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Optimization of the Prestress Value in Binding of Reinforced Concrete Structures

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Abstract

The value of prestress when manufacturing reinforced concrete structures is given in an arbitrary way within limits of maximum and minimum values that are recommended as normal values. While making calculations, it is necessary to ensure the implementation of a number of requirements that guarantee the operability of structures. Increasing prestress level leads to the upgrade of the cracking resistance and rigidity performance, and at the same time gives rise to the danger of destruction resulting from the concrete compression and destruction of bonds in a stress transfer zone from reinforcement to concrete at the manufacturing stage, as well as reduces the strength of a combined action of the bending moment and the transverse force near supporting structures. These problems become relevant during the production of hollow-core slabs in the process of formless molding as the installation of additional reinforcement in these slabs for increasing the bond strength near the supporting structures is impossible because of technological molding peculiarities. The need to optimize the value of prestress is conditioned by the occurrence of damages that appear at the manufacturing stage of intermediate hollow-core slabs of formless molding. The algorithm of optimization is introduced taking into account the ensuring of requirements of bond strength of reinforcement and concrete in the stress transfer zone, compression strength and cracking resistance.

Keywords: reinforced concrete; prestress; optimization; bond strength; compression strength; cracking resistance.

In the process of manufacturing of hollow-core reinforcement slabs, ceilings with formless molding at the stage of compression, some damages as longitudinal cracks in concrete cover and reinforcement slipping (displacement) in the zone of transfer of its concrete pretensioning occur.

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When assigning the value of reinforcement prestress in projects, the Russian rules and regulations [1,2] require fulfillment of the following conditions:

1. Ensuring the cracking resistance by the moment of formation of cracks and their width;
2. Ensuring the bond strength of reinforcement and concrete in the zone of transfer of presstresses;
3. Ensuring the restrictions of concrete compression stresses at the moment of compression resulted from prestress;
4. Ensuring the deflection value and angular deflection rigidity;
5. Ensuring the limits of value change of reinforcement prestress.

The formation of damages indicated above is connected with the failure to comply with the requirement as defined in claim 2.

To fulfill all these requirements it is necessary to set the value of prestress initially, and then verify that all the requirements are fulfilled. To do this, we need to repeat calculations many times. Usually in the process of design the greatest value of reinforcement prestress is set. In this case, the requirements 1,4,5 are fulfilled but the requirements 2 and 3 may be failed, which leads to the damages that cause nonoperability of the structure. During the process of formless molding of slabs, the requirement 3 can be satisfied by setting the upper prestressed reinforcement, and to meet the requirement 2 the extra reinforcement cannot be set. The requirement 2 is also connected with ensuring the strength of the oblique section in the zone of the joint action of bending moment and transverse force, where the setting of extra reinforcement is also impossible for the formless molding technology.

The rules on determining the optimal value of prestress were not discovered by the author in foreign scientific literature [3-6]. In foreign literature it is also recommended to assign the value of the prestress reinforcement arbitrarily, and then perform calculations to comply with various restrictions.

The article proposes the problem statement of determining the optimal values of reinforcement prestress and provides a way to solve it. General principles for solving such problems are introduced in the scientific literature [7-10]. The main issue in the problem of determining the optimal prestress of reinforcement is the formation of an objective function. In order to form the objective function it is proposed to consider the combination of different values of prestress:

\[
\begin{align*}
    z_1 &= \sigma_{sp.1} - \sigma_{sp.2} \\
    z_2 &= \sigma_{sp.1} - \sigma_{sp.3} \\
    z_3 &= \sigma_{sp.1} - \sigma_{sp.4} \\
    z_4 &= \sigma_{sp.2} - \sigma_{sp.3}
\end{align*}
\]  

(1)

where \( \sigma_{sp.j} \) - functions determining the values of reinforcement prestress that meet the requirements 1…5 (cracking resistance, bond strength, restrictions of concrete compression stresses, …) in the form of dependencies on the rules and regulations. Then the system of objective functions is obtained where the optimal value of parameter is determined in accordance with the following system of equations:

\[
\sum_{j=1}^{5} \frac{dz_j}{d\sigma_{sp}} = 0
\]  

(2)

Thus, in accordance with the importance of individual requirements it is assumed that some of the parameters included in the formulas of norms depend on the value of unknown argument \( \sigma_{sp} \), the optimal value of which is sought. Then, when differentiating the objective functions, the rule for differentiation of the product of two functions of one argument and the rule for differentiation of implicit functions are applied.

Below is an algorithm for solving a simplified problem. The simplification is in that three most important requirements are selected (points 1,2,3). The priority of these requirements is conditional on the fact that the failure
to comply with the requirement 1 does not provide a durability of structures, and non-compliance with the requirements 2 and 3 leads to the destruction of individual zones of the structure at the stage of its manufacture.

Since the bond strength of concrete and reinforcement depends on the strength of the concrete and the value of reinforcement prestress, let us assume the existence of dependence:

\[ R_b = R_b(\xi), \]  
(3)

where \( \xi = \sigma_{sp}/R_{sn} \) - relative value of reinforcement prestress, \( R_{sn} \) - characteristic strength of reinforcement.

The optimization in this problem is derived according to the same allowance (coefficient) of cracking resistance and bond strength:

\[ K_{crc}(\xi) = K_{bond}(\xi); \]
\[ z(\xi) = K_{crc}(\xi) - K_{bond}(\xi) = 0 \]  
(4)

According to the condition of cracking resistance [1,2]:

\[ k_{tp} = \frac{M_{crc}}{M_n} = a + \xi \cdot b, \]  
(5)

where \( a = \frac{R_{bt.n} W}{M_n} \); \( b = \frac{R_{sn} A_{sp}}{M_n} \) and \( \xi = \frac{\sigma_{sp,1}}{R_{sn}} \).

The condition of bond strength in the limiting state is written on the basis of equilibrium of the reinforcement section in the transmission zone of prestresses of [1] long on the assumption of linear distribution of stresses in this zone [11] and calculated resistance \( R_{bond} \) [2]:

\[ T_{bond} = \sigma_{sp,2} A_{sp} \leq n u_s \left[ \left( \frac{\sigma_{sp}}{R_{bp}} + \lambda p \right) d_s \right] R_{bond}, \]  
(6)

where \( u_s \) - armature perimeter, \( n \) - number of bars; \( d_s \) - diameter; \( A_{sp} \) - total area.

Then from the equation (6) the coefficient:

\[ K_{bond} = c + \frac{k}{\xi}, \]

where \( c = \frac{n u_s d_s \omega p R_{bond}}{R_{bp} A_{sp}} \); \( k = \frac{n u_s d_s \lambda p R_{bond}}{R_{sn} A_{sp}} \)

The existence of dependencies is taken into account

\[ R_b = \eta_1 R_{bt,n} = \eta_2 R_{bp} = \eta_3 R_{bond} \]
Taking into account that \( R_b = R_b(\xi) \), we will obtain the expression for objective function:

\[
z(\xi) = a_1 R_b(\xi) + \xi b - c_1 \frac{k_1 R_b(\xi)}{\xi},
\]

where \( a_1 = \frac{W}{\eta_1 M_n} \), \( c_1 = \frac{nu_s d_s \omega_p \eta_2}{A_{sp} \eta_3} \), \( k_1 = \frac{nu_s d_s \lambda_p}{A_{sp} \eta_3 R_{sn}} \).

The differentiation of objective function (7) is then derived:

\[
\frac{dz}{d\xi} = a_1 \frac{dR_b(\xi)}{d\xi} + b - k_1 R_b(\xi)(-\frac{1}{\xi^2}) + k_1 \frac{dR_b(\xi)}{d\xi} \frac{1}{\xi},
\]

The expression \( \frac{dR_b(\xi)}{d\xi} \) is determined considering the restriction according to the point 3 [2]:

\[
\sigma_{sp} = \frac{A_{sp} \sigma_{sp,3}}{A_b} + \frac{A_{sp} \sigma_{sp,3} h_{sp}}{I_b} \leq \frac{0.5 R_b(\xi)}{\eta_2}
\]

At the following reductions \( r \approx 0.24 h_s \), \( I_b \approx 0.06 A_b h_s^2 \) and notation \( \mu_s = A_{sp}/A_b \) the dependence is obtained:

\[
\frac{dR_b(\xi)}{d\xi} = \xi m
\]

where \( m = 13 \cdot \eta_2 R_{sn} \mu_s \).

Taking into account (10), we will obtain the equation:

\[
a_1 m \xi^2 + \xi (k_1 m + b) + mk_1 = 0
\]

The solution of the equation is determined by the optimal value \( \xi = \xi_{opt} \), taking into account the requirements 1…3.

The issue of ensuring the requirements on the restriction of the width of cracks and rigidity, including the restrictions on strength of the oblique sections in the simplified statement can be solved by the introduction of the coefficient of cracking resistance allowance and bond strength of reinforcement and concrete. The solution in the general case according to the system (2) is complicated, but still is possible.

In simplified form of the problem solution the coefficients \( \eta_1, \eta_2 \) and \( \eta_3 \) from [1,2] have the following sense:

- coefficient \( \eta_1 \) is determined on the basis of known ratio between concrete compression and expansion strengths with account of static variability;
– coefficient $\eta_2$ is determined as the result of ratio of concrete calculated resistance of concrete to compression and concrete compression strength;

– coefficient $\eta_3$ expresses the ratio of concrete strength and bond strength.

After the calculation of optimal value of reinforcement prestress in the simplified form, the width of cracks and curvature is suggested to determine from the stress increment $\Delta \sigma_S$ in the reinforcement as follows.

The dependence of 'stress in the reinforcement - bending moment' is considered. It is assumed that between the points defining the moment of crack formation and limit torque the dependence is linear and its incline is determined by the expression:

$$
tg \alpha = \frac{R_S - 300}{M_{ult} - M_{cr,0}},
$$

where 300 kgf/cm$^2$ – value of stress in reinforcement at the moment of crack formation for the structure without prestress. Consequently, $M_{cr,0}$ - cracking moment.

Then, the stress increment involving the prestress equals:

$$
\Delta \sigma_S = (M_{ult} - M)tg \alpha,
$$

where $M$ – the moment under the action of which the values of cracking width and curvature are determined.

The assumption of linear dependence gives some leeway, since the value $\Delta \sigma_S$ is increased due to a small line convex connecting the points mentioned above.

The consideration of the duration of stresses during the calculation of the cracking width and curvature shall be determined finally by the method of recommendations [2].

**Conclusion**

1. Statement of the problem of determining the optimal value of the prestress of reinforcement of concrete structures is a new problem and has a practical use because it allows reducing the amount of calculations and providing a certain and equivalent leeway according to set requirements and restrictions.

2. The calculation algorithm is proposed, on the basis of which it is possible to obtain general and specific solutions under various restrictions connected with the value assignment of reinforcement prestress.

**References**

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