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Neutrino lensing and modification of Newtonian gravity at large distances

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Abstract

The nature of the modification to neutrino lensing from galaxies, as caused by possible modifications to Newtonian gravity at large distances, is studied.

1. Introduction

The validity of Newtonian gravity laws have been experimentally verified to distances as short as about a tenth of a millimeter [1]. On the other hand there has been no direct experimental verification beyond stellar scale distances. Possible modification of Newtonian gravity both at small and large distances has become the subject of many new theories [2]. These are theories in which the universe is no longer four-dimensional, the extra dimensions being invoked to solve the so called 'hierarchy problem' between scales involved in the Standard Model of particle interactions and the four-dimensional 'Planck scale' [3]. Kogan et al. [4] proposed in this connection a three-brane model of the universe which removes some of the drawbacks of the earlier brane model of Randall and Sundrum [3], which had only two branes. This model has the distinct feature, unlike the two-brane model of Randall and Sundrum, of having possible modifications to Newtonian gravity at large distances into a Yukawa type of interaction, rather than the $1/r$ gravitational potential. The theory of course gives no indication of the range of such a Yukawa type potential but of course the success of Einsteinian general relativity demands that the range of such a Yukawa interaction be much larger than the size of the solar system. The Yukawa type of gravitational interaction also results from other approaches, like one involving higher terms in the gravitational interaction [5].

Theoretically, in the context of brane world scenarios in higher dimensions, there are problems associated with massive gravitons which have not yet been resolved. There are issues arising out of the fact that the number of degrees of freedom associated with gravitons in massive graviton theories changes from

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five in a linearized theory to six when one goes beyond the linearized version. Nonetheless, Damour, Kogan and Papazoglou [6] have demonstrated the possibility of solutions of a massive gravity theory which effectively corresponds to a Yukawa type of gravity at large distances from the source. Although it is by no means true that a Yukawa type of interaction is the only type of modification of Newtonian gravity at large distances, we will for analytic simplicity consider only a Yukawa type of modification for gravitational interactions at large distances. Remarks on the nature of changes of our results for other possibilities will be mentioned at the end.

Phenomenologically the modification to Newtonian gravity over large distances can be studied through the lensing of light and neutrinos, where we have their origins as far away from us. Lensing of optical images has of course been studied widely in the context of standard Einsteinian gravity. Some time back McKellar, Mahajan and two of us (S.R.C., G.C.J.) [7], studied the possibility of using detailed data from the lensing of distant stars as a tool for studying possible modifications of Newtonian gravity at cosmological scales. Lately, neutrino lensing and their possible observation has attracted much attention. In two recent papers Escribano, Frere, Monderen and Van Elewyck [8] have performed detailed analysis of magnitudes of neutrino lensing that can be expected from normal gravitational interactions with sources such as stars and galaxies. In line with our earlier study we devote this Letter to explicitly recording the nature of the modification of such lensing predictions of Newtonian gravity for a sphere of radius R with a finite range Yukawa type interaction with a range in the cosmological scale.

The energy–momentum tensor of an ideal fluid is given by

$$T_{\mu\nu} = p g_{\mu\nu} + (p+\rho) U_\mu U_\nu, \tag{1}$$

where U_μ is the four velocity. For spherically symmetric systems that we consider, ρ, p and U are functions of r only. Use of such an energy–momentum tensor in the standard Einstein equations for a spherical mass leads to a Newtonian gravity at large distances from the sphere. We use an ad hoc prescription to simulate an effective Yukawa type of gravity rather than Newtonian by adding to $T_{\mu\nu}$ above an extra term such that the new energy–momentum reads

$$T'_{\mu\nu} = T_{\mu\nu} + \frac{\Lambda(r)}{r^2} g_{\mu\nu} \tag{2}$$

and suitably adjust the value of $\Lambda(r)$ so that at large distances we end up with Yukawa type of gravity. Writing the four-dimensional line element in the standard form

$$ds^2 = B(r)\,dt^2 - A(r)\,dr^2 - r^2\,d\theta^2 - r^2 \sin^2\theta\,d\phi^2 \tag{3}$$

we obtain the following equations for the components of the curvature tensor

$$R_{rr} = -4\pi G(\rho - p)A(r) + 8\pi G \frac{\Lambda(r)}{r^2} A(r), \tag{4}$$

$$R_{\theta\theta} = -4\pi G(\rho - p) r^2 + 8\pi G \Lambda(r), \tag{5}$$

$$R_{tt} = -4\pi G(\rho + 3p) B(r) - 8\pi G \frac{\Lambda(r)}{r^2} B(r). \tag{6}$$

In the matter free region, $\rho = p = 0$, we continue to have

$$\frac{R_{rr}}{A(r)} + \frac{R_{tt}}{B(r)} = 0 \tag{7}$$

so that in this region we still have $A(r) = 1/B(r)$.

In the presence of matter the equation for $A(r)$, by standard manipulation of Eqs. (4)–(6) to eliminate pressure, becomes,

$$\frac{d}{dr}\left(\frac{r}{A(r)}\right) = 1 - 8\pi G\rho r^2 + 8\pi G \Lambda(r). \tag{8}$$

We can now assume a form for $\Lambda(r)$:

$$\Lambda(r) = \Lambda e^{-\lambda r} \tag{9}$$

with λ and Λ being constants. Eq. (8) is then easily integrated yielding

$$\frac{1}{A(r)} = 1 - \frac{2GM(r)}{r} - \frac{8\pi G\Lambda}{\lambda r}\left(e^{-\lambda r} - 1\right), \tag{10}$$

where the constant of integration has been adjusted to make $A(r)$ finite at $r = 0$ and $M(r)$ is the mass contained between $r = 0$ and r. Identifying

$$8\pi\Lambda = 2M\lambda \tag{11}$$

we get for values of r outside R

$$\frac{1}{A(r)} = 1 - \frac{2GM}{r} e^{-\lambda r} \tag{12}$$

which is the Yukawa form of gravitation that we desired to reproduce. The form of $A(r)$ given in Eq. (10) is of course valid for all r and for values of r much smaller than R, it can be approximated by

$$\frac{1}{A(r)} = 1 - \frac{2GM(r)}{r} + 2G\lambda. \quad (13)$$

The value of the function $B(r)$ for $r > R$ is of course the inverse of $A(r)$. To obtain values of the function $B(r)$ inside the spherical core, we assume that the matter distribution in the core is Gaussian, in which case we can make the approximation $\rho \ll p$ and the function $B(r)$ there satisfies

$$\frac{B'}{B} = \frac{2M(r)}{r^2}. \quad (14)$$

As we have assumed a Gaussian matter distribution the density inside the core is of the form

$$\rho(r) = \rho_0 e^{-r^2/r_0^2}. \quad (15)$$

The mass $M(r)$ for such a density turns out to be

$$M(r) = \frac{M}{R} r e^{(R^2-r^2)/r_0^2} \frac{f(r_0/r)}{f(r_0/R)},$$
$$f(x) = xe^{x^2} \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) - 1. \quad (16)$$

For $r < R$ the function $B(r)$ is then

$$B(r) = 1 - \frac{2M}{R} e^{(R^2-r^2)/r_0^2} \frac{f(r_0/r)}{f(r_0/R)} + O(M^2). \quad (17)$$

Given the functions $A(r)$ and $B(r)$, the deflection of a beam of neutrinos (assumed massless) initially moving along the direction $\phi = 0$ is given by

$$\phi(r) = \int_{\infty}^{r} \frac{A^{1/2}(r)\,dr}{r^2} \left(\frac{1}{b^2 B(r)} - \frac{1}{b^2} - \frac{1}{r^2} \right), \quad (18)$$

where b in the equation above is the impact parameter. The total deviation of the beam is then given by

$$\Delta\phi = 2\phi(r = r_{\min}) - \pi. \quad (19)$$

For neutrino trajectories which lie totally outside the core, the above equation simplifies since $A(r) = 1/B(r)$ there. We get

$$y = \frac{1}{r} - M \frac{e^{-\lambda/r}}{r^2}, \quad (20)$$

$$\phi(y) = \int_0^y dy\, \frac{1 + (2y - \lambda) M e^{-\lambda/y}}{\sqrt{1/b^2 - y^2}} + O(M^2). \quad (21)$$

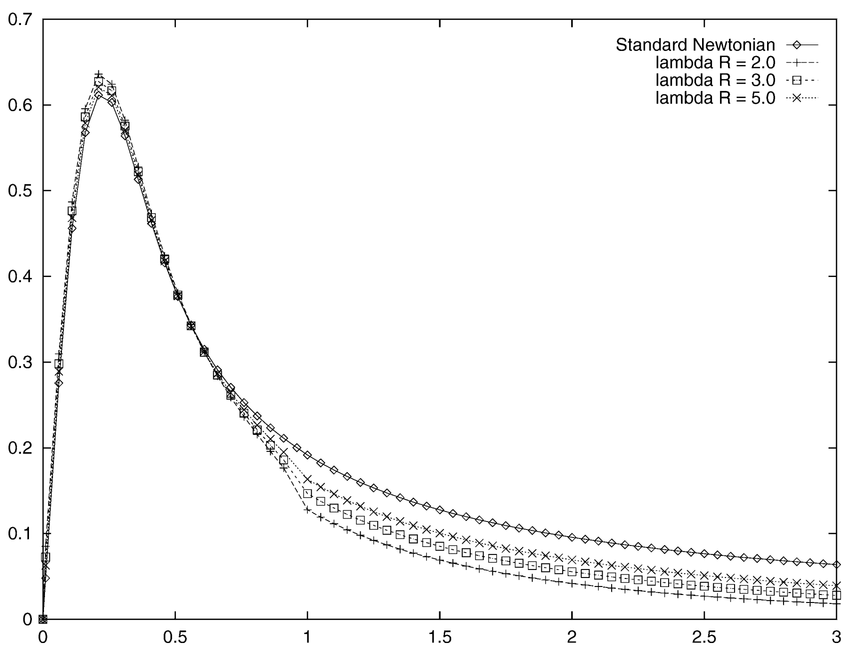

Fig. 1. Graphs showing the deflection $\Delta\phi$ in units of 10^{-5} radians for various choices of λR, where $M = M_{\text{Galaxy}} = 10^{12} M_{\odot}$, $R = R_{\text{Galaxy}} = 100$ kpc and $r_0 = 0.2\, R_{\text{Galaxy}}$. Also shown is the graph for standard Newtonian gravity.

The maximum value of y is $1/b$ and the integral above can be easily numerically integrated to give the value of $\Delta\phi$ by Eq. (19). For trajectories that pass through the region $r < R$, one has to numerically integrate using the value of $A(r)$ from Eq. (13) above and $B(r)$ from Eq. (17).

We display our results in Fig. 1 for a typical galactic mass and radius. As a variant of the Yukawa form of the interaction, we have also performed calculations using Newtonian gravity up to a certain cutoff distance followed abruptly by a gravity free region beyond that. The results qualitatively are similar but to the one above, but due to the discontinuous nature of the potential, the graphs show some ugly discontinuities which of course have no physical significance.

As expected, for neutrino trajectories that come deep through the mass of the galaxy, the Yukawa nature of the gravitational force does not cause any change in the deflection of the beam. However, there is considerable change in the deflection of the beam, as expected, if the impact parameter is substantially greater than the core radius. Of course, in those regions the deflection of the beam also decreases, making observations there more difficult than trajectories through the core. It seems, therefore, that it would require some fortuitous circumstances of a suitable set of sources, from where neutrino beams would be lensed through galactic masses, for observations to possibly detect changes in the law of gravitation for neutrinos. Most certainly, the optical analog of such observations present a more promising scenario for future experiments.

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References

[1] C.D. Hoyle, et al., Phys. Rev. Lett. 86 (2001) 1418.
[2] N. Arkani-Hamed, et al., Phys. Lett. B 429 (1998) 263; N. Arkani-Hamed, et al., Phys. Rev. D 59 (1999) 086004; I. Antoniadis, et al., Phys. Rev. B 436 (1998) 257.
[3] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370.
[4] I. Kogan, et al., Nucl. Phys. B 595 (2001) 225.
[5] K.S. Steele, Gen. Relativ. Gravit. 9 (1978) 353.
[6] T. Damour, et al., Phys. Rev. D 67 (2002) 064009.
[7] S.R. Choudhury, G.C. Joshi, S. Mahajan, B. Mckellar, hep-ph/0204161.
[8] R. Escribano, J.M. Frere, D. Monderen, V. Van Elewyck, hep-ph/9910510; R. Escribano, J.M. Frere, D. Monderen, V. Van Elewyck, Phys. Lett. B 512 (2001) 8.