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Research article

Prediction of gas compressibility factor using intelligent models

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Abstract

The gas compressibility factor, also known as Z-factor, plays the determinative role for obtaining thermodynamic properties of gas reservoir. Typically, empirical correlations have been applied to determine this important property. However, weak performance and some limitations of these correlations have persuaded the researchers to use intelligent models instead. In this work, prediction of Z-factor is aimed using different popular intelligent models in order to find the accurate one. The developed intelligent models are including Artificial Neural Network (ANN), Fuzzy Interface System (FIS) and Adaptive Neuro-Fuzzy System (ANFIS). Also optimization of equation of state (EOS) by Genetic Algorithm (GA) is done as well. The validity of developed intelligent models was tested using 1038 series of published data points in literature. It was observed that the accuracy of intelligent predicting models for Z-factor is significantly better than conventional empirical models. Also, results showed the improvement of optimized EOS predictions when coupled with GA optimization. Moreover, of the three intelligent models, ANN model outperforms other models considering all data and 263 field data points of an Iranian offshore gas condensate with R^2 of 0.9999, while the R^2 for best empirical correlation was about 0.8334.

Keywords: Z-factor; Gas condensate; Empirical correlation; Intelligent models

1. Introduction

Obtaining fluid properties from gas and oil reservoirs has been of great importance to many researchers and petroleum engineers. The significance of this knowledge becomes more brilliant when the oil and gas capacity of reservoirs, dissolved gas, aquifer model and other reservoir properties depends directly or indirectly on fluid properties [1]. For this purpose, the pressure, volume and temperature (PVT) analysis should be applied to find the aforementioned parameters. This can be made in PVT laboratory or by using proper correlations [2].

For the case of gas condensate and gas reservoir, estimation of Z-factor plays a key role for determination of other properties. Obtaining the accurate Z-factor has been the subject of controversy among researchers. High expenses and inaccessibility to some well-equipped laboratories are the reasons for researchers to be reluctant to use the direct measurement of Z-factor. The common ways for prediction of Z-factor are EOS and empirical correlations. The EOS have been developed and extended for vapor liquid equilibrium (VLE) calculations [3], estimation of critical properties [4] and prediction of volumetric properties of gas mixture as well [5,6]. The point that should be considered about EOS is that despite the accurate results attained from developed and modified EOS in comparison to empirical correlations, a bit more difficulties are involved in solution process and more involving parameters are dealt with. On the other hand, the foible point of empirical

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correlations is that they are usually developed based on specific data set. A good illustration is Sanjari and Lay investigation which concluded to an empirical correlation for Z-factor using Khangiran Refinery data set [7]. Another example is Heidarian et al. study on gas compressibility factor which led to empirical correlation based on limited experimental data [8]. Likewise, Azizi et al. [9]generated a correlation using extracted data from Standing-Katz chart [10] or investigation of Farzaneh-Gord and Rahbari [11]who used measurable real time properties for developing the empirical correlation. An interesting example is Jarrahian and Heidarian [12] study in which they proposed a new EOS for sour and sweet natural gases when the composition is unknown. They tried to lessen the input variables in compare to other empirical correlations.

The fundamental tool for estimation of thermophysical properties of hydrocarbon fluids is EOS. Overall, EOS have their own mixing rules which cause complexity in solution process. The EOS based on statistical-mechanical theory yield more accurate results. On the other hand, empirical correlations are widely used in petroleum engineering applications simply as they are practical and easy to use. That is to say, the chief reason which makes the petroleum engineers tend to deal with these kind of correlations is that they are explicit in Z with straight forward solution procedure [13].

The complexity of EOS makes them difficult to apply especially for mixtures with large number of components. Also, questionable and unreliable predictions of Z-factor using empirical correlations at some pressures and temperatures have led the researchers to seek for easier, more reliable and valid prediction for z-factor. On the other hand, application of intelligent models becomes important to compensate weakness of conventional methods. The intelligent systems are widely used as robust tools to predict the petroleum properties and also other engineering parameters [14–16]. A good example of using intelligent models in reservoir engineering is Saemi et al. work [17], in which they predicted reservoir permeability using linked Adaptive Neural Network and Genetic Algorithm (GA). Other examples of intelligent models usage in reservoir fluid properties are prediction of bubble point pressure by ANN [18], minimum miscibility pressure (MMP) by least square support vector machine (LSSVM) [19], dew point pressure using Fuzzy Logic model [20], Z-factor of natural gas [21] and sour gasusing Adaptive Neuro Fuzzy Inference System (ANFIS) and ANN model [22] and condensate to gas ratio by LSSVM model [23]. In another study, Ganji-Azad et al. applied the ANFIS model to predict reservoir fluid PVT properties [24]. Moreover, Fayazi et al. [25] and Rafiee-Taghanaki [26] proposed a robust model for prediction of gas compressibility factor by application LSSVM.

In this study, experimental PVT data of gas condensate reservoir are used to compare and analyze accuracy of empirical correlations and EOS coupled with intelligent models. In the following sections, the application of intelligent models will be presented in two parts. The first part includes improvement and optimization of Van Der Waals and Redlich Kwong equation of state by implementation of experimental data using Genetic Algorithm [27]. Second part is allocated to employ the Fuzzy Logic (FIS), ANFIS and ANN predicting models and suggest the best intelligent model for predicting gas Z-factor. These intelligent models are trained by share of experimental data, while the remaining data are used for validation and test. Some of these intelligent models are utilized to predict the gas Z-factor in previous works, and their ability will be evaluated and compared with empirical correlations comprehensively in the current study.

2. Empirical correlations and equations of state

2.1. Empirical equations

Several empirical correlations have been developed yet to predict Z-factor. These correlations relate the critical properties of mixture, temperate and pressure of reservoir to the Z-factor.

The regression approach is frequently used to generate empirical correlations such as that of Sanjari and Lay (SL) in 2012. They generated an empirical predicting correlation of gas compressibility. They have developed their correlation based on Virial equation of state. They proposed correlation as a function of p_{pr} and T_{pr} within the range of $0.01 \le p_{pr} \le 15$ and $1.01 \le T_{pr} \le 3$ [7].

$$Z = 1 + A1 p_{pr} + A2 p_{pr^{2}} + \frac{A3 p_{pr^{A4}}}{T_{pr^{A5}}} + \frac{A6 p_{pr^{A4+1}}}{T_{pr^{A7}}} + \frac{A8 p_{pr^{A4+2}}}{T_{pr}(A7+1)} \tag{1}$$

Many empirical correlations are adjusted by pseudo reduced temperature and pressure such as that of Shell Oil Company (SOC) which was referenced by Kumar [28].

$$Z = A + B p_{pr} + (1-A)\exp(-C) - D\left(\frac{p_{pr}}{10}\right)^{4} \tag{2}$$

2.2. Equations of state

Generally, cubic EOS originated from Van Der Waals equation of state are more applicable for industrial proposes [29]. These EOS are commonly rewritten in cubic polynomial form. Vander Waals (VdW) equation is the basic cubic EOS which modified the ideal gas PVT relations [30]. The cubic polynomial form of VdW EOS, equation (8), can be solved to find the Z-factor:

$$Z^3 - (1+B)Z^2 + AZ - AB = 0 \tag{3}$$

where, $A = \frac{ap}{R^2T^2}$ and $B = \frac{bp}{RT}$. The coefficients a and b are defined as follow:

$$a = 0.421875 \frac{R^2 T_c^2}{p_c} \tag{4}$$

$$b = 0.125\frac{RT_c}{p_c} \quad (5)$$

Redlich and Kwong (RK) in 1949 improved the VdW EOS to predict more accurate compressibility of vapor phase. They considered a generalized temperature dependence term as modification of attraction pressure term in their correlation [31].

$$Z^3 - Z^2 + \left(A - B - B^2\right)Z - AB = 0 \quad (6)$$

where, $A = \frac{ap}{R^2T^{2.5}}$ and $B = \frac{bp}{RT}$. The coefficients a and b are obtained by equations 12 and 13:

$$a = 0.42747\frac{R^2T_c^{2.5}}{p_c} \quad (7)$$

$$b = 0.08664\frac{RT_c}{p_c} \quad (8)$$

The evolutionary path of VdW type EOS has reached to Soave-Redlich-Kwong (SRK) equation of state in 1972 and Peng-Robinson (PR) equation in 1976 [32,33]. For the cases in which the composition of gas mixture is unknown (like this study), the use of SRK and PR equation of state is impossible.

3. Intelligent models

The chief purpose of an intelligent software is to bridge sets of input and output variables to each other considering the system specifications [34]. Application of intelligent-based models is more efficient in such cases which are time consuming and involve non-linear mathematical modeling, adaptive learning and when there is not any meaningful relation between input and output of a system. The intelligent models developed in this study include ANN, ANFIS (including FIS) and GA.

3.1. Artificial Neural Network

An ANN is a network of interconnected nodes exhibiting the process of biological neurons in a brain. The artificial neurons lie in constitutive layers of the network. Each layer is linked to the next by specific weights (w) [35]. One of the most practical structures of ANN is Multi-Layer Perceptron (MLP) in which the input and output layers are connected to each other by an additional layer called hidden layer. The hidden layers do the processing step and output layer gathers the signals and distributes [36]. A MLP network may have one or more hidden layers; however, it is seen that a network with one hidden layer can predict the performance of a system as well [15]. The network adopts the weights of neurons based on error between outputs and targets in training steps. Moreover, for constructing robust design, some of unused data in training step are used for validation, which makes the model more accurate. The structure of ANN is illustrated in Fig. 1.

It is seen from figure that the networks consist of three layers i, j and k where the weights between layers is designated by w_{ij} and w_{jk}. The initial weighted values are modified

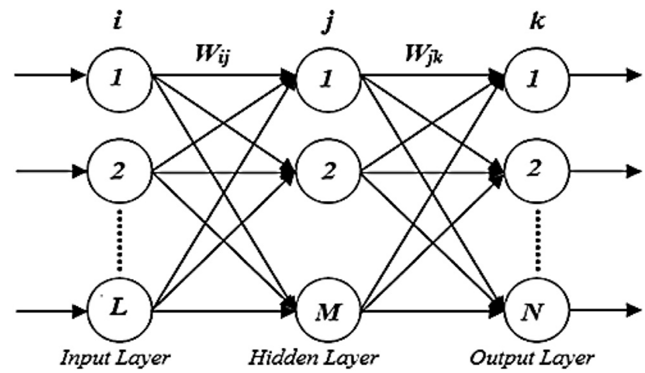

Fig. 1. Schematic of MLP structure.

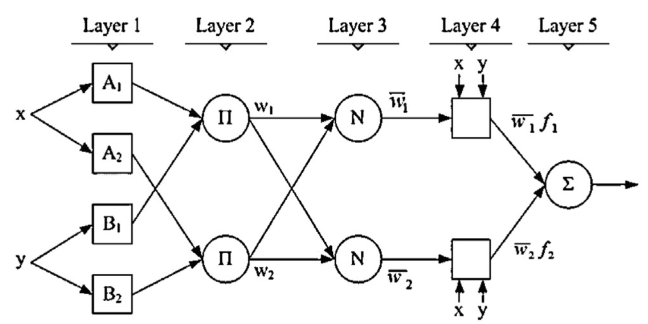

Fig. 2. The structure of ANFIS model.

during training process by the comparison made between predicted and real values [37]. Among various training algorithm Levenberg-Marquart (LM) is commonly used for training system due to its stability and swift convergence [38]. In the current study, the LM algorithm will be utilized where the weights are computed by:

$$W_{k+1} = W_k - \left[J_{W_k}^T J_{W_k} + \mu_k I\right]^{-1} * J_{Wk}^T V_{W_k} \quad (9)$$

where the weighted matrix are symbolized by W_{k+1} and W_K during $K+1^{th}$ and K^{th} repetitions, J is the Jacobian matrix, V is the accumulated errors vector, I is the identity matrix, and μ_k is the parameter to express the ability of LM algorithm for altering the searching method. In the present study, the input

Table 1
Statistical information of data points.

Property	Max.	Min.	Avg.	SD[a]
Temperature/R	681.03	515.07	615.653	56.133
Pressure/psia	9104.536	1175.69	5391.07	1390.637
Z-factor	1.374	0.71	0.9896	0.1159
T_{pc}/R	427.2004	385.4219	4.09E+02	1.03E+01
p_{pc}/psia	663.6487	650.0807	6.67E+02	1.56E+00
MW	2.62E+01	2.06E+01	2.21E+01	9.74E-01
Specific volume/$m^3\ kg^{-1}$	1.46E-02	2.34E-03	3.81E-03	1.65E-03
Gas gravity	9.06E-01	7.11E-01	7.63E-01	3.38E-02

[a] denotes Standard deviation.

Table 2
Calculated errors of empirical correlations considering all experimental data.

	R^2	ARE %	AARE %	RMSE	RSS	MSE
BB	0.833467	1.855974	3.670279	0.001201	0.612853	0.002403
DA	0.7476682	0.2914712	3.9412058	0.0015169	0.7736368	0.0030338
HY	0.830662	2.992630	4.030546	0.001338	0.682459	0.002676
HD	0.8330268	3.0963429	3.9694283	0.0012782	0.6519107	0.0025565
PP	0.379220	17.39785	18.12433	0.038935	19.85726	0.077871
SL	0.7956868	3.1706066	4.2371119	0.0016540	0.8435509	0.00330804
SOC	0.7903665	0.7361394	5.1792569	0.0023512	1.1991327	0.0047024

layer L is consist of three variables which are T_{pr}, p_{pr} and γ_g also output layer K is allocated to target value, i.e. Z-factor.

3.2. Adaptive Neuro-Fuzzy Inference System (ANFIS)

The ANFIS is the combination of neural networks and fuzzy modeling in training step in order to improve the ability of learning [39]. The ANFIS applies the beneficial features of ANN and fuzzy model by a hybrid structure and modifies the inappropriate properties. In other words, ANFIS combines the low level calculation of ANN and the powerful reasoning ability of a fuzzy logic system. Based on ANFIS modeling for non-linear systems, the input space is divided into many local areas. In this regard, the modest local is developed by linear functions or adjustable coefficients; next the ANFIS uses the membership function (MF) to determine the dimension of each input. Hence, the MFs and the hidden layers play a key role in estimation of ANFIS model ability. The five layers of ANFIS modeling is shown in Fig. 2 [39].

The adaptive nodes of the first layers are equated as:

$$\mu_{Ai}(x) = \exp^{-\left(\frac{x-x^*}{\sigma^2}\right)} \tag{10}$$

where x^* and σ^* are premise parameters which are adapted by a hybrid algorithm and x is the input variable. In the present study, the three input variables are T_{pr}, p_{pr} and γ_g.

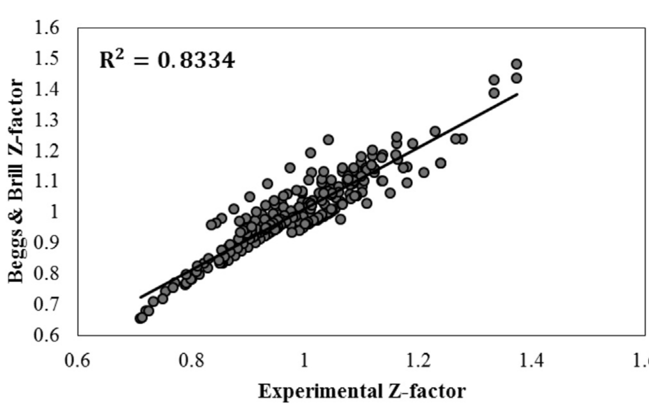

Fig. 3. Comparison between the best empirical correlation and experimental data.

The firing strength of each rule is determined in the second layer by quantifying the extent of each rule's input data. The output of a layer is the algebraic product of input signals:

$$O_{2,i} = \omega_i = \mu_{Ai}(x_1) \times \ldots \times \mu_{Ci}(x_n) \tag{11}$$

The third layer is responsible of normalization by calculating ratio of ith rule's firing strength to the summation result of all rule's firing strength:

$$O_{3,i} = \overline{\omega}_i = \frac{\omega_i}{(\omega_i + \ldots + \omega_n)} \tag{12}$$

The calculation of output is done by the fourth layer:

$$O_{4,i} = \sum \overline{\omega}_i f_i \tag{13}$$

where the total output is obtained as the summation of all input signals in the fifth layer by calculation of wave height as follow [40]:

$$O_{5,i} = \frac{\sum_{i=1}^{n} \omega_i f_i}{\sum_{i=1}^{n} \omega_i} \tag{14}$$

$O_{5,i}$ is Z-factor in this study.

Table 3
Modification of EOS coefficients using GA.

	a	b	Modified a	Modified b
VdW	0.421875	0.125	0.3619423	0.1016583
RK	0.42747	0.08664	0.4995234	0.0897378

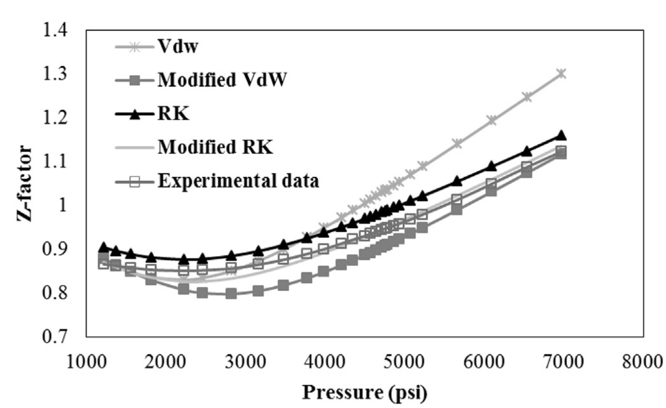

Fig. 4. Z-factor variation versus pressure at 667.67 R for all 263 data points.

Table 4
The statistical errors of EOSs and Modified EOSs using GA.

	R^2	ARE %	AARE %	RMSE	RSS	MSE
VdW	0.4181896	14.381814	15.160368	0.0159576	8.1383792	0.0319152
Modified VdW	0.8038455	−0.659714	4.8360067	0.0017642	0.8997736	0.0035285
RK	0.7717325	4.0203441	4.7016636	0.0015935	0.8127042	0.0031870
Modified RK	0.8669765	3.3925143	3.3925143	0.0010056	0.5127304	0.0020103

The adaptive layers are first and the fourth. A_i, C_i and σ_i are premise parameters of input fuzzy MFs in first layer. It is worth mentioning that the Gaussian MFs are used in this work.

3.3. Genetic Algorithm (GA)

Relying on Darwin's theory, it is claimed that species of organisms have evolved over a long period of time through natural selection while all of them share a common ancestor [41]. The key observation is that there are limited resources for the population of all organisms existing in nature, and this leads to competition between individuals of different species. Those fitter individuals to the environment have more chances for survival and reproduction. Consequently, the process of natural selection along with random modifications cause a rise in the fitness of the population and the developments of species. Genetic Algorithm (GA) is a search heuristic in computer science which first introduced by J. Holland [42] to solve optimization problems. Inspiring from the biological evolution, the main idea of GA is based on the survival of the fittest among individuals where each one represents a possible solution to a given problem. In order to optimize the given problem, GA starts from an initial population of randomly generated individuals and proceeds in an iterative process resembling the genome evolution. Each iteration of the algorithm generates a new population by first performing crossover operator on elder populations and second applying mutation on new generation which called offspring. Note that a fitness proportionate selection is applied to recombination phase where the more fit individuals are stochastically selected from the current population as parents. To this end, the value of the objective function should be determined to measure the fitness of each individual in the population. The iterative routine of optimization stops when GA converges to a good enough individual or visits the maximum number of generations.

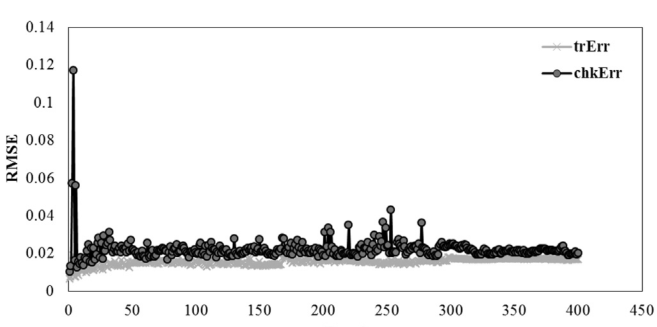

Fig. 5. Learning process of ANFIS model considering training and checking errors.

4. Results and discussion

In order to compare empirical correlations, EOS, modified EOS and application of intelligent models including FIS, ANFIS and ANN, 263 experimental data points were used. These data were extracted from gas condensate reservoir. The statistical properties of data points are given in Table 1. For the comparison purposes, various types of errors are employed to determine the best predicting model. The applied errors are: Coefficient of determination (R^2), Average relative error (ARE), Average absolute relative error (AARE), Root mean square error (RMSE), Residual sum of square (RMSE) and Mean squared error (MSE). These statistical errors are presented in Appendix B. It should be mentioned the limitations of empirical correlations if there were any, are considered in calculation of Z-factor.

Quantitative comparison between correlations is summarized in Table 2 for all experimental data. It is observed that BB correlation has the highest value of R^2 (0.8334) and least MSE (0.002.40). On the other hand, PP has the least R^2 (0.3792) and maximum MSE (0.0778). According to Table 2, deviation of PP [43] and DA [44] correlations from experimental data are more than other correlations, while BB [45], HD [8] and HY [46] correlations give more accurate predictions. However, although these correlations show a good

Table 5
Statistical errors reported for developed FUZZY model.

	Data point	RMSE	MSE	R^2
Train	185	4.585782E-3	2.102940E-5	9.983545E-1
Validation	39	9.599466E-2	9.214976E-3	6.661773E-1
Test	39	2.132217E-2	4.546349E-4	9.666858E-1
Total	263	3.773630E-2	1.424029E-3	9.051515E-1

Table 6
Statistical errors reported for developed ANFIS model.

	Data point	RMSE	MSE	R^2
Train	185	6.398754E-3	4.094405E-5	9.967936E-1
Validation	39	1.015981E-2	1.032216E-4	9.938876E-1
Test	39	9.142918E-3	8.359295E-5	9.931449E-1
Total	263	7.538195E-3	5.682438E-5	9.957756E-1

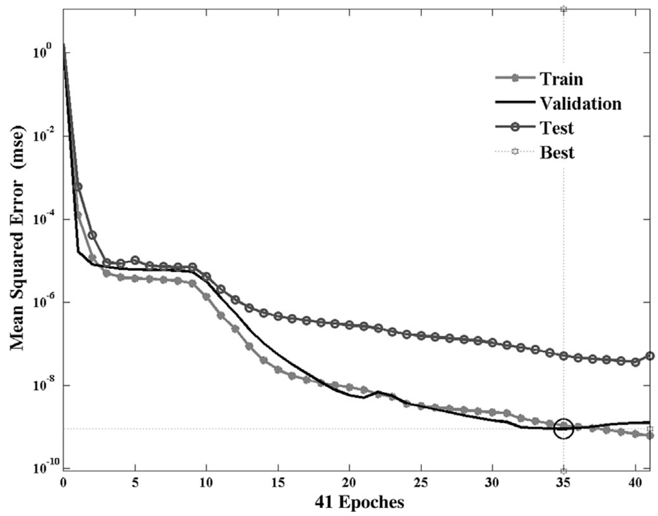

Fig. 6. Variation MSE values versus training Epoches.

agreement with experimental data, they do not convince the petroleum engineering needs for predicting the accurate and reliable Z-factor.

Comparison of experimental data and BB model as the best predicting correlation for all 263 data points is shown in Fig. 3 by scatter diagram. The BB model shows the closest agreement with experimental data; however, predicted results are still far from the set point data.

As the gas mixture components and their acentric factors are not available, the VdW and RK EOS were optimized using GA. To do this, results of VdW and RK EOS were chosen as fitness functions. The fitness functions have been compared with experimental data to reach the optimum results. The objective functions were function of four variables, i.e. temperature, pressure, pseudo critical temperature and pressure. It is worth noting that 50 generations were used to find the optimum results which are two coefficients of EOS. The Modified coefficients are summarized in Table 3. Effects of modification on VdW and RK EOS are shown in Fig. 4, where the variations of Z-factor versus pressure are displayed.

The statistical errors are reported in Table 4. As seen, the R^2 improved from 0.4181 to 0.8038 and from 0.7717 to 0.8669 for VdW and RK EOS, respectively as a result of GA optimization. Accordingly, the least values of AARE and MSE errors belong to Modified RK, which are 3.3925 and 0.002 respectively. On the other hand, the VdW EOS allocates the maximum AARE and MSE errors 15.1603 and 0.0319. It is clear from Fig. 5 and Table 4 that the accuracy of results from VdW EOS is much less than those of RK equation. The considerable difference between concluded results from two methods still remains even after the modification.

4.1. Development of ANFIS

The developed ANFIS is used to present an intelligent predicting model for Z-factor. In fact, the ANFIS system is a combination of FIS and ANN model. In other words, the Fuzzy model parameters are being optimized by Neural Network. The ANFIS model compensates some weaknesses of FIS system. The Sugeno-type Fuzzy Inference System settles down to present a predicting model using training data without any check and testing. Therefore, it is common to tune the model with the least error in training step, but unusual errors in tests and validation steps which results in error propagation. In other words, if there is any checking step, it will prevent over fitting the model on training data.

The Sugeno-type Fuzzy Interface System generates clusters for introducing its rules. Therefore, determining the radius of clusters, which specifies the number of clusters, is essential in obtaining the number of rules and developing the Fuzzy model. The less radius results more clusters and also more rules. In other words, the larger the radius, the less is the number of clusters.

For this study, all 263 series of data points were first randomly divided in three parts where 70% of whole data used for training, 15% for validation and 15% for test. Next, the Sugeno-type Fuzzy system was generated using 70% of whole data which were trained. For this purpose, the initial radius of input data, which are temperature, pressure and specific gas gravity, should be determined. The applied initial input variables were 5, 0.5, 0.05 for temperature, pressure and specific gas gravity respectively and also 0.05 for Z-factor which is output variable. The statistical errors related to Fuzzy

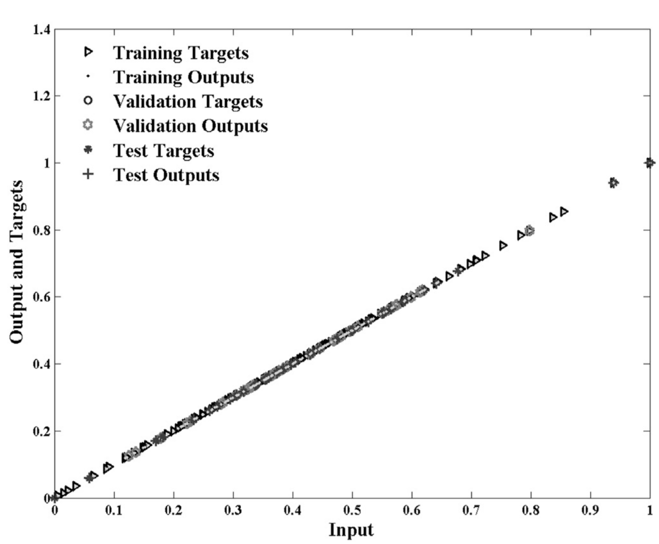

Fig. 7. Comparison of output and target values related to training, validation and test.

Table 7
Statistical errors reported for developed ANN model.

	Data point	MSE	R^2
Train	185	0.11031E-10	0.99999
Validation	39	9.12044E-10	0.99999
Test	39	5.26094E-08	0.99999
Total	263	8.75017E-09	0.99999

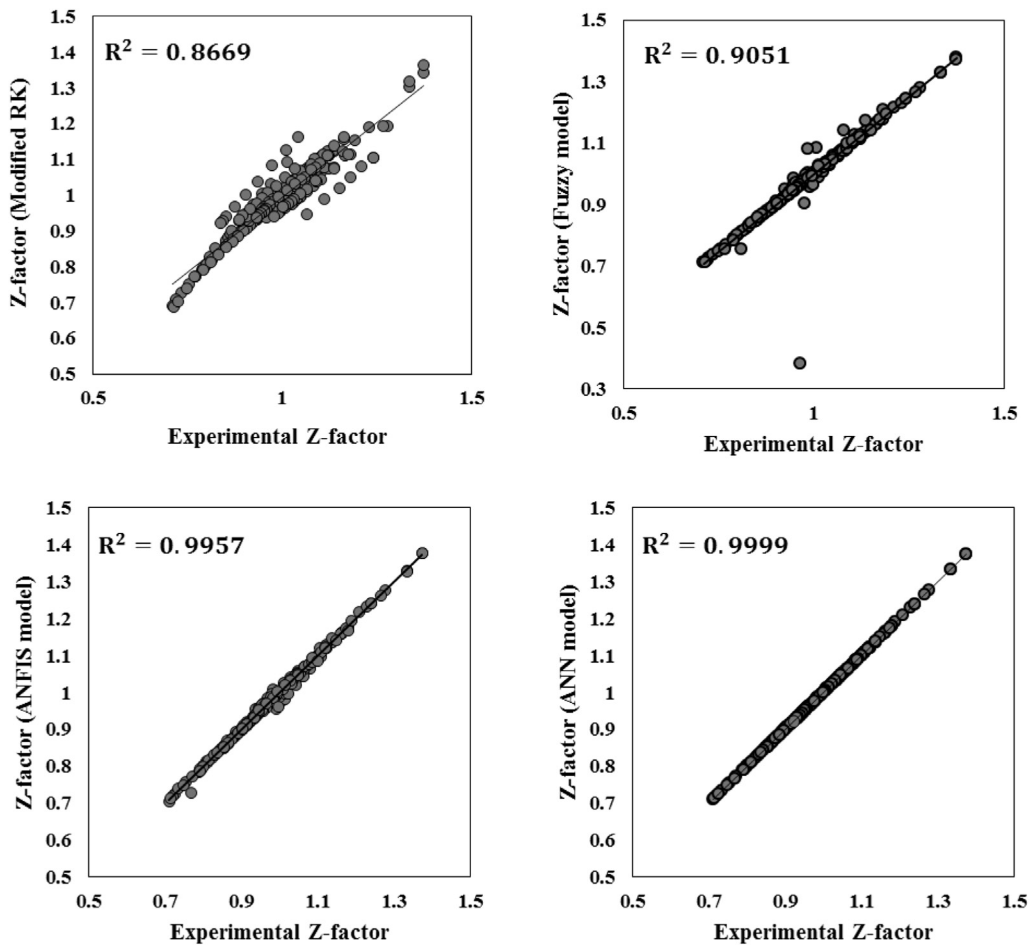

Fig. 8. Comparison of experimental data and ANN results.

Table 8
Comparison of statistical errors corresponded to three intelligent models.

	R^2	ARE/%	AARE/%	RMSE	RSS	MSE
FIS	0.90183886	−0.1846262	0.71927844	7.458890E-4	0.380403403	0.001491778
ANFIS	0.99577559	−0.0395657	0.42042680	2.841218E-5	0.014490215	5.6824372E-5
ANN	0.99999999	7.6890E-04	0.00223901	1.92894E-09	9.83761E-07	8.75017E-09

Table 9
Properties of other data base extracted from literature.

Ref.	Data points	No. Gas mixtures	P/psia	T/R	MW	Z
[48]	47	5	97.02−1106.90	558.36−646.92	51.56−16.60	0.86−0.99
[49]	165	5	1039.29−7120.68	559.67−619.66	23.67−18.17	0.67−1.14
[50]	100	4	3238.41−1747.83	545.76−753.48	17.05−20.51	1.28−2.04
[51]	84	3	145.53−2207.94	455.67−581.67	16.31−17.85	0.59−0.94
[52]	234	2	1470.00−17125.50	563.76−795.24	17.09−16.35	0.92−1.42
[53]	241	3	132.20−2950.29	432.00−720.12	18.43−17.24	0.64−1.91
[54]	105	5	1039.29−7120.68	560.77−620.76	20.85−18.80	0.71−1.14
[55]	61	6	98.49−1265.67	509.65−599.70	29.92−31.30	0.66−0.99
Total	1038	33				

predicting model is reported in Table 5. As seen, the maximum R^2 belongs to training section as expected, and the least value for validation. It is worth noting that the most important type of reported error is that of check or test which determine the ability of model for new unused data while the validity error shows the generality of proposed model. The R^2 value of 0.9051 obtained from Fuzzy model is more than that of best empirical correlation and modified RK EOS; however, predicted values do not match the target values with high accuracy.

In order to improve the ability of FIS predicting model, the generated Fuzzy model parameters were optimized by ANN. The ANFIS model considers 70% of data for training and the 15% for validation and the remains for test. It should be mentioned that the three parts of implemented data were completely explicit and there was not any mutual node in three parts. The learning process of ANFIS model along with considering training and checking errors is shown in Fig. 5 which indicates that the values of errors in training and checking steps are close to each other at the end of process.

The statistical errors of output ANFIS model are reported in Table 6. It is seen that total R^2 value (0.99577) is more than that of FIS model where the MSE value of Fuzzy model (1.42E-3) was reduced to 5.683E-5. Results of Table 6 indicate that the proposed model works better in comparison to other methods for prediction of Z-factor.

4.2. Development of ANN

For developing the ANN model, all 263 series of data point were normalized between 0 and 1. The input variables are pressure, temperature and specific gas gravity, and Z-factor is considered as a function of these parameters:

$$Z = f(T, p, \gamma_g) \tag{17}$$

It is worth mentioning that selection of input variables affects the reliability and performance of any predicting model; hence, it should reflect the physical properties and the nature of system. The network consists of two layers, i.e. input layer and hidden layers. The input layer included three nods regarding temperature, pressure and gas gravity. These nodes are bridged to the hidden layer by specific weights. This layer is responsible for the main data processing. On the other hand, the output of this network has one nod corresponding to normalized Z-factor. For the training purpose, 70% of 263 series of data points were chosen randomly. In addition, half of the remaining data was used for test and half for the validation of constructed model. It should be noted that the number of hidden layer neurons should be determined to lessen the deviation of output network and validation data. The adjusted number for this study was 20 neurons. Several training algorithms were applied, including Levenberg-Marquardt algorithm (LM), Scaled Conjugate Gradient, Gradient Descent

Table 10
Comparison of AARE of intelligent system with other predicting model using reported data in Table 9.

Ref.	AARE/%							
	PP	SL	DA	HD	HY	FIS	ANFIS	ANN
[48]	0.87	0.32	14.71	0.55	1.32	0.894	0.76	0.0045
[49]	5.40	1.52	9.69	2.36	2.79	2.89	3.44	0.0986
[50]	85.62	6.04	5.52	35.57	5.12	0.343	0.742	0.0042
[51]	32.10	1.28	20.12	1.33	1.40	1.65	2.028	0.0730
[52]	24.78	1.58	1.56	2.41	2.43	1.11	0.998	0.0060
[53]	1.46	0.94	13.14	2.09	2.14	0.815	0.694	2.3E-04
[54]	7.32	2.00	8.96	2.47	2.77	1.81	1.38	0.0164
[55]	2.044	1.28	33.42	0.9116	1.09	0.880	0.976	0.1274
Total	159.59	14.96	107.12	47.69	19.06	10.39	11.018	0.329

Table 11
Comparison of MSE of intelligent system with other predicting model using reported data in Table 9.

Ref.	MSE							
	PP	SL	DA	HD	HY	FIS	ANFIS	ANN
[48]	9.97E-05	2.14E-05	0.0202	4.29E-05	3.78E-04	2.81E-04	1.34E-04	1.74E-08
[49]	0.0088	3.33E-04	0.0098	5.89E-04	7.54E-04	9.98E-04	1.190E-3	1.52E-06
[50]	2.5694	0.0191	0.0214	0.4589	0.017	6.843E-4	1.21E-03	9.41E-08
[51]	0.1175	2.84E-04	0.0329	1.39E-04	1.38E-04	3.22E-04	3.71E-04	1.07E-06
[52]	0.1936	4.82E-04	5.02E-04	9.65E-04	9.67E-04	7.43E-04	5.46E-04	3.40E-06
[53]	0.0042	0.004	0.0209	0.0043	0.0043	1.33E-03	8.16E-05	5.68E-12
[54]	0.0144	5.90E-04	0.0091	8.52E-04	9.51E-04	8.81E-03	3.43E-04	1.05E-06
[55]	5.88E-04	3.44E-04	0.1353	3.50E-04	3.65E-04	2.95E-04	3.12E-04	1.16E-05
Total	2.91	2.52E-02	0.25	0.466	2.49E-02	1.34E-02	4.18E-03	1.88E-05

with Momentum, adaptive learning rate Back-propagation and Resilient Back-propagation [15,47]. The best model was presented by LM algorithm, so this algorithm was utilized for the last network training. Fig. 6 shows the performance of ANN model by means of MSE values related to training, validation and test. As seen, the training stopped after 35 epoches where the MSE value of validation started to rise. The output of ANN model including three sets of data (train, validation and test) are shown in Fig. 7. For comparison, corresponding experimental data of ANN predicting model are shown in the same figure.

The MSE and R^2 values of output results and target data which consist of training, validation and test are reported in Table 7. The high values of R^2 errors and the least values of MSE (8.7501E-09) confirm that the output track the target well enough.

The scattered diagrams plotted in Fig. 8 show comparison between intelligent models. According to this figure, accuracy of models increases as the nodes become more concentrated. The evolutionary trend of improvement is clearly understandable by comparing the scattered diagrams of FIS and ANFIS model. Also, there are some poorly predicted nodes that approach their corresponding experimental values by modifications applied by ANFIS. Fig. 8 clarifies the robustness of ANN developed model, since the target and output values cover each other completely. The points are placed on nearly 45° which proves the accuracy of predicting ANN model.

Among three intelligent systems used for prediction of Z-factor, the ANN model is the most accurate model. As realized from Table 8 the maximum R^2 value belongs to ANN (0.9999), while the FIS model has the least R^2 (0.9018). It should be considered that improvement of Fuzzy model by application of ANFIS is significant when the MSE value reduces from 0.00149 (FIS) to 5.6824E-5 (ANFIS).

To confirm the advantage of intelligent systems over conventional predicting correlations, 1038 data points of different gas mixtures were used to obtain Z-factor. The thermophysical properties of gas samples are listed in Table 9. The sum of AARE and MSE values given in Tables 10 and 11 prove the accuracy of intelligent systems for predicting Z-factor. However, one should note that the number of data bank directly affects the accuracy of intelligent systems. Precision of ANN model is highly significant among all three types of predicting methods, even those with fewer data points [48–55].

5. Conclusion

In this study, the application of several intelligent systems was investigated to find the most powerful model for prediction of Z-factor. The applied intelligent systems were GA, FIS, ANFIS and ANN model. Several statistical errors were calculated to determine the accuracy of each one. The developed intelligent models show high accuracy over empirical correlations. In addition, ANN model showed the most accurate prediction in comparison with other intelligent models for all data. Also, GA was used to optimize the parameters of VdW and RK EOS. Results shows that RK EOS responses better to parametric optimization compared to VdW EOS and its modified parameters resulted in better Z-factor predictions.

Nomenclature

I	identity matrix
J	Jacobian matrix
O	*i*th layer output
P	gas pressure, psi
T	gas temperature, R
T	reciprocal of the pseudo-reduced temperature
V	vector of accumulated errors
X	Fuzzy linguistic variables
W	matrix of weights
Y	Fuzzy linguistic variables

Greek symbols

Σ Fuzzy ordered parameter
M model Specification
ω firing Strength
P density

Subscripts

A	Fuzzy set
B	Fuzzy set
Pc	pseudo critical
Pr	pseudo reduced
Tc	pseudo critical
Tr	pseudo reduced
R	reduced
I	input

Abbreviations

AARE	average absolute error
ARE	average relative error
ANFIS	Adaptive Neuro-Fuzzy Inference System
ANN	Artificial Neural Network
FIS	Fuzzy interface system
LM	Levenberg-Marquardt
MSE	mean squared error
MF	membership function
MLP	multi-layer perceptron
NN	neural network
R^2	squared correlation coefficient
RMSE	root mean squared error
RSS	residual sum of square
RSS	residual sum of square

Appendix A. Empirical correlations

Hall-Yarborough's (HY) proposed an empirical correlation in 1973 [46].

$$Z=\left[\frac{0.06125tp_{pr}}{Y}\right]\exp\left[-1.2(1-t)^2\right] \quad (A-1)$$

The required parameters of Hall-Yarborough's Method are shown in equations (A-3)–(A-6), where the Y parameter should be obtained by solving equation (A-2).

$$F(Y)=X_1+\frac{Y+Y^2+Y^3-Y^4}{1-Y}-(X_2)Y^2+(X_3)Y^{X_4} \quad (A-2)$$

$$X_1=-0.06125Pprtexp\left[-1.2(1-t)^2\right] \quad (A-3)$$

$$X_2=\left(14.76t-9.76t^2+4.58t^3\right) \quad (A-4)$$

$$X_3=\left(90.7t-242.2t^2+42.4t^3\right) \quad (A-5)$$

$$X_4=(2.18+2.82t) \quad (A-6)$$

Also, Dranchuk and Abu-Kassem (DA) correlated Z-factor in 1975 [44]. This correlation is used for determination of gas compressibility of dry gas.

$$Z=\left[A_1+\frac{A_2}{T_{\mathrm{pr}}}+\frac{A_3}{T_{\mathrm{pr}}^3}+\frac{A_4}{T_{\mathrm{pr}}^4}+\frac{A_5}{T_{\mathrm{pr}}^5}\right]\rho_r+\left[A_6+\frac{A_7}{T_{\mathrm{pr}}}+\frac{A_8}{T_{\mathrm{pr}}^2}\right]\rho_r^2 - A_9\left[\frac{A_7}{T_{\mathrm{pr}}}+\frac{A_8}{T_{\mathrm{pr}}^2}\right]\rho_r^5+A_{10}\left(1+A_{11}\rho_r^2\right)\frac{\rho_r^2}{T_{\mathrm{pr}}^3}\exp\left[-A_{11}\rho_r^2\right]+1 \quad (A-7)$$

The empirical correlation can also be obtained by regression methods and use of several experimental data. Take Heidarian (HD) et al. correlation as an example which is determined by multiple regression analysis. They used 1220 data points in specific range of $0.2\le p_{\mathrm{pr}}\le 15$ and $1.2\le T_{\mathrm{pr}}\le 3$ [8].

$$Z=\ln\left[\frac{A_1+A_3\ln\left(p_{\mathrm{pr}}\right)+\frac{A_5}{T_{\mathrm{pr}}}+A7\left(\ln p_{\mathrm{pr}}\right)^2+\frac{A_9}{T_{\mathrm{pr}}^2}+\frac{A_{11}}{T_{\mathrm{pr}}}\ln\left(p_{\mathrm{pr}}\right)}{1+A_2\ln\left(p_{\mathrm{pr}}\right)+\frac{A_4}{T_{\mathrm{pr}}}+A6\left(\ln p_{\mathrm{pr}}\right)^2+\frac{A_8}{T_{\mathrm{pr}}^2}+\frac{A_{10}}{T_{\mathrm{pr}}}\ln\left(p_{\mathrm{pr}}\right)}\right] \quad (A-8)$$

The required coefficients of Dranchuk-Abukassem [44], Heidarian et al. [8] and Sanjari and Lay [7] are listed in Table (A-1) and (A-2):

Table (A-2)

Parameters	A7	A8	A9	A10	A11
DA	−0.7361	0.1844	0.1056	0.6134	0.721
HD($0.2 < p_{\mathrm{pr}}<3$)	0.190387	0.620009	1.838479	0.405237	1.073574
HD($0.3 < p_{\mathrm{pr}}<15$)	0.066006	0.612078	2.317431	0.163222	0.56606
SL ($0.01 < p_{\mathrm{pr}}<3$)	7.138305	0.08344	–	–	–
SL($3 < p_{\mathrm{pr}}<15$)	3.543614	0.134041	–	–	–

The related parameters of Shell oil company correlation are shown by equations (A-9)–(A-15):

$$A=-0.101-0.36T_{\mathrm{pr}}+1.3868\sqrt{T_{\mathrm{pr}}-0.919} \quad (A-9)$$

$$B=0.021+\frac{0.04275}{T_{\mathrm{pr}}-0.65} \quad (A-10)$$

$$C=p_{\mathrm{pr}}\left(E+Fp_{\mathrm{pr}}+Gp_{\mathrm{pr}}^4\right) \quad (A-11)$$

$$D=0.122\exp\left[-11.3\left(T_{\mathrm{pr}}-1\right)\right] \quad (A-12)$$

$$E=-0.6222-0.224T_{\mathrm{pr}} \quad (A-13)$$

$$F=\frac{0.0657}{T_{pr}-0.85}-0.037 \quad (A-14)$$

$$G=0.32\exp\left[-19.53\left(T_{\mathrm{pr}}-1\right)\right] \quad (A-15)$$

A similar empirical equation was introduced by Beggs and Brill (BB) to predict gas Z-factor as function of pseudo critical pressure and temperature [45]. Their method is not appropriate for the pseudo pressure less than 0.92.

$$Z=A+(1-A)\exp(-B)+Cp_{\mathrm{pr}}^D \quad (A-16)$$

$$A=1.39\left(T_{\mathrm{pr}}-0.92\right)^{0.5}-0.36T_{\mathrm{pr}}-0.101 \quad (A-17)$$

$$B=\left(0.62-0.23T_{pr}\right)p_{pr}+\left(\frac{0.066}{T_{pr}-0.86}-0.037\right)p_{pr}^2+\left(\frac{0.32}{10^{\left(9\left(T_{pr}-1\right)\right)}}\right)p_{pr}^6 \quad (A-18)$$

$$C=0.132-0.32\log\left(T_{pr}\right) \quad (A-19)$$

Table (A-1)

Parameters	A1	A2	A3	A4	A5	A6
DA	0.3265	−1.07	−0.5339	0.01569	−0.0517	0.5475
HD ($0.2 < p_{\mathrm{pr}} < 3$)	2.827793	−0.46882	−1.26229	−1.53652	−4.53505	0.068951
HD($0.3 < p_{\mathrm{pr}} < 15$)	3.252838	−0.13064	−0.64492	−1.51803	−5.39102	−0.0138
SL($0.01 < p_{\mathrm{pr}} < 3$)	0.007698	0.003839	−0.46721	1.018801	3.805723	−0.08736
SL($3 < p_{\mathrm{pr}} < 15$)	0.015642	0.000701	2.341511	−0.6579	8.902112	−1.136

$$D = 10^{\left(0.3106-0.49T_{pr}+0.1824T_{pr}^2\right)} \quad (A-20)$$

Papay (PP) proposed simple correlation for estimation of gas compressibility factor [43]. The correlation was adopted as a function of pseudo reduced pressure and temperature.

$$Z = 1 - \frac{3.53p_{pr}}{10^{0.9813T_{pr}}} + \frac{0.274p_{pr}^2}{10^{0.8157T_{pr}}} \quad (A-21)$$

Appendix B. Types of errors

Coefficient of determination

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(Z_i^{Pred} - Z_i^{exp}\right)^2}{\sum_{i=1}^{N}\left(Z_i^{Pred} - average(Z_i^{exp})\right)^2} \quad (B-1)$$

Average relative error

$$ARE\% = \frac{100}{N}\sum_{i=1}^{N}\left(\frac{Z_i^{Pred} - Z_i^{exp}}{Z_i^{exp}}\right) \quad (B-2)$$

Average absolute relative error

$$AARE\% = \frac{100}{N}\sum_{i=1}^{N}\left(\left|\frac{Z_i^{Pred} - Z_i^{exp}}{Z_i^{exp}}\right|\right) \quad (B-3)$$

Root mean square error

$$RMSE = \left(\frac{\sum_{i=1}^{N}\left(Z_i^{Pred} - Z_i^{exp}\right)^2}{N}\right)^{\frac{1}{2}} \quad (B-4)$$

Residual sum of square

$$RSS = \sum_{i=1}^{N}\left(Z_i^{Pred} - Z_i^{exp}\right)^2 \quad (B-5)$$

Mean squared error

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(Z_i^{Pred} - Z_i^{exp}\right)^2 \quad (B-6)$$

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