Probabilistic pseudostatic analysis of pile in laterally spreading ground: Two layer soil profile

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KEYWORDS
Reliability analysis; Response surface method; Single pile; Lateral spread; Pseudostatic approach

Abstract  Coupling the finite element model of pile under lateral spread with the Monte Carlo Simulation is frequently prohibited by excessive lengthy computations. In the present paper, a simplified pseudostatic method is integrated with an improved response surface scheme to evaluate the reliability of pile subjected to lateral spread. The pseudostatic model takes both geometric and soil nonlinearity into account, while, the response surface formulation takes load, geometry, material and model uncertainties into consideration. First, the improved response surface scheme is suggested and validated with the help of a simple example. Then, the pseudostatic model of a full size pile under lateral spread is integrated with the improved response surface scheme in order to assess the pile reliability. In the considered example, for both operational and structural possible modes of failure, it has been found that the most influential random variables are lateral displacement, and pile radius, respectively.

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1. Introduction

Liquefaction-induced lateral spread can cause substantial amount of damage to pile-foundations of buildings and bridge piers. The lateral spread is very unpredictable and its kinematic interaction with the pile may induce significant residual horizontal deflections, shear forces and bending moments to the pile. The analysis and design procedure of pile in liquefying grounds is inherently burdened by many uncertainties such as; ground motion induced loads and displacements, material properties of piles and the pile–soil interaction characteristics. Therefore, rational design decision cannot be made without taking these uncertainties into account. In other words, to obtain a least-cost pile which recognizes the presence of uncertainties over its expected life time, the design of pile should be based on reliability concept, where the uncertainties can be recognized and treated adequately in a probabilistic-based format.

Bradley et al. [1] have proposed a probabilistic framework for pseudostatic analysis of pile foundations in liquefied and lateral spreading soils. Where a pseudostatic method involves applying static displacements and forces to a typical beam-spring/Winkler model, has been integrated with Monte Carlo Simulation. It has been observed that the significant uncertainties involved in pile in laterally spread soil result in significant uncertainty in pile-head displacement and pile bending moment for a given level of input ground motion. Consequently the
decision making based on a single reference model is potentially erroneous.

Although, the Winkler model is simple and can be practically coupled with Monte Carlo Simulation, it needs a soil resistance–lateral displacement curve ($p-y$ curve). This curve should be back-figured from either the field or a model test. Also, the beam-spring model is clearly a gross simplification of the highly non-linear dynamic response of an entire soil–pile system. The uncertainty of force–displacement response can be accounted as uncertainty in both the equivalent stiffness and strength.

To the author knowledge, the above mentioned method is the only method in the literature to determine the reliability of pile under lateral spread. As an alternative to the spring model-based simulation method, the present paper aims to integrate an improved response surface scheme with a pseudostatic based 3-D elasto-plastic model of pile under lateral spread to compute the pile reliability.

First, an improvement in the response surface scheme of Lee and Haldar [2] is initially suggested and verified using a simple example, (example 1) [3]. Then, the probability of failure is computed for a pseudostatic based 3-D elastoplastic model of pile under lateral spread from the literature Hussein et al. [4], (example 2). This model is chosen to avoid complexity and lengthily time consuming in long running of the finite element code which governs the reliability assessment. Moreover, both the geometric and soil nonlinearities are taken into account. In the formulation of response surface, the uncertainties of loads, geometrical details, material properties and modeling are explicitly incorporated. Finally, the most influential random variables are determined.

In other words, the paper suggests an improvement of the response surface scheme of Lee and Haldar [2], then integrates the improved scheme with a simplified pseudostatic-based model of pile under lateral spread of Hussein et al. [4] to compute an approximated value of the probability of failure in one computer session.

2. Pile embedded in two layer soil profile

In practice, two cases are commonly encountered; a 2-layer soil profile and a 3-layer soil profile. While, the 2-layer soil profile is manipulated in the present paper, the pile embedded in 3-layer soil profile is handled in another ongoing paper. A 2-layer soil profile represents a thick liquefiable soil layer which lies upon a non-liquefiable bed. To resist deformations of the lateral spread, free head piles are driven through the liquefiable soil layer and firmly embedded into the non-liquefiable bed. This case is usually encountered in practice when river or lake banks, is covered by poorly consolidated natural deposits or fills [5], as shown in Fig. 1a. This design case can be represented by a simple model called a limit equilibrium model which was suggested by Dobry et al. [6]. In this model, the pile will respond as a partially fixed column of length equal to the thickness of the liquefiable soil layer $H_{\text{liq}}$ and with rotational spring at the base of rotational stiffness, $k_r$, as shown in
3. The response surface approximation

In general, the reliability analysis is difficult when the limit state function is implicit. For such complex structural systems that have no explicit limit state function, the reliability analysis is accomplished by coupling the FE model with a simulation based method. However, the excessive length computations of numerous FE calls frequently prohibits the reliability evaluations. So, the response surface method (RSM) is utilized [7-10]. The used procedure of RSM can be seen in Appendix B. The RSM-based approach has the potential to generate an equivalent limit state function by simple, approximated and explicit polynomial. Hence, the reliability calculation can be performed using the first or the second order reliability method FORM/SORM [3,11,12]. A second-order polynomial without or with cross terms are usually used:

\[ g(X) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} \sum_{j=1}^{k} b_{ij} X_i X_j \]

where \( X_i \) is the \( i \)th random variable, \( k \) is the number of random variables in the formulation and \( b_0, b_i, b_{ii}, \) and \( b_{ij} \) are unknown coefficients to be determined. The numbers of coefficients necessary to define Eqs. (1) and (2) are \( p = 2k + 1 \) and \( = (k + 1)(k + 2)/2 \), respectively. The coefficients can be fully defined either by solving a set of linear equations or from regression analysis using responses at specific data points called experimental sampling points. They can be defined using the uncertainty of the random variables and a center point as follows:

\[ X_i = X_i^c \pm h_i \sigma_{X_i} \quad i = 1, 2, \ldots, k \]

where \( X_i^c \) and \( \sigma_{X_i} \) are the coordinates of the center point and the standard deviation of a random variable \( X_i \), respectively. \( h_i \) is an arbitrary factor that defines the sampling/experimental region. Its value can be assumed from 1 to 3. It may be taken constant throughout all the iterations or may be taken large in the first iterations and small at the last iterations [7]. In the present work it is assumed to be constant and equal 1 throughout all the iterations.

3.1. The location of the sample points/experimental designs

The location of the center point should be at failure point, a point which is not at a hand. To determine the location of the failure point, the initial center point is taken as the mean value point [8,9]. Then an iterative linear interpolation scheme is used as elaborated in the following:

A response surface \( g(X) \) can be generated explicitly in terms of the random variables \( X_i \)’s by conducting deterministic finite element method analyses at all the experimental sampling points around the center point. Once an explicit expression of the limit state function \( g(X) \) is obtained, the coordinates of the checking point \( x_{n_i} \) can be estimated using FORM/SORM, and all the statistical information on the \( X_i \)’s. The actual response can be evaluated again at the checking point \( x_{n_i} \), i.e., \( g(x_{n_i}) \) and a new center point \( x_{C_i} \) can be selected as:

\[ x_{C_i} = x_{C_i} + (x_{n_i} - x_{C_i}) \times g(x_{C_i})/g(x_{C_i}) - g(x_{n_i}) \]

if \( g(x_{n_i}) \geq g(x_{C_i}) \)
\[
\mathbf{x}_C = \mathbf{x}_D + (\mathbf{x}_C - \mathbf{x}_D) \times g(\mathbf{x}_D) / (g(\mathbf{x}_D) - g(\mathbf{x}_C)) \\
\text{if } g(\mathbf{x}_D) < g(\mathbf{x}_C)
\]

(5)

Then, the new center point \( \mathbf{x}_C \) can be used to develop an explicit performance function for the next iteration. This iterative strategy can be repeated until a preselected convergence criterion of \( (\mathbf{x}_{C_{i+1}} - \mathbf{x}_{C_i}) / \mathbf{x}_{C_i} \leq \varepsilon \) is satisfied. In the present work, \( \varepsilon \) is considered to be \( 0.05 \). The iterative strategy was suggested by Bucher and Bourgund [8] and applied systematically by Rajashekhar and Ellingwood [9]. A detailed description of the RSM is available in Haldar and Mahadevan [7].

4. Efficiency and accuracy of RSM

Since the proposed algorithm is iterative and the basic SD and CCD require different amount of computational effort, Lee and Haldar [2], studied several schemes considering efficiency without compromising accuracy. Three schemes are of interest:

1. Scheme 0: SD using quadratic polynomial without the cross terms throughout all the iterations. This scheme may be called as the classical response surface. It is the most efficient but least accurate in estimating the probability of failure, \( P_f \) and reliability index, \( \beta \)-index.

To improve the accuracy, Lee and Haldar [2], have recommended the following two schemes:

2. Scheme 1: SD using quadratic polynomial without the cross terms in intermediate iterations and SD (with edge points) using full quadratic polynomial in the final iteration.

3. Scheme 2: SD using quadratic polynomial without the cross terms in intermediate iterations and CCD using full quadratic polynomial in the final iteration.

Considering the above three schemes, the total number of FE analyses required to generate the necessary response surface are \( 2k + 1 \), \( (k + 1)(k + 2)/2 \) and \( 2^k + 2k + 1 \), respectively, where \( k \) is the total number of random variables in the formulation. The three schemes require variant implementation effort. For example for \( k = 9 \), the number of required FE analyses will be 19, 55, and 531, respectively. Fig. 2 shows a diagram for the algorithm of the three schemes.

4.1. Improvement in the response surface schemes

In the present work, the simplified pseudostatic 3-D elastoplastic finite element model which takes about 2 h is chosen for the sake of simplicity. However, the FE model of a full size soil-pile system is usually a long running FE code which often lasts for long time. Conducting this model for few tens of runs continues for days or perhaps weeks. So, there is a need to improve the algorithm without compromising the accuracy. To meet this objective, two improvements have been suggested as follow [10]:

4.1.1. Scheme M1

To improve the efficiency of Scheme 1, it is suggested to add the cross terms (edge points), \( k(k - 1) \), only of the most sensitive variables, i.e., in the last iteration, the cross terms are added only for the most sensitive random variable integrated with the corresponding edge point, to calculate the corresponding reliability index. Similarly, other less sensitive random variables can be added one by one integrated with their edge points in a sequence and the reliability index can be calculated until the changes in the reliability index become negligible. For an example, suppose the total number of basic variables is \( k \) and the total number of most sensitive random variable is \( m \), then the total number of FE analyses required for Scheme 1 and Scheme M1 are \( (k + 1)(k + 2)/2 \) and \( 2k + 1 + m(2k - m - 1)/2 \), respectively. For \( k = 9 \) and \( m = 3 \), the total number of FE analyses will be 55 and 40, respectively, for the two schemes indicating the improvement in the efficiency.

4.1.2. Scheme M2

In Scheme 2, instead of using the full factorial plan in CCD, Raymond [15] recently demonstrated using half or quarter...
factorial plan, as shown in Fig. 2 in the coded variable space. This improved version of Scheme 2 will be denoted hereafter as Scheme M2. In Scheme M2, it is proposed that only one half or quarter of the factorial points corresponding to the most sensitive random variables are to be considered. As an example for a problem with \( k = 4 \), the required number of sampling points will be 25, 17, and 13, for scheme CCD, Scheme M2 of half, and quarter factorial plan, respectively.

In curve fitting operation, it is self-evident that the accuracy of the obtained curve is increased as the number of the sample points increases. Based on the required accuracy, one of the above schemes can be chosen, i.e. the analyst can choose to make \( 2n + 1 \) to obtain the lowest accuracy, if it is sufficient and meet the analyst purpose or if the analyst has limited time. Better accuracy can be obtained by addition edge points or factorial points, for important purpose.

To compare the efficiency of these schemes, the number of the required samples versus \( k \) is plotted in Fig. 3. The curve between the points is just to show the trend. In generating curves for Scheme M1, \( m \) is assumed to be \( k/2 \) when \( k \) is an even number and \( (k + 1)/2 \) when \( k \) is an odd number. The figure shows the improvement in the efficiency (Scheme M1 is more efficient than Scheme 1 and Scheme M2 is more efficient than Scheme 2). Also, it can be noted that for \( k < 6 \), Scheme M2 is more efficient than Scheme 1 but for \( k \geq 6 \), Scheme 1 becomes more efficient than Scheme M2. On the other hand, the accuracy is validated in the first simple example, i.e. the results of Scheme M1 and M2 are in good agreement with those of Scheme 1 and 2; respectively. Finally, a full size example is analyzed using a pseudostatic method. Three commercial codes COSMOS/M [16], STATISTICA [17] and COMREL [12], are used in finite element, regression and reliability analysis, respectively.

5. Limit states

In order to avoid structural or operational failure of the foundation and the supported structure, there are two basic components of pile response that need to be calculated; the maximum moment developing along the pile and the associated maximum pile deflection. The two accompanied structural and operational limit states can be expressed as:

\[
g_m(X) = M_u - a_m g_m(X) \tag{6}
\]

\[
g_u(X) = X_{all} - a_u g_u(X) \tag{7}
\]

where \( g_m(X), \hat{g}_m(X), g_u(X) \) and \( \hat{g}_u(X) \) are the limit state function and the response function of moment and drift, respectively, \( a_m \) and \( a_u \) are the model correction factors for the estimation of moment and drift, respectively, \( M_u \) and \( X_{all} \) are the moment capacity of the pile section and the allowable drift, respectively. In the present work, \( X_{all} \) is assumed 50 cm for the two examples.

6. Pseudostatic method

The used pseudostatic method was adopted by Hussein et al. [4]. It can be described in short as follow:

In this method, a pre-estimated or a given liquefaction-induced lateral displacement \( Dh \) is applied as external loads to a three dimensional FE model as shown in Fig. 4a. The soil domain is assumed sufficient where its dimensions in the space are, \( X_d = Y_d = Z_d = 50.0 \) m. The soil nonlinear behavior of the non-liquefied layer is represented by elastoplastic Drucker–Prager material while, the liquefied soil is horizontally left free in \( x \)-direction and modeled using elastic material with reduced stiffness. The stiffness degradation factor ranges from
from the pile centerline where 

\[ N = 6.00 \text{ m} \]

and can be 

depend on the section of 

\[ LN = 0.30 \text{ m} \]

are the cross sectional 

\[ LN = 3.4 \text{ cm} \]

and 

\[ LN = 7.5 \text{ cm} \]

are the sec-

\[ LN = 7.5 \text{ cm} \]

ments, respectively; and \( A, A_b, \) and \( A_s \) are the cross sectional area of pile, beam and solid elements, respectively. 

\[ EI \text{ and } EA \text{ are given and } I_b \text{ and } A_s \text{ depend on the section of pile. The reduced Young's modulus, } E' \text{ and the cross sectional area of the beam element } A_b \text{, are expressed as} \]

\[ E' = E/(I_b/I_s + 1) \quad \text{and} \quad A_b = A(I_b/I_s) \]  

The ratio of \( I_b/I_s \) is considered to be 9, as recommended by 

Zhang et al. [19].

Eventually, the soil displacement is simplified as a linear displacement pattern with maximum value at the surface and a zero value at the bottom of the liquefied layer, as shown in 

Fig. 2b. This displacement pattern is applied to all the nodes 

with maximum value at the surface and 

in the liquefied zone except the nodes at a distance less than 

1.5D_p from the pile centerline where \( D_p \) is the pile diameter.

7. Application examples

As mentioned earlier, the suggested schemes are elaborated further and verified with the help of a simple example that has an explicit limit state. Then, a full size example which has no explicit limit state is analyzed. The first example represents limit equilibrium model [6], while the second example is a full size problem.

7.1. Example 1: Limit equilibrium model

A reinforced concrete pile driven in a 6.00 m thick liquefiable layer and embedded in a non-liquefiable bed is considered. For the sake of simplification, the pile is represented according to the limit equilibrium analysis developed by Dobry et al. [6]. This model is a partially fixed column with column length equals to the thickness of the liquefiable soil layer \( H_{liq} = 6.00 \text{ m} \), rotational stiffness at base, \( k_r \), and subjected to uniform pressure of the liquefied soil \( q \), as shown in Fig. 4. Both \( k_r \) and \( q \) have predetermined test-based values 5738 kN m/rad and 10.50 kN/m², respectively. The statistical properties of the involved random variables are assumed according to the literature as given in Table 1. The pile head deflection can be expressed as

\[ u_x = qD_pH_{liq}^4/(8EI) + 0.5qD_pH_{liq}^2/K_r \]  

where \( E, I \) are the pile elastic modulus and pile second moment of inertia, respectively, the other variables are defined before.

7.1.1. Problem simplification

In this example, as the number of variables is relatively large, a sensitivity analysis is carried out using the first order polynomial to reduce the number of variables, \( k \) and simplify the stochastic model. From the sensitivity analysis, it is observed that, \( E \) and \( t \) can be considered as deterministic variables (their sensitivities are \( \leq 5\% \)), reducing the number of variables to four variables. This step is termed as first order polynomial in Table 2.

7.1.2. Monte Carlo Simulation

To verify the improved schemes, the \( \beta \)-index using 100,000 Monte Carlo Simulations (MCS) and using SORM, is found 2.236 and 2.235, respectively. To show the immaterial effect of considering \( E \) and \( t \) as deterministic variables, the same values are recalculated but assuming \( E \) and \( t \) as random variables. (Monte Carlo full model and SORM full model in Table 2). The change in \( \beta \)-index is found 1.21% and 1.03% for MCS and SORM, respectively.

7.1.3. Basic reliability analysis using quadratic response surface

Using the quadratic polynomial as a response surface, the \( \beta \)-index of scheme 0, is found 1.916. Then, this value is recalculated using Scheme 1, Scheme M1-1 and Scheme M1-2. Their values are found to be 1.997, 1.993 and 2.067, (10.69%, 10.87% and 7.56% more than \( \beta \) of Monte Carlo) using 15, 12 and 14 function calls, respectively. Where Scheme M1-1 terms to Scheme M1 when only the cross terms of the most important random variable is added, \( q \). If more accuracy than Scheme M1-1 is desired, the cross terms of the second important variable \( H_{liq} \) can be added using 2 more function calls (Scheme M1-2) and so on. This means that the suggested Schemes M1-1 and Scheme M1-2 are in good agreement with Scheme 1.

While, using Scheme 2 and Scheme M2, yields \( \beta \)-index = 2.006, 1.934 and 1.784 (10.29%, 13.51% and 20.21% more than \( \beta \)-Monte Carlo) using 25, 17 and 13 function calls, respectively, i.e., the suggested Scheme M2 is in good agreement with Scheme 2, as given in Table 2. The improved schemes yield approximately the same accuracy as Scheme 1 and Scheme 2, respectively, but with less number of function calls.

<table>
<thead>
<tr>
<th>Random variables</th>
<th>Symbol</th>
<th>Distribution</th>
<th>Nominal</th>
<th>Mean</th>
<th>Bias</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Lateral pressure</td>
<td>( q )</td>
<td>EV-I</td>
<td>10.5 kN/m²</td>
<td>10.5</td>
<td>1.0</td>
<td>0.25*</td>
</tr>
<tr>
<td>2 Radius</td>
<td>( r )</td>
<td>LN</td>
<td>0.30 m</td>
<td>0.30</td>
<td>1.0</td>
<td>0.10</td>
</tr>
<tr>
<td>3 Thickness</td>
<td>( t )</td>
<td>LN</td>
<td>3.4 cm</td>
<td>3.4 cm</td>
<td>1.0</td>
<td>0.05</td>
</tr>
<tr>
<td>4 Length</td>
<td>( H_{liq} )</td>
<td>N</td>
<td>6.00 m</td>
<td>6.00 m</td>
<td>1.0</td>
<td>0.04*</td>
</tr>
<tr>
<td>5 Flexural modulus of pile</td>
<td>( E )</td>
<td>LN</td>
<td>3300 Mpa</td>
<td>3300</td>
<td>1.0</td>
<td>0.06*</td>
</tr>
<tr>
<td>6 Rotational spring</td>
<td>( K_r )</td>
<td>LN</td>
<td>5738 kN m/rad</td>
<td>5738</td>
<td>1.0</td>
<td>0.21*</td>
</tr>
</tbody>
</table>

* Data not available. Assumed parameters are based on engineering judgment.
Table 2  Lateral spread results of reliability analysis – example 1.

<table>
<thead>
<tr>
<th>Variables sensitivities</th>
<th>β</th>
<th>Error %</th>
<th>Pf</th>
<th>No. of calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>q, H_{Liq}, k_r, r, E, t</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 First order polynomial</td>
<td>−0.946</td>
<td>−0.245</td>
<td>0.163</td>
<td>0.116</td>
</tr>
<tr>
<td>2 Scheme 0</td>
<td>−0.843</td>
<td>−0.340</td>
<td>0.305</td>
<td>0.211</td>
</tr>
<tr>
<td>3 Scheme M1-1 q</td>
<td>−0.837</td>
<td>−0.397</td>
<td>0.310</td>
<td>0.214</td>
</tr>
<tr>
<td>4 Scheme M1-2 q, H_{Liq}</td>
<td>−0.837</td>
<td>−0.403</td>
<td>0.305</td>
<td>0.210</td>
</tr>
<tr>
<td>5 Scheme M1-3 q, k_r</td>
<td>−0.838</td>
<td>−0.395</td>
<td>0.311</td>
<td>0.214</td>
</tr>
<tr>
<td>6 Scheme 1</td>
<td>−0.838</td>
<td>−0.395</td>
<td>0.311</td>
<td>0.214</td>
</tr>
<tr>
<td>7 Scheme 2,</td>
<td>−0.835</td>
<td>−0.392</td>
<td>0.326</td>
<td>0.208</td>
</tr>
<tr>
<td>8 Scheme M2 Half</td>
<td>−0.800</td>
<td>−0.388</td>
<td>0.390</td>
<td>0.240</td>
</tr>
<tr>
<td>9 Scheme M2 Quarter</td>
<td>−0.139</td>
<td>−0.457</td>
<td>0.865</td>
<td>0.154</td>
</tr>
</tbody>
</table>

(ii) Explicit limit state

<table>
<thead>
<tr>
<th>No.</th>
<th>Monte Carlo (full model)</th>
<th>Monte Carlo Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2.209 2.12 1.36 × 10⁻²</td>
<td>100,000</td>
</tr>
<tr>
<td>8</td>
<td>2.236 – 1.27 × 10⁻²</td>
<td>100,000</td>
</tr>
</tbody>
</table>

8 SORM (full model) 2.213 1.03 1.35 × 10⁻² 0
8 SORM 2.235 0.04 1.27 × 10⁻² 0

7.2. Example 2: Full size pile embedded in two soil-layer profile

A 15.00 m long pile subjected to horizontal lateral spreading \(D_h = 30\) cm, is chosen to be studied, Hussein et al. [4]. The pile has a circular cross section with outside diameter \(D_p = 0.75\) m, driven in liquefiable layer of \(5.00\) m thickness which has an assumed reduction factor \(0.001\). The statistical descriptions of all the random variables are collected from the literature and listed in Table 3. Some values of the coefficient of variation are reasonably assumed as they are not available in the literature.

Building the 3-D FE model using COSMOS [16], the drift and moment are in good agreement with Hussein et al. [4], as shown in Fig. 5a and b, respectively. This case of analysis is termed as \(O_{wb} + O_{L} + D_h\). Finally, the maximum moment value, which governs the analysis, is found to be 100 kN m.

However, this 3-D FE model is criticized as it is weightless model. Therefore, it is suggested to use an equivalent model which takes the own weight into consideration and yield the same maximum moment value (100 kN m). This can be easily accomplished by using new value of the reduction factor, i.e. using the back analysis technique. Hence, the own weight is incorporated in two steps. First, the own weight of the non-liquefied bed is incorporated in a FE run termed as \(O_{wb}\) and \(D_h\). Then, the weight of the liquefied layer is incorporated but as a uniform load over the non-liquefied bed in another FE run termed as \(O_{wb} + O_{L} + D_h\). Finally, the maximum moment value, 100 kN m, is kept by determining a new value for the reduction factor using the back analysis technique. The new value is found to be 0.0019 as shown in Fig. 6. This case is termed as “current \(R_f''\)” in Fig. 5, and used hereafter.

As the limit state is implicit in this example, the simulation of the above validated FE model is used in the above mentioned response surface algorithm in Section 4 and Appendix B, to drive drift and moment limit state. For the drift limit state, based on the sensitivity analysis of the preliminary reli-

Table 3  Statistical characteristics of random variables – example 2.

<table>
<thead>
<tr>
<th>Loads</th>
<th>No</th>
<th>Random variables</th>
<th>Symbol</th>
<th>Dist.</th>
<th>Nominal</th>
<th>Bias</th>
<th>Mean</th>
<th>COV</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pile</td>
<td>1</td>
<td>Lateral spread</td>
<td>(D_h)</td>
<td>EV-I</td>
<td>0.30 m</td>
<td>1.00</td>
<td>0.30</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Radius</td>
<td>(r)</td>
<td>N</td>
<td>0.375 m</td>
<td>1.00</td>
<td>0.375</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>E-modulus</td>
<td>(E)</td>
<td>LN</td>
<td>2.2 × 10⁷ kN/m²</td>
<td>1.00</td>
<td>2.2 × 10⁷</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Concrete density</td>
<td>(\gamma_c)</td>
<td>N</td>
<td>25 kN/m³</td>
<td>1.00</td>
<td>25</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Poisson's ratio</td>
<td>(v)</td>
<td>LN</td>
<td>0.2</td>
<td>1.00</td>
<td>0.2</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>Ultimate moment capacity</td>
<td>(M_u)</td>
<td>N</td>
<td>880 kN m</td>
<td>1.10</td>
<td>968</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Layer _1: Liquefied</td>
<td>7</td>
<td>Soil E-modulus</td>
<td>(E_1)</td>
<td>LN</td>
<td>14.25 kN/m²</td>
<td>1.00</td>
<td>16.3875</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>Soil density</td>
<td>(\gamma_1)</td>
<td>LN</td>
<td>17 kN/m³</td>
<td>1.00</td>
<td>1.7</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>Reduction factor</td>
<td>(R_f)</td>
<td>N</td>
<td>0.0019</td>
<td>1.00</td>
<td>0.0019</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Layer _2: Non-liquefied</td>
<td>10</td>
<td>Soil E-modulus</td>
<td>(E_2)</td>
<td>LN</td>
<td>7500 kN/m²</td>
<td>1.15</td>
<td>8625</td>
<td>0.21</td>
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<tr>
<td></td>
<td>11</td>
<td>Friction angle</td>
<td>(\phi_f)</td>
<td>LN</td>
<td>35°</td>
<td>1.03</td>
<td>36.05</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>Poisson's ratio</td>
<td>(v_2)</td>
<td>LN</td>
<td>0.4</td>
<td>1.00</td>
<td>0.4</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>Soil density</td>
<td>(\gamma_2)</td>
<td>LN</td>
<td>17 kN/m³</td>
<td>1.00</td>
<td>17</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Model coefficient</td>
<td>14</td>
<td>Drift</td>
<td>(x_d)</td>
<td>N</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.10²</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>Moment</td>
<td>(x_m)</td>
<td>N</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.10²</td>
<td></td>
</tr>
</tbody>
</table>

* Data not available. Assumed parameters are based on engineering judgment.
ability analysis, only four of the random variables are considered in the formulation. Then, $\beta$-index for Scheme M1-1 and Scheme M1-2 are found to be 5.037 and 5.165, respectively. As the change in $\beta$-index is less than 5%, no more improvements are performed, as can be seen in Table 4. The table shows also the probability of failure, the reliability index and the sensitivities of variables. It is obvious that, the most sensitive design variables are the lateral displacement and the pile radius.

For the strength limit state, the number of variables is still relatively large even after the simplification. Using the classical scheme, the large value of the probability of failure (0.94) ensures that the pile will fail in this limit state. So, it is assumed here that low accuracy is required (Scheme 0). Consequently, no more improvement is performed. The results are listed in Table 4. The sensitivities show that the most critical variables are the pile radius and the lateral shift.

8. Conclusion

In the literature, the reliability of pile subjected to lateral spread can be determined by conducting the typical beam-spring/ Winkler model with the Monte Carlo Simulation. As an alternative to this approach, the present paper introduces another method in which an improved response surface scheme is integrated with a pseudostatic-based 3-D finite element model. Initially, the improvement which based on sensitivity analysis in the response surface scheme is suggested and validated. Then, the improved scheme is applied to a full size simplified pseudostatic pile model under lateral spread.

The 3-D elastoplastic FE model takes both geometric and soil nonlinearities into account, while, the response surface formulation takes uncertainties; geometry, load, material and model uncertainties into consideration. Finally, the
reliability corresponding to operational and structural limit states are evaluated. For the considered example, it has been found that the most influential variables are lateral displacement and the pile radius.

Appendix A. Methods of reliability analyses [3]

A.1. Definitions

A.1.1. Reliability
It is known that the reliability is defined as the probability of safety or the complement of the probability of failure. Sometimes reliability and safety are used as synonyms.

A.1.2. Safety margin

\[ Z = R - S \]  

(A.1)

where \( R, S \) is the resistance and stress resultant and \( Z \) is a point of failure with a unique/invariant value.

A.1.3. Probability of failure (\( Pf \))
If the allowable resistance is \( R \) and the applied stress is \( S \) with probability density function \( f_R \) and \( f_S \), respectively, then the probability of failure is the amount of overlap of the probability density functions \( f_R \) and \( f_S \) (in this work, \( f_R, f_S \) and the amount of overlap are assumed to be time independent). In another form, let \( f_R \) and \( f_S \) be two marginal density functions as shown in Fig. A.1 where \( f_R \) is the resistance cumulative function and both \( R \) and \( S \) are independent then,

\[ P_f = P(R - S \leq 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{S_R} f_R(r)f_S(s)drds \]

\[ = \int_{-\infty}^{\infty} F_R(x)f_S(x)dx \]  

(A.2)

In a more general form, the random variables affecting the response are grouped in a vector called the vector of basic random variables \( X \).

\[ P_f = P(G(X) \leq 0) = \prod_{G(X_i) \leq 0} f_X(x)dx \]  

(A.3)

where \( f_X(x) \) is the joint probability density function of \( n \) basic variables \( X, G(x) \) is the limit state function.

A.1.4. Calculation of probability of failure \( P_f \)
As previously mentioned, the probability of failure can be defined as

\[ P_f = \int_{G(X) \leq 0} \cdots \int f_X(x)dx \]  

(A.4)

The above multidimensional probability convolution integral is rather tedious. However, \( P_f \) may be directly calculated by numerical methods for simple cases otherwise two main categories of methods may be used. They are denoted as the simulation methods and the fast integration methods.

A.2. Simulation methods

The Monte Carlo Simulation (MCS) technique involves sampling process randomly to simulate a large number of experiments and observe the result. If the number of sampling \( N \) with \( n \) failure states, then

\[ P_f \approx n(G \leq 0)/N \]  

(A.5)

\[ P_f \approx \hat{P}_f = \frac{1}{N} \sum_{i=1}^{N} I[G(x_i) \leq 0] \]  

(A.6)

where \( I[G(X_i) \leq 0] \) is an indicator function of \( G(x) \) equal one if \( X \) lies in the failure domain and zero otherwise. \( N \) depends on the required accuracy.

The sampling is obtained randomly using tables of random numbers or using a pseudo random number generator which uses the local time as a seed value to avoid any reproducibly. However, using the tables is very slow and using the pseudo random number generator may be criticized as it is no longer random as the sequence of the numbers is determined. So, it may be called quasi MCS.

The pervious MCS technique is the simplest form and may be called “direct sampling” or “Crude Monte Carlo”. Other modified methods such as variance reduction, importance sampling, and adaptive Monte Carlo are found in Melchers [3].

A.3. Fast integration methods

These methods are based on the simplicity of finding the integral in the standardized space. So, all basic variables \( X_i \) are transformed to uncorrelated standardized distributed variables \( U_i \). Also, the limit state function \( G(x) \) is transformed to \( G(u) \) [25]. Hence, \( P_f \) may be estimated by one of the following methods.

A.3.1. First Order Reliability Method (FORM)
Hasfor and Lind [26] have initially proposed this method in 1974. In 1978 Rackwitz and Fieessler [27] have put the solution in an algorithmic form. In the basic FORM [28], the limit state \( G(u) \) in \( u \)-space is approximated by its hyperplane in the \( G(u) \) at a point \( (u') \) closest to the origin. By this way the multidimensional integral problem is converted to an optimization problem for finding the shortest distance between the origin and the hyperplane which is called the reliability index \( \beta \)
\( P_f \approx \Phi(-\beta) \) \hspace{1cm} (A.7)
\[
\beta = \|U^*\| \hspace{1cm} (A.8)
\]
where \( u^* = \min |u| \) for \( \{x \mid g(x) \leq 0\} \).

The optimization problem requires that the distribution of \( X \) and \( G(u) \) should be differentiable. This method yields sufficiently accurate probability of failure estimation for most engineering proposes, COMREL [12]. Through this method, the probability of failure for concave and convex limit state function is the same as that of linear limit state function provided that they have the same check point as shown in Fig. A.2.

A.3.2. Second Order Reliability Method (SORM)

Obviously, the linear approximation of the true failure surface in FORM appears to be rather crude. Breitung in 1984 [11] has given a sound theoretical basis SORM using a quadratic approximation of the failure surface by use of asymptotic consideration which has been modified in COMREL [12] according to the following formula

\[
P_f \approx \Phi(-\beta) \prod_{i=1}^{n-1} \left(1 - \frac{g_i(-\beta)}{\Phi(-\beta)} \right) \hspace{1cm} (A.9)
\]

where \( \beta = \|U^*\| \) in which \( u^* \) is found from \( u^* = \min |u| \) for \( \{x \mid g(x) \leq 0\} \).

The difference between linear and quadratic approximation of nonlinear surface increases with problem dimensions and safety index. It drastically depends on the mathematical calculations. This simplified or approximated function is called the response surface function [7-10].

Appendix B. Response surface method

As it is mentioned in Appendix A, the reliability can be estimated by either simulation or by First or Second order reliability methods, FORM/SORM. The first method requires repeated calls for the limit state function, a requirement which is so expensive, especially in case of large-scale structural system. The later, FORM/SORM, assume that the limit state function and its derivatives are available. Generally, this condition is only available for some linear and simple structural problems. The use of nonlinear model is the only way to obtain reliable relationship describing the behavior of the complex structure which has implicit limit state. For the above reasons, an explicit approximated function should be used to simplify the mathematical calculations. This simplified or approximated function is called the response surface function [7-10].

B.1. Algorithm of the used RSM

The classical response surface algorithm in its simplest form – in the present work-consists of four main phases as shown in Fig. B.1

Phase I:
1. Prepare the FEM using COSMOS/M [16].
2. Perform \((2n+1)\) numerical experiments around the mean value of the random variables \( X_i \) as a center point \( x_c \).

Phase II:
1. Using first order polynomial (linear polynomial Eq. (B.1)) to approximate the actual response, LRF, in STATISTICA [17].
2. Use the limit state function, LSF in COMREL [12], to eliminate the variables of low sensitivities. (This step is optional).
3. Using quadratic polynomial without cross terms Eq. (1), approximate the actual response to a response function, RF, in STATISTICA [17].

Phase III:
1. Prepare the limit state function LSF in a suitable form for COMREL using the RF built in II-2 to calculate \( P_f, \beta \) and the design point.
2. Repeat the above procedure for each limit state.

In the first phase: the FEM is run using mean values of the variables. Then, two runs for each variable is performed at axial points according to Eq. (3) and the response, such as displacement, internal forces or stresses are recorded. In the second phase: the \( 2n+1 \) response points are used to formulate a response surface function using a quadratic polynomial without cross terms, Eq. (1). In the third phase: the limit state function is built in COMREL, the \( \beta \)-index, the probability of failure \( P_f \), and the sensitivities of the variables are computed. Unfortunately, these computed values are not correct as the samples are not around the failure point. Therefore, the available information from this step (the \( \beta \)-index and the checking point) can be used in selecting a new center point [8,27]. Then, the procedure is repeated in an iterative strategy until convergence. This systematic approach is an iterative linear interpolation scheme, Eqs. (4) and (5).

\[
g(X) = b_0 + \sum_{j=1}^{k} b_j X_j \hspace{1cm} (B.1)
\]

In reliability analysis of such complex structural system, it is a good practice to use, at the beginning, a first order polynomial function, Eq. (B.1). This polynomial is easily to be applied in a...
Preliminary reliability analysis to eliminate the non-important variables to simplify the problem (Step II-1). In other words, this step distinguishes the random variables and the deterministic variables. At the beginning, all the variables are assumed to be random. Then, each variable that has sensitivity less than 5% (in the present work) is considered to be as a deterministic variable in the following reliability computations.

**References**


[16] COSMOS/M 2.6, Structural research and analysis corporation, Santa Monica, California; 2000.