A COMPARATIVE STUDY OF A HEAT AND FLUID FLOW PROBLEM USING THREE MODELS OF DIFFERENT LEVELS OF SOPHISTICATION

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Abstract. Three mathematical models of different levels of sophistication have been used to study a practical problem on underground heat and fluid flow, associated with the seasonal storage of hot water in an aquifer. A number of scenarios have been examined using the three models. For the basic problem the three models yield similar results, so use of the simplest is preferred. For several variations on the problem, only the more complicated models are adequate to properly address the problem. In general, the choice of an appropriate model is very problem-specific and requires not only experience with modelling methods, but also an understanding of the physics of the problem.

Keywords. Code comparison; optimal code choice; underground heat and fluid flow; aquifer thermal energy storage.

INTRODUCTION

In applying mathematical models it is usually desirable to use the simplest model that can adequately solve the problem being considered. This is especially true for optimization studies and problems involving statistical variables, where multiple cases must be investigated. In such studies, the time and effort required to use a particular mathematical model may become the dominant factor in determining whether or not it can be of practical use.

Three mathematical models of different levels of sophistication have been used to study an underground heat and fluid flow problem, involving the seasonal storage of hot water in an aquifer, a water-saturated porous geological formation. The storage system includes a pair of wells drilled into a shallow aquifer. During summer, water is pumped out of the aquifer through one well (the supply well), heated using solar collectors, and injected back into the aquifer through the other well (the storage well) located some distance away. The hot water is stored in the aquifer until winter, when it is pumped out of the storage well, cooled as it supplies heat to a space-heating system, and reinjected through the supply well. The efficiency of the storage system is quantified by the energy recovery factor, ϵ, defined as the ratio of the energy extracted during winter to the energy stored during summer. The basic processes which must be considered when modelling such an aquifer storage system are fluid flow caused by pressure gradients and gravity, convective heat flow (heat carried with the moving fluid), and conductive heat flow caused by temperature gradients. These processes are highly coupled, and the three mathematical models considered differ in the manner in which they treat the coupling.

The simplest model (SFM-Graph) utilizes a graphical technique that does not require the use of a computer. The parameters of the problem are combined into dimensionless groups. A series of graphs show ϵ and the temperature variation of the extracted water, T_p, as a function of these groups. The method assumes that the fluid flow field can be completely uncoupled from the temperature distribution in the aquifer. Furthermore, the storage system is assumed to be radially symmetric around the storage well, so the influence of the supply well on the temperature and flow fields is ignored. The second model (SFM-ANGLE) uses two simple computer programs. First, some key parameters of the fluid flow field are calculated semianalytically for given temperature boundary conditions. Second, the resulting fluid flow field is used in the numerical calculation of the heat flow, which yields ϵ and T_p. As in the graphical method, the storage system is assumed to be radially symmetric. The third model (PT) is a sophisticated computer code that calculates fully coupled heat and fluid flows for one-, two-, or three-dimensional geometries. In general, a more complex model requires more time and effort to use, but is applicable to a wider range of problems.

In the following two sections, the governing equations for heat and fluid flow in a porous medium are given, and the three models discussed. Next, a basic problem involving hot water storage in an aquifer is posed, and studied using the three models. Then variations on the basic problem are used to introduce a number of complications to the analysis. Based on these variations, comments are made on the limitations and range of applicability of each model.

GOVERNING EQUATIONS

The equations governing the behavior of an aquifer energy storage system are the continuity equations for fluid mass and energy. Darcy's law is used to describe macroscopic fluid flow through a porous medium.

Darcy's law is expressed by

$$\mathbf{q} = -\frac{k}{\mu}(\nabla P - \rho_w g \hat{z}) \quad (1)$$

where $\mathbf{q}$ is the flux of fluid flowing relative to the solid rock, k is the permeability of the aquifer formation (a second rank tensor), μ is the viscosity of the fluid, and the driving forces for fluid motion are the pressure gradient, ∇P, and the gravitational body force, $\rho_w g$ (ρ_w is fluid density, g is gravitational acceleration). In most applications, k is assumed to be a scalar or a diagonal tensor. Darcy's law is an approximation of the general Navier-Stokes equation for momentum conservation. The average microscopic velocity $\mathbf{v}$ within individual pores is related to the macroscopic velocity $\mathbf{q}$ by $\mathbf{v}=\mathbf{q}/\phi$, where ϕ is aquifer porosity.

The continuity equation for fluid mass is given by

$$\frac{\partial(\phi\rho_w)}{\partial t} = -\nabla\cdot(\rho_w \mathbf{q}) + Q_f \quad (2)$$

where t is time and Q_f describes fluid sources or sinks. In terms of pressure P and temperature T this expression becomes

$$\phi\rho_w\left(\beta\frac{\partial P}{\partial t} + \alpha\frac{\partial T}{\partial t}\right) = -\nabla\cdot(\rho_w \mathbf{q}) + Q_f \quad (3)$$

where β and α are aquifer (rock plus water) compressibility and expansivity, respectively.

Conservation of energy gives

$$C_a\frac{\partial T}{\partial t} = -\nabla\cdot(C_w T \mathbf{q}) + \nabla\cdot(\lambda_a \nabla T) + Q_h \quad (4)$$

where C_a is the aquifer volumetric heat capacity, C_w is the volumetric heat capacity of water, λ_a is the thermal conductivity of the aquifer (a second rank tensor), and Q_h describes heat sources or sinks. In most applications, λ_a is assumed to be a scalar. The parameter C_a is defined as $\phi C_w+(1-\phi)C_r$, where C_r is the rock matrix heat capacity. Both C_r and C_w

are assumed to be constants. The left-hand side of (4) gives the time rate of change of internal energy of the aquifer. The first term on the right-hand side represents heat flow by convection and the second term heat flow by conduction. Thermal hydrodynamic dispersion due to flow through a heterogeneous aquifer is sometimes accounted for by using a large effective value of λ_a.

In the aquifer, the rock and surrounding water are assumed to be in point local thermal equilibrium at all times. Energy changes due to fluid compressibility, acceleration, and viscous dissipation are neglected. The fluid flow and heat flow equations are coupled through the Darcy velocity $\mathbf{q}$, the temperature term in the fluid flow equation, and the pressure and temperature dependences of the fluid properties. For typical aquifer heat storage conditions, 0 to 150°C, ρ_w is strongly dependent on T and somewhat dependent on P, and μ is a strong function of T.

MATHEMATICAL MODELS

SFM-Graph

The simplest model, SFM-Graph, considers ρ_w, μ, and ϕ to be constant, so β and α in (3) are zero and the Darcy velocity $\mathbf{q}$ is independent of temperature.

$$\rho_w \nabla \cdot \mathbf{q} = Q_f \tag{5}$$

The fluid source term Q_f is constant over each of the seasonal periods of the storage cycle: positive for hot water injection during summer, negative for production during winter, and zero for the storage periods during spring and fall. Thus within each period the fluid flow field is steady. The influence of the supply well on the flow around the storage well is assumed to be negligible; so a radially symmetric aquifer centered at the storage well is considered. Further, the aquifer is assumed to be laterally infinite, of constant thickness, and homogeneous. For this simple geometry the Darcy velocity can be calculated directly as

$$\mathbf{q} = q_r = \frac{Q}{2\pi H r} \tag{6}$$

where H is the aquifer thickness, r is the distance from the storage well, and $Q = (Q_f / \rho_w) V_w$, is the volumetric flow rate through the well (Q_f is a mass flow rate per unit volume and V_w is the well volume). This expression for $\mathbf{q}$ can be substituted into (4) for heat flow in the aquifer.

The SFM-Graph model considers the aquifer to be bounded above and below by impermeable confining layers into which heat may flow, but not fluid. The heat flow in these regions is calculated from

$$C_c \frac{\partial T}{\partial t} = \nabla \cdot (\lambda_c \nabla T) \tag{7}$$

where the confining layer material properties are given by C_c for volumetric heat capacity and λ_c for thermal conductivity. The parameter C_c is defined analogously to C_a, combining rock and water properties. Both λ_a and λ_c are assumed to be scalars. The temperature and the conductive heat flow are continuous at the boundaries between the aquifer and confining layers. Above the caprock, which is of thickness D, lies a constant temperature boundary, held at T_0, the initial temperature of the system. The bedrock is assumed to be infinitely thick.

The governing equations (4) and (7) and the boundary conditions can be non-dimensionalized (Doughty et al., 1982), yielding the following parameters which govern heat flow for the energy storage system.

$$\mathrm{Pe} = \frac{QC_w}{2\pi\lambda_a H} \qquad \Lambda = \frac{2C_a^2 H^2}{C_c \lambda_c \tau} \tag{8}$$

$$\lambda_a / \lambda_c \qquad C_a / C_c \qquad d = D/H \tag{9}$$

The parameter τ in Λ is the average length of time spent in the aquifer by a fluid particle, and is defined by $\tau = 0.5(t_i + t_p) + t_s$, where t_i, t_s, and t_p are the durations of the injection, storage, and production periods, respectively.

A simple numerical model called the SFM (Steady Flow Model) (Doughty et al., 1982) has been used to calculate ϵ and T_p for injection-storage-production cycles for a wide range of the parameters Pe, Λ, λ_a / λ_c, and d. For typical aquifer storage problems, the parameter C_a / C_c is found not to vary appreciably, so a single value is used. The ratio λ_a / λ_c may become large if λ_a includes a contribution for hydrodynamic dispersion. Selected results are presented graphically in Figures 1, 2, and 3 for $\lambda_a / \lambda_c = 2$. Graphs for $\lambda_a / \lambda_c = 1$ and 10 have also been made (see (Doughty et al., 1982)). The ϵ value read off Figure 1 considers $d = \infty$. To account for a finite thickness caprock, ϵ should be multiplied by the appropriate correction factor from Figure 3.

The SFM uses a fully explicit finite-difference scheme to calculate conductive heat flow from (4) and (7). Convection is modelled by translating the temperature field in the aquifer radially in accordance with Darcy velocity q_r given by (6). Translation is away from the well during the injection period and toward the well during the production period. During the injection period, the source term Q_h is accounted for by assigning the injection temperature T_1 to the innermost column of mesh elements. During the production period, T_p is calculated by averaging the temperature of the elements in that column. The calculational mesh is specifically designed to minimize the numerical dispersion that typically arises in convection-conduction problems.

The major limitations of the SFM-Graph model are due to the assumptions of constant fluid density and a radially symmetric aquifer. The constant density assumption means that the model neglects buoyancy flow (natural convection) which causes the lighter warm water to rise above the heavier cool water. The radial geometry assumption implies that the presence of the supply well, which distorts the hot plume, is neglected.

SFM-ANGLE

The second model, SFM-ANGLE also uses the SFM to calculate the heat flow for a given steady fluid flow field, but in this case the fluid flow field is designed to represent buoyancy flow in an approximate way. A semianalytical model called ANGLE has been developed (Hellstrom et al., 1979) to study the motion of a two-fluid interface in a porous medium. ANGLE considers the motion of an initially vertical interface between two fluids with different densities and viscosities, due to buoyancy flow and forced-convection tilting. Buoyancy causes the fluid of lower density to flow towards the upper part of the aquifer; forced-convection tilting acts on the differences in viscosity, and hence in flow resistance, along different flow paths and thereby influences the tilting. ANGLE can predict the angle of tilt of a thermal front after periods of injection, storage, or production of hot or chilled water in an aquifer. For use in the SFM, the predicted tilt angle can be approximated by multiple levels of radial flow at different rates, as shown in Figure 4.

When running the SFM rather than simply using the graphical results, several additional features may be included: the caprock and bedrock may be heterogeneous, an anisotropic aquifer thermal conductivity may be used, and a time-varying temperature boundary above the caprock may be included.

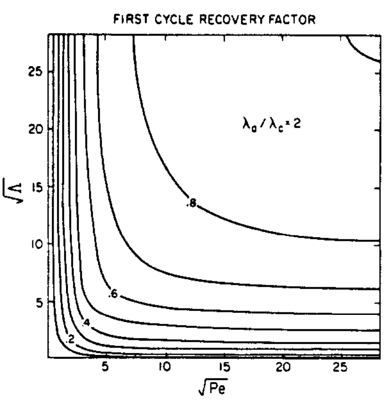

Fig. 1. Recovery factor ϵ as a function of $\sqrt{\mathrm{Pe}}$ and $\sqrt{\Lambda}$ for $\lambda_a / \lambda_c = 2$.

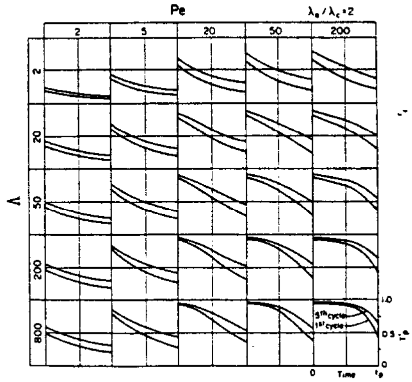

Fig. 2. First and fifth cycle dimensionless production temperature T_p' versus time for a range of Pe and Λ for $\lambda_a/\lambda_c = 2$. T_p' is defined as $T_p' = (T_p - T_0)/(T_1 - T_0)$.

PT

The third model, PT (Pressure-Temperature), developed at Lawrence Berkeley Laboratory (Bodvarsson, 1982), calculates fully coupled heat and fluid flows through water-saturated porous or fractured-porous media, using equations (1), (3), and (4). The following physical effects can be included in PT calculations: (1) heat convection and conduction; (2) regional groundwater flow; (3) multiple heat and/or mass sources and sinks; (4) constant pressure or temperature boundaries; (5) hydrologic or thermal barriers; (6) gravitational effects (buoyancy); (7) complex geometries due to heterogeneous materials; and (8) anisotropic permeability and thermal conductivity. The discretized equations used in PT are based upon the Integral-Finite-Difference method (Narasimhan and Witherspoon, 1976). This method treats one-, two-, or three-dimensional problems equivalently. An efficient sparse solver (Duff, 1977) is used to solve the linearized mass and energy matrix equations. The equations are solved implicitly to allow for large time steps. PT adjusts the time step automatically, so that the temperature or pressure changes in any node during one time step are within user-specified limits. Mass and energy balances are calculated for each node every time step. PT has been verified against a number of analytical solutions and field experiments (Tsang and Doughty, 1985).

THE BASIC PROBLEM

The International Energy Agency has created several test problems (Hadorn and Chuard, 1983) involving the seasonal storage of hot water, in order to evaluate mathematical models from a number of research institutions world-wide, and to find the best ones to incorporate as subroutines in an economic analysis program for thermal energy storage systems. The particular problem involving hot water seasonal storage in an aquifer is outlined below. It considers a storage cycle that lasts only 80 days instead of the usual one year, and does not include the above-ground components of the storage system such as the solar collectors supplying the heat and the space-heating system utilizing it.

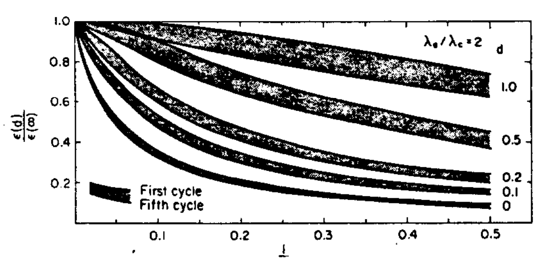

Fig. 3. Finite thickness caprock correction factor for the first and fifth cycles.

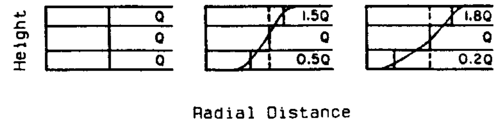

Fig. 4. A tilted thermal front and its approximation using three layers of radial flow.

A 10-m-thick aquifer is penetrated by a storage-supply well doublet, with a spacing of 50 m. The horizontal permeability of the aquifer is 10^{-11} m^2 (10 darcies). The aquifer is bounded above and a below by a caprock and a bedrock, each 20 m thick, with permeabilities 100 and 1000 times less than the aquifer value, respectively. In each layer, the vertical permeability is one tenth the horizontal permeability. The thermal properties of each layer are summarized in Table 1. Below the bedrock the temperature remains constant at 12 °C. Initially the water table is 1 m below the ground surface, in the caprock. The initial temperature of the system is 11 °C. The ground surface temperature varies sinusoidally, with an amplitude of 5 °C, a mean of 10 °C, a period of 1 day, and a maximum temperature at 12:00 hours. The 80 day storage cycle consists of 30 days of injection, 20 days of rest, and 30 days of production. During the injection period the well flow rates are 30 m^3/hr between 8:00 and 16:00 hours each day, and zero at other times. The injection temperature increases in hourly 10 degree steps from 30 to 60 °C between 8:00 and 12:00 hours, then similarly decreases between 12:00 and 16:00 hours. During the production period the well flow rates are constant at 10 m^3/hr and the produced water is reinjected directly into the supply well.

PRELIMINARY ANALYSIS

Before applying the three mathematical models to the basic problem, a number of preliminary calculations were made using simple analytical solutions to determine whether any features of the problem could be neglected.

Some general results from the theory of flow in leaky aquifers can be used to determine if the fluid flow into the confining layers is large enough so that convective heat transfer is significant there. A leaky aquifer is an aquifer that is bounded vertically by confining layers that are somewhat permeable. The pressure change felt in a confining layer due to pumping in an adjacent leaky aquifer is always smaller than or equal to that felt in the aquifer. To a good approximation, when a well penetrating through a leaky aquifer is pumped, the fluid flow is horizontal in the aquifer and vertical in the confining layer. Therefore, equation (1) indicates that the fluid flux in the aquifer will be at least 1,000 times greater than that in the caprock (permeability contrast 100, anisotropy factor 10) and at least 10,000 times greater than that in the bedrock (permeability contrast 1,000, anisotropy factor 10). Hence convective heat transfer into the confining layers will be very small compared to that in the aquifer. Additionally, the water-table level will not change appreciably from its initial value 1 m below the ground surface.

Because heat flow into the confining layers appears to be conduction-dominated, the vertical extent of temperature

Table 1. Thermal properties for the basic problem.

Layer	Property	Value
Bedrock	Heat capacity	2.84×10^6 J/m^{3}K
	Hor. thermal conductivity	1.5 W/mK
	Vert. thermal conductivity	1.5
Aquifer	Heat capacity	2.82×10^6
	Hor. thermal conductivity	3.0
	Vert. thermal conductivity	2.0
Caprock	Heat capacity	2.66×10^6
	Hor. thermal conductivity	1.3
	Vert. thermal conductivity	1.5
Unsaturated zone	Heat capacity	1.0×10^6
	Hor. thermal conductivity	2.0
	Vert. thermal conductivity	2.0

changes can be estimated by the conduction length $l = \sqrt{4\lambda t / C}$, where t is the duration of the storage cycle, 80 days. For the caprock $l = 3.9$ m and for the bedrock $l = 3.8$ m. The confining layer thickness, 20 m, is much larger than l, so the effect of the ground surface, unsaturated zone, and lower boundary at 12 °C can safely be neglected in modelling the storage system.

The daily injection temperature variation creates a series of pulses of hot water moving out into the aquifer. In the absence of conductive heat transfer, the radial width of each pulse would decrease as it moved away from the storage well, since Darcy velocity decreases according to (6). By the end of the injection period, 30 days, the outermost pulse would be only 0.1 m wide, while the innermost pulse would be 3.3 m wide. At the end of the injection period the conduction length, l, is about 3 m for the outer pulses, which have been in the aquifer for almost 30 days, but less than 1 m for the inner pulses, which have been in the aquifer just a few days. Thus for the outer pulses, conduction completely smooths out the injection temperature variation by the end of the injection period, while the inner pulses are still somewhat defined. By the end of the storage period, another 20 days, the conduction distance for the inner pulses is 2.7 m, so even near the storage well there is little evidence of the variable injection temperature. Therefore a constant average injection flow rate of 10 m^3/hr and an average temperature of 45 °C are expected to yield reasonable results.

In order to ascertain whether or not buoyancy flow is significant in the problem, we use an analytical expression (Hellstrom et al., 1979) for the initial rate of tilting of a vertical thermal front in an aquifer

$$\omega_0 = 29.1\sqrt{k'k}\,\frac{C_w(\rho_0-\rho_1)}{HC_a(\mu_0+\mu_1)} \quad \text{rad/s} \qquad (10)$$

where k and k' are the horizontal and vertical permeabilities in the aquifer and the subscripts 1 and 0 refer to the warm and cool sides of the thermal front. The formula assumes an infinite plane aquifer with a sharp vertical front in the absence of forced convection, while in fact, we have a radially symmetric aquifer bounded by an injection well with a diffuse, tilting front that is moving due to forced convection. A more elaborate semianalytical expression for the tilting rate, given in program ANGLE, does include these effects and it indicates that the formula given above provides a conservative upper limit for the tilting rate. For the present problem we use $T_1 = 45$ °C and $T_0 = 11$ °C and the material properties given previously, and get $\omega_0 = 0.31$ degrees per day, or a maximum tilting of 25 degrees during the entire 80 day cycle. This is a rather modest tilting, so use of the simpler two models, which do not account for buoyancy flow may provide useful results.

The influence of the supply well on the temperature and fluid flow fields around the storage well can be studied through the complex velocity potential formulation (Javandel et al., 1984), which calculates two-dimensional steady flow fields in laterally infinite aquifers with multiple sources and sinks. The streamline pattern for the injection period (assuming a constant flow rate of 10 m^3/hr) is shown in Figure 5. Also shown is the location of the thermal front at the end of the injection period.

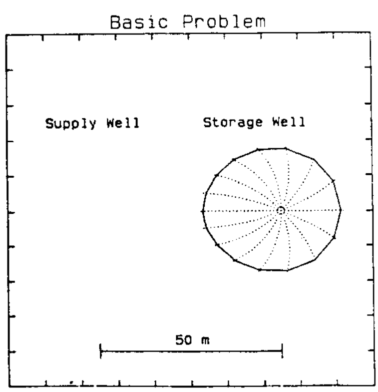

Fig. 5. Streamline pattern for flow from the storage well, and the thermal front at the end of the injection period.

The small departure from radial symmetry indicates that the simpler two models, which assume radial symmetry, may be used with little error.

In summary, the preliminary calculations indicate that the fluid flow into the confining layers can be neglected; the ground surface, unsaturated zone, and lower constant-temperature boundary are far enough away to be ignored; the thermal behavior of the caprock and bedrock are quite similar; the daily temperature and flow rate variation can be averaged out; the thermal front tilting is moderate; and the departure from radial symmetry is small.

RESULTS FOR THE BASIC PROBLEM

SFM-Graph

The preliminary calculations above indicate that the SFM-Graph model should be applicable to the basic problem. To use this model one first forms the dimensionless parameters Pe, Λ, λ_a/λ_c, and d defined in (8) and (9). The anisotropic thermal conductivities (Table 1) cannot be included explicitly, so the arithmetic average of horizontal and vertical values are used. The values for the caprock and bedrock are similarly averaged to represent the confining layer properties. We have the following parameters:

$Q = 10\ m^3/hr$	$\lambda_a = 2.5$ W/mK	Pe = 73
$\tau = 50$ d	$\lambda_c = 1.5$	$\Lambda = 88$
$H = 10$ m	$C_a = 2.82\times10^6\ J/m^3K$	$\lambda_a/\lambda_c = 1.67$
$D = 20$ m	$C_c = 2.75\times10^6$	$d = D/H$
	$C_w = 4.1\times10^6$	

Figure 1 gives $\epsilon \approx 0.73$-0.74. Figure 3 indicates that the finite caprock correction factor is 1, as expected from the small conduction length l. The Figure 2 curve for Pe=50, Λ=50 shows the general shape of the T_p curve.

SFM-ANGLE

The simplified tilting rate formula (10) predicts tilt angles of 9.8 and 15.5 degrees at the end of the injection and storage periods, respectively. The more accurate program ANGLE predicts corresponding tilt angles of 5.4 and 10.2 degrees. These angles are small enough so that a uniform radial fluid flow field can be used in the SFM. Only minor differences from the SFM-Graph model are expected, due to the following added features. The thermal properties for the caprock and bedrock can be considered individually and the anisotropic thermal conductivity in the aquifer can be incorporated in the SFM-ANGLE model in place of the averaged values used in the SFM-Graph model. The unequal duration of the injection and storage periods can also be considered, rather than combining them into the parameter τ. The calculational mesh is designed so that all the elements of a given row have the same volume, which implies decreasing radial dimension for increasing distance from the well. Thus the temperature field can be translated at uniform time intervals, to reflect the steady fluid flow, without introducing numerical dispersion. The mesh is shown in Figure 6. The calculated temperature field at the

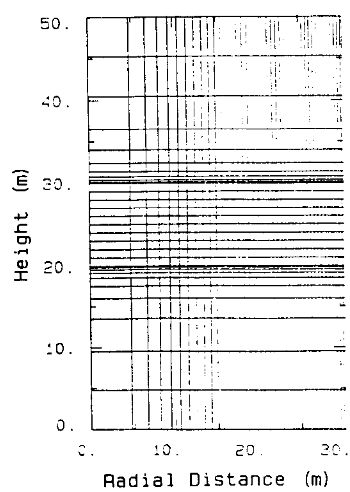

Fig. 6. Calculational mesh used for the SFM-ANGLE model.

end of the injection period is shown in Figure 7. The temperature fields in the caprock and bedrock are virtually identical, indicating that the small difference in these thermal properties is unimportant. The calculation yields ϵ=0.74 and a T_p curve as shown in Figure 8.

The SFM has also been run incorporating the daily injection temperature and flow rate variation. The resulting values of ϵ and T_p are identical to those found above. The temperature distributions in the aquifer at the end of the injection and storage periods conform to the descriptions given in the preliminary calculations. The computer time required for this case is much greater than that for the constant injection temperature case due to the smaller timesteps and finer mesh spacing needed to resolve the pulses.

PT

In theory, the PT model can be used with an arbitrarily detailed, three-dimensional calcuiational mesh to represent complicated field problems accurately. However, a mesh with many elements may require prohibitively large amounts of computer memory and time, so simplifications are often necessary. Two simplified approaches are taken for the basic problem, in order to reduce the problem to a manageable size.

Radially Symmetric Model. The first PT model uses a two-dimensional $r-z$ mesh, part of which is shown in Figure 9, to represent a cross-section of radially symmetric aquifer and confining layers centered at the storage well. The mesh extends farther radially than is shown, with increasing size elements. Recall that, in contrast to the SFM, the fluid density and viscosity vary with temperature and the caprock and bedrock are permeable. Thus this model may be used to examine buoyancy flow and fluid flow into the caprock and bedrock, to determine whether the results calculated with the SFM are valid. The calculated temperature field at the end of the injection period is shown in Figure 10. The temperature distribution shows that the thermal front tilting due to buoyancy flow is small. Careful measurement indicates a tilt angle of 5 degrees at the end of the injection period, in good agreement with the prediction of the program ANGLE (5.4 degrees). The extent of the heat flows into the caprock and bedrock are quite similar, although the permeability of the caprock is 10 times higher, indicating that convective heat flow is negligibly small. The extent of the heat flow is comparable to that predicted by the conduction length, l=3.8 m. Overall, the temperature distributions from the SFM and PT look quite similar, justifying the use of the SFM. The radial diffuseness of the thermal front is greater for PT than for the SFM, demonstrating the effect of numerical dispersion, which is inherent in PT. Increasing numerical dispersion is comparable to increasing λ_s, it increases radial heat loss. The T_p curve (Figure 8) and recovery factor (ϵ=0.70) calculated by PT are therefore lower than the values calculated by the SFM.

Areal Model. The second PT model uses a two-dimensional areal mesh, shown in Figure 11, that includes both the storage and supply wells. In the region around the storage well vertical heat loss to the confining layers is also calculated. The temperature distribution at the end of the injection period is shown in Figure 12. The deviation from radial symmetry is modest, as predicted by the streamline pattern shown in Figure 5. The areal mesh is quite coarse, so numerical dispersion

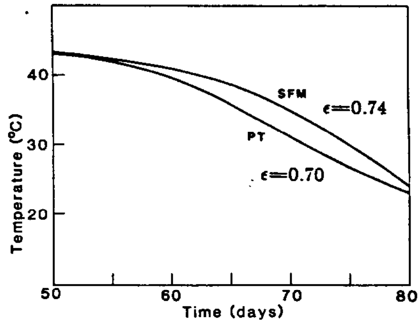

Fig. 8. The production temperature as a function of time for the two radial models.

greatly widens the thermal front, making this model inadequate for predicting recovery factor. However, the areal model can be used to indicate whether or not the radially symmetric models are valid by calculating two storage cycles, one with and the other without the supply well. The production temperatures and recovery factors for the two cycles are almost identical, indicating little net effect of the supply well. Although the hot plume is pulled toward the supply well during the injection period, it is pushed back toward the storage well during the production period.

VARIATIONS ON THE BASIC PROBLEM

Problems addressable by SFM-Graph

The basic problem can be studied by the the SFM-Graph model because buoyancy flow is sufficiently small. By examining the simple tilting rate formula (10), we see that buoyancy flow decreases with larger aquifer thickness, H, lower vertical or horizontal permeability, k' or k, smaller temperature difference (smaller $\rho_0-\rho_1$), or lower overall temperature levels (larger $\mu_0+\mu_1$). Thus such variations from the basic problem result in problems that even the simplest model can address.

If the caprock thickness is decreased, the SFM-Graph model can still be used provided that the unsaturated zone thermal properties are similar to those of the caprock and the daily ground surface temperature variation can be neglected. The conduction penetration depth for a sinusoidal surface temperature with period τ_0 is given by $l'=\sqrt{\lambda\tau_0/\pi C}$. For the basic problem l'=0.16 m.

Problems requiring SFM-ANGLE

For any change in (10) that increases the tilting rate substantially (smaller H, larger k or k', a larger temperature difference, or higher overall temperature levels), the SFM-ANGLE model becomes necessary. Buoyancy flow influences the recovery factor by distorting the hot plume and increasing its surface to volume ratio. Since heat loss is proportional to

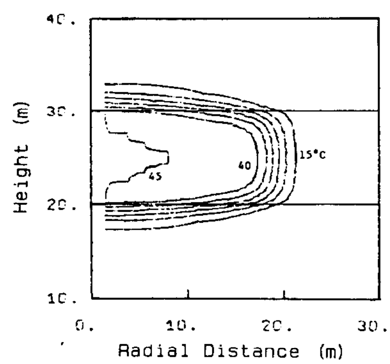

Fig. 7. The temperature distribution in the aquifer at the end of the injection period, calculated by the SFM-ANGLE model.

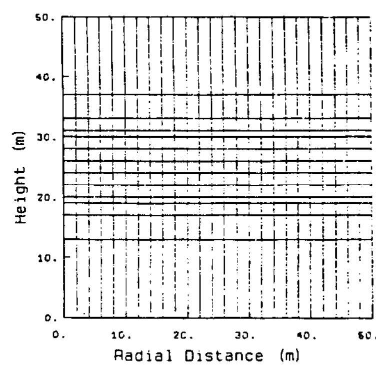

Fig. 9. The calculational mesh used for the PT radial model.

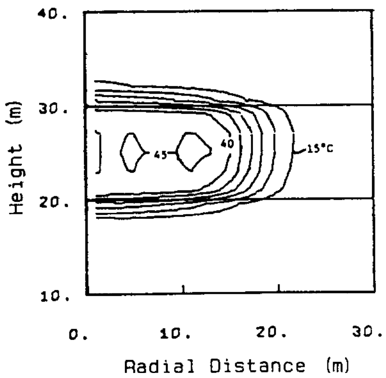

Fig. 10. The temperature distribution in the aquifer at the end of the injection period, calculated by the PT radial model.

plume surface area, and heat stored is proportional to plume volume, a larger surface to volume ratio results in a lower value of ϵ. Still lower values of ϵ would result if buoyancy flow were large enough to cause cool water to be produced from the lower part of the well during the production period. For example, if k and k' increase by a factor of five (with $k'=0.1k$) and if the injection temperature increases from 45 °C to 75 °C, (10) gives a tilting rate of 4.5 degrees per day. The program ANGLE predicts large tilt angles of 57 and 70 degrees at the end of the injection and storage periods for this case. Using 10 layers of radial flow to approximate these angles as shown in Figure 4, the SFM then calculates ϵ=0.59. The values of the dimensionless parameters remain unchanged from the basic problem, but the SFM-Graph model ($\epsilon\approx$0.73–0.74) no longer provides a very good representation of the system.

If the caprock thickness is decreased and the unsaturated zone thermal properties are sufficiently different from those of the caprock so that it needs to be treated as a separate layer, then the SFM-ANGLE model can be used so long as fluid flow into the caprock remains small. Similarly, ground surface temperature variations can be included.

If the aquifer is non-uniform, with horizontal layers of different permeabilities, the SFM-ANGLE model can be used with the flow into each layer specified as proportional to the permeability-thickness product of that layer.

Problems requiring PT

The streamline plot for a proposed problem can be used to indicate how distorted a plume is. If the separation between the storage and supply wells is decreased or the well flow rates increased, the supply well will distort the hot plume more than in the basic problem, as shown in Figure 13. The distortion will increase the plume surface to volume ratio and decrease recovery factor, so use of PT with a non-radial mesh is necessary to treat the problem.

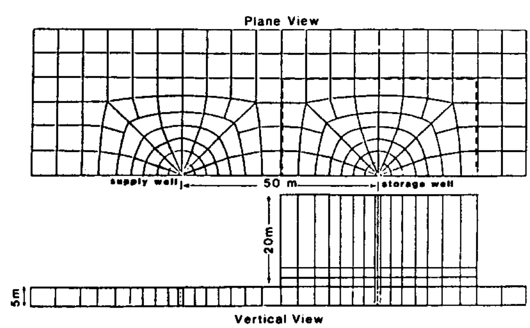

Fig. 11. The calculational mesh used for the PT areal model. Due to symmetry, the mesh discretizes only one quarter of the problem: the upper half of the aquifer/confining layer system, and one half of the area between the storage and supply wells.

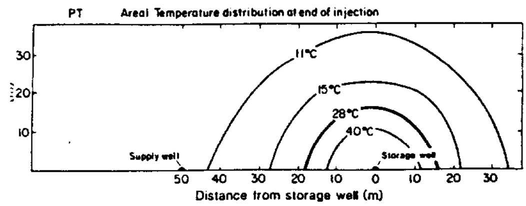

Fig. 12. The temperature distribution in the aquifer at the end of the injection period, calculated by the PT areal model.

If the confining layer vertical permeability is within a factor of 10 or so of the aquifer horizontal permeability, a substantial amount of fluid will flow into the confining layer. The aquifer flow will have a vertical component, so PT must be used instead of the the SFM-ANGLE model. A similar situation exists if the storage well only partially penetrates the aquifer thickness, or if permeability heterogeneities exist in the aquifer.

Problems requiring other models

A number of physical effects which are not included in the PT model may be important in certain cases. If the storage system involves an unsaturated zone, then a model that calculates the interactions of liquid water, water vapor, air, and energy becomes necessary. If the injection temperature is increased above 100 °C steam-liquid water phase transitions must be considered. Chemical reactions may occur in the aquifer when hot water is injected, resulting in changes in fluid density or aquifer permeability, which in turn affect the fluid flow field. In all these cases more sophisticated models than PT may be necessary. Additionally, in some seasonal energy storage systems, it may be necessary to model processes taking place in the aquifer in conjunction with the operation of the rest of the storage system. This requires a total system model, in which PT (or SFM-Graph or SFM-ANGLE) is just a part.

CONCLUSIONS

We have illustrated the use of three mathematical models of widely varying complexity to study a basic heat and fluid flow problem, and discussed the limits of the range of applicability of each model. For the basic problem, the three models yield similar results, so the use of the simplest model is preferred. For a number of variations on the basic problem the simpler models are found to be inadequate to address the problem, and the more complicated models must be used. While the simpler models suffer from their approximate treatment of buoyancy flow and neglect of non-radial effects, the more complicated numerical model suffers from numerical dispersion. Minimization of numerical dispersion may require considerable additional computational effort. The trade-off between the fully numerical and the simpler models must be considered carefully. In general, we may make the following two observations. Firstly, the choice of the simplest appropriate model is very problem-specific and requires not only experience with modelling methods, but also an understanding of the physics of the problem. Secondly, even when a simple model can be used, a sophisticated model is required to validate the simpler model and to establish its range of applicability.

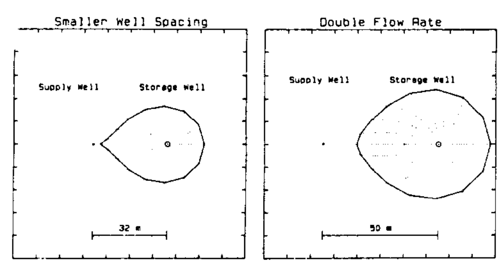

Fig. 13. Streamline pattern and thermal front for two variations on the basic problem.

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