(\theta) = \frac{1}{2 \cosh \frac{\theta}{2}} \exp \int_0^\infty dt \frac{\cosh(\frac{1}{2} - \nu)t - \cosh \frac{1}{2}t}{t \sinh t \cosh t/2} \cosh t \left(1 - \frac{\theta}{i\pi} \right).$$

The latter form factor has asymptotic behaviour when $\theta \rightarrow -\infty$

$$f(\theta) \sim e^{\theta/2} \left(1 + (A + A'\theta)e^\theta + \left(\frac{A^2}{2} + B + AA'\theta + \frac{(A')^2\theta^2}{2} \right) e^{2\theta} \right), \quad (3)$$

¹ We refer the reader to [6] for a discussion on form factors in massless QFT.

where $A = (1 - 2\nu) \cos \pi \nu - 1 + 2i \sin \pi \nu$, $A' = -\frac{2}{\pi} \sin \pi \nu$, $B = \frac{1}{2}(\cos 2\pi \nu - 1)$. The logarithmic contributions come from resonances.

The residue condition (2) written in terms of the function $Q_{r,l}$ reads

$$\begin{aligned} & \text{Res}_{\theta_{12}=i\pi} Q_{r,l}(\theta_1, \theta_2, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l) \\ &= Q_{r-2,l}(\theta_3, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l) \times (-)^{r-1} (2i)^{l+r-1} \\ & \times \prod_{j=3}^r \frac{1}{\sinh \theta_{2j}} \left(\prod_{k=1}^l (\sinh(\theta_2 - \theta'_k) + i \sin \pi \nu) - (-1)^{r+l} \prod_{k=1}^l (\sinh(\theta_2 - \theta'_k) - i \sin \pi \nu) \right). \end{aligned} \quad (4)$$

Let us introduce now the functions

$$\phi(\theta_{ij}) \equiv \frac{S_{RR}}{f_{RR}(\theta_{ij}) f_{RR}(\theta_{ij} + i\pi)} = \frac{2i}{\sinh \theta_{ij}}, \quad \phi(\theta'_{ij}) \equiv \frac{S_{LL}}{f_{LL}(\theta'_{ij}) f_{LL}(\theta'_{ij} + i\pi)} = \frac{2i}{\sinh \theta'_{ij}}$$

as well as

$$\Phi(\theta_i - \theta'_j) \equiv \frac{S_{RL}(\theta_i - \theta'_j)}{f_{RL}(\theta_i - \theta'_j) f_{RL}(\theta_i - \theta'_j + i\pi)} = -2i (\sinh(\theta_i - \theta'_j) - i \sin \pi \nu),$$

and

$$\tilde{\Phi}(\theta_i - \theta'_j) \equiv \Phi(\theta_i - \theta'_j + i\pi) = 2i (\sinh(\theta_i - \theta'_j) + i \sin \pi \nu).$$

We assign odd Z_2 -parity to both right and left movers ($\psi_R \rightarrow -\psi_R$, $\psi_L \rightarrow -\psi_L$, $\phi \rightarrow \phi$) and even (odd) parity to right (left) movers under duality transformations ($\psi_R \rightarrow \psi_R$, $\psi_L \rightarrow -\psi_L$, $\phi \rightarrow -\phi$).

2.1. Neveu–Schwarz sector: trace of the stress-energy tensor

The operator Θ has nonzero matrix elements on (even, even) number of particles. The first form factor is determined by using the Lagrangian: $Q_{2,2} = -4\pi M^2$. We introduce the sets $S = (1, \dots, 2r)$ and $S' = (1, \dots, 2l)$, and propose

$$\begin{aligned} & Q_{2r,2l}(\theta_1, \theta_2, \dots, \theta_{2r}; \theta'_1, \theta'_2, \dots, \theta'_{2l}) \\ &= -4\pi M^2 \sum_{\substack{T \in S, \\ \#T=r-1}} \sum_{\substack{T' \in S', \\ \#T'=l-1}} \prod_{i \in T, k \in \bar{T}} \phi(\theta_{ik}) \prod_{i \in T', k \in \bar{T}'} \phi(\theta'_{ik}) \prod_{i \in T, k \in \bar{T}} \Phi(\theta_i - \theta'_k) \prod_{i \in T', k \in \bar{T}'} \tilde{\Phi}(\theta_k - \theta'_i), \end{aligned}$$

where T, \bar{T} are respectively subsets of S and \bar{S} , the notation ‘#’ stands for ‘number of elements’, and by definition $\bar{T} = S \setminus T$, $\bar{T}' = S' \setminus T'$.

Let us show that this representation does indeed satisfy the residue condition (4): only two cases will contribute to this computation, namely when $1 \in T, 2 \in \bar{T}$ and $2 \in T, 1 \in \bar{T}$. It amounts to evaluate the residue at $\theta_{12} = i\pi$ of the quantity:

$$\begin{aligned} & \left[\phi(\theta_{12}) \prod_{k \in \bar{T} - \{2\}} \phi(\theta_{1k}) \prod_{i \in T - \{1\}} \phi(\theta_{i2}) \prod_{k \in \bar{T}'} \Phi(\theta_1 - \theta'_k) \prod_{i \in T'} \tilde{\Phi}(\theta_2 - \theta'_i) \right. \\ & \left. + \phi(\theta_{21}) \prod_{k \in \bar{T} - \{1\}} \phi(\theta_{2k}) \prod_{i \in T - \{2\}} \phi(\theta_{i1}) \prod_{k \in \bar{T}'} \Phi(\theta_2 - \theta'_k) \prod_{i \in T'} \tilde{\Phi}(\theta_1 - \theta'_i) \right] \\ & \times -4\pi M^2 \sum_{\substack{U \in S - \{1,2\}, \\ \#U=r-2}} \sum_{\substack{T' \in S', \\ \#T'=l-1}} \prod_{i \in U, k \in \bar{U}} \phi(\theta_{ik}) \prod_{i \in T', k \in \bar{T}'} \phi(\theta'_{ik}) \prod_{i \in U, k \in \bar{U}} \Phi(\theta_i - \theta'_k) \prod_{i \in T', k \in \bar{T}'} \tilde{\Phi}(\theta_k - \theta'_i). \end{aligned}$$

The last line is nothing but $Q_{2r-2,2l}(\theta_3, \theta_4, \dots, \theta_{2r}; \theta'_1, \theta'_2, \dots, \theta'_{2l})$; the evaluation of the residue at $\theta_{12} = i\pi$ of the term into brackets gives explicitly

$$(2i)^{2r+2l-1} \prod_{j=3}^{2r} \frac{1}{\sinh \theta_{2j}} \left[(-1)^{2r-2} \prod_{k=1}^{2l} (\sinh(\theta_2 - \theta'_k) - i \sin \pi \nu) - (-1)^{2l} \prod_{k=1}^{2l} (\sinh(\theta_2 - \theta'_k) + i \sin \pi \nu) \right],$$

and Eq. (4) is satisfied.

As a remark, we would like to note that the leading infrared behaviour of $F_{2,2}$ is given by $T\bar{T}$, which defines the direction of the flow in the IR region. To determine the subleading IR terms that appear in the expansion (1), one uses the asymptotic development for f_{RL} given by Eq. (3). For example (up to the logarithmic terms):

$$\begin{aligned} & f_{RL}(\theta_1 - \theta'_1) f_{RL}(\theta_1 - \theta'_2) f_{RL}(\theta_2 - \theta'_1) f_{RL}(\theta_2 - \theta'_2) \\ & \sim e^{\theta_1 + \theta_2 - \theta'_1 - \theta'_2} \left[1 + A e^{\theta_1 - \theta'_1} + \left(\frac{A^2}{2} + B \right) e^{2\theta_1 - 2\theta'_1} \right] \left[1 + A e^{\theta_1 - \theta'_2} + \left(\frac{A^2}{2} + B \right) e^{2\theta_1 - 2\theta'_2} \right] \\ & \quad \times \left[1 + A e^{\theta_2 - \theta'_1} + \left(\frac{A^2}{2} + B \right) e^{2\theta_2 - 2\theta'_1} \right] \left[1 + A e^{\theta_2 - \theta'_2} + \left(\frac{A^2}{2} + B \right) e^{2\theta_2 - 2\theta'_2} \right]. \end{aligned}$$

The terms into brackets give

$$\begin{aligned} & 1 + A(e^{\theta_1} + e^{\theta_2})(e^{-\theta'_1} + e^{-\theta'_2}) + \left(\frac{A^2}{2} + B \right) (e^{2\theta_1 - 2\theta'_1} + e^{2\theta_1 - 2\theta'_2} + e^{2\theta_2 - 2\theta'_1} + e^{2\theta_2 - 2\theta'_2}) \\ & \quad + A^2(e^{2\theta_1 - \theta'_1 - \theta'_2} + e^{\theta_1 + \theta_2 - 2\theta'_1} + e^{\theta_1 + \theta_2 - 2\theta'_2} + e^{2\theta_2 - \theta'_1 - \theta'_2} + 2e^{\theta_1 + \theta_2 - \theta'_1 - \theta'_2}) + \dots \\ & = 1 + \frac{A}{M^2} L_{-1} \bar{L}_{-1} + \frac{A^2}{2M^4} L_{-1}^2 \bar{L}_{-1}^2 + \frac{B}{M^4} L_{-2} \bar{L}_{-2} + \dots, \end{aligned}$$

where $L_{-1} = e^{\theta_1} + e^{\theta_2}$ and $L_{-2} = e^{2\theta_1} + e^{2\theta_2}$. So the next irrelevant operator appearing in (1) is $T^2\bar{T}^2$ (up to derivatives).

2.1.1. Form factors of the energy operator ϵ

The number of left movers and right movers is odd. Let $S = (1, \dots, 2r + 1)$, $S' = (1, \dots, 2l + 1)$. The lowest form factor is $Q_{1,1} = 1$. We propose

$$\begin{aligned} & Q_{2r+1,2l+1}(\theta_1, \theta_2, \dots, \theta_{2r+1}; \theta'_1, \theta'_2, \dots, \theta'_{2l+1}) \\ & = \sum_{\substack{T \in S, \\ \#T=r}} \sum_{\substack{T' \in S', \\ \#T'=l}} \prod_{k \in \bar{T}} \phi(\theta_{ik}) \prod_{\substack{i \in T', \\ k \in \bar{T}'}} \phi(\theta'_{ik}) \prod_{\substack{i \in T, \\ k \in \bar{T}'}} \Phi(\theta_i - \theta'_k) \prod_{\substack{i \in T', \\ k \in \bar{T}}} \tilde{\Phi}(\theta_k - \theta'_i). \end{aligned}$$

The proof that this expression satisfies Eq. (4) is the same as above.

2.2. Ramond sector

2.2.1. Order operator σ

It has nonvanishing matrix elements when the sum of left movers and right movers is odd. Let $S = (1, \dots, 2r + 1)$, $S' = (1, \dots, 2l)$. The lowest form factors are $Q_{1,0} = Q_{0,1} = 1$. We propose:

$$\begin{aligned} & Q_{2r+1,2l}(\theta_1, \theta_2, \dots, \theta_{2r+1}; \theta'_1, \theta'_2, \dots, \theta'_{2l}) \\ & = \sum_{\substack{T \in S, \\ \#T=r}} \sum_{\substack{T' \in S', \\ \#T'=l}} \prod_{k \in \bar{T}} \phi(\theta_{ik}) \prod_{\substack{i \in T', \\ k \in \bar{T}'}} \phi(\theta'_{ik}) \prod_{\substack{i \in T, \\ k \in \bar{T}'}} \Phi(\theta_i - \theta'_k) \prod_{\substack{i \in T', \\ k \in \bar{T}}} \tilde{\Phi}(\theta_k - \theta'_i). \end{aligned}$$

2.2.2. Disorder operator μ

It has nonvanishing matrix elements when the sum of left and right movers is even. As it is explained in [9], there is an additional minus sign in front of the product of S matrices in the residue condition (2).

- The number of left and right movers are both even.

Let $S = (1, \dots, 2r)$, $S' = (1, \dots, 2l)$ and the lowest form factor $Q_{0,0} = 1$. We propose:

$$Q_{2r,2l}(\theta_1, \theta_2, \dots, \theta_{2r}; \theta'_1, \theta'_2, \dots, \theta'_{2l}) = (-i)^{r+l} \sum_{\substack{T \in S, \\ \#T=r}} \sum_{\substack{T' \in S', \\ \#T'=l}} \prod_{\substack{i \in T, \\ k \in \bar{T}}} \phi(\theta_{ik}) e^{\frac{1}{2} \sum \theta_{ki}} \prod_{\substack{i \in T', \\ k \in \bar{T}'}} \phi(\theta'_{ik}) e^{\frac{1}{2} \sum \theta'_{ki}} \prod_{\substack{i \in T, \\ k \in \bar{T}'}} \Phi(\theta_i - \theta'_k) \prod_{\substack{i \in T', \\ k \in \bar{T}}} \tilde{\Phi}(\theta_k - \theta'_i);$$

- The number of left and right movers are both odd.

Let $S = (1, \dots, 2r + 1)$, $S' = (1, \dots, 2l + 1)$. The lowest form factor is $Q_{1,1} = e^{(\theta'_1 - \theta_1)/2}$. We propose:

$$Q_{2r+1,2l+1}(\theta_1, \theta_2, \dots, \theta_{2r+1}; \theta'_1, \theta'_2, \dots, \theta'_{2l+1}) = (-i)^{r+l} \sum_{\substack{T \in S, \\ \#T=r}} \sum_{\substack{T' \in S', \\ \#T'=l}} \prod_{\substack{i \in T, \\ k \in \bar{T}}} \phi(\theta_{ik}) e^{\frac{1}{2} \sum \theta_{ki}} \prod_{\substack{i \in T', \\ k \in \bar{T}'}} \phi(\theta'_{ik}) e^{\frac{1}{2} \sum \theta'_{ki}} \prod_{\substack{i \in T, \\ k \in \bar{T}'}} \Phi(\theta_i - \theta'_k) \prod_{\substack{i \in T', \\ k \in \bar{T}}} \tilde{\Phi}(\theta_k - \theta'_i).$$

Let us note that the exponentials will be responsible for the additional minus sign in the residue condition (4).

2.2.3. Remarks

- One can check that in the IR, the form factors of the operator $\mathcal{O} = \sigma + \mu$ satisfy the cluster property like an exponential of a Bose field [8], for example:

$$\mathcal{O}_{r,l}(\theta_1, \theta_2, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l) \sim \mathcal{O}_{1,0}(\theta_1) \mathcal{O}_{r-1,l}(\theta_2, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l) \quad \text{for } \theta_1 \rightarrow -\infty.$$

- The expressions for the form factors of σ and μ give the expected leading IR behaviour [4,9,10]:

$$F_{r,l}^{\text{IR}}(\theta_1, \theta_2, \dots, \theta_r; \theta'_1, \theta'_2, \dots, \theta'_l) \sim \prod_{i < j} \tanh \frac{\theta_{ij}}{2} \tanh \frac{\theta'_{ij}}{2}, \text{ where } r+l \text{ is odd for } \sigma \text{ and even for } \mu.$$

3. Concluding remarks

We understand it is important to check the UV properties of the form factors proposed in this Letter; we hope to present numerical checks in a future publication. As far as the operators Θ, σ, μ are concerned, we expect our representation to be the correct answer to the problem: indeed, the form factors of the operators σ and μ have the expected leading IR behaviour; moreover we recover immediately the form factors of the operators Θ, σ, μ in the tricritical Ising model perturbed by the subenergy that defines a massless flow to the Ising model [12,13], simply by replacing in our formulae S_{RL} and f_{RL} by their corresponding values that can be found in [6,14]. We checked for a low number of particles that they correctly reproduce the results of [6] where the first form factors of the operators Θ, σ, μ are computed in terms of symmetric polynomials.² We also obtained agreement (again for a low number of particles) with [7], where an expression quite similar to ours for the form factors of the operator Θ is proposed (with an arbitrary number of intermediate particles). The case of the energy operator ϵ could be slightly more tricky: although it is evoked in [6], only its lowest form factor with one left mover and one right mover is explicitly given there.

² The authors of [6] checked numerically the UV properties of their form factors, including the c-theorem.

Finally, the representations we provide for the functions $Q_{r,l}$ are in principle general enough³ to provide results for other massless models flowing to the Ising model, but where the S -matrix has a more complicated structure of resonance poles [14].

Acknowledgements

I am grateful to A.I.B. Zamolodchikov for suggesting the problem. Discussions with D. Bernard, V.A. Fateev, G. Mussardo, H. Saleur, F.A. Smirnov and especially G. Delfino are acknowledged. Work supported by the CEA and the Euclid Network HPRN-CT-2002-00325.

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³ Their architecture is very similar to the one found for the form factors of the operator $e^{\alpha\phi}$ in the bosonic sinh-Gordon model in [11], Eq. (61)—let us recall that another representation in terms of determinant formula was first proposed in the prior work [8].